On escort distributions, q-gaussians and Fisher information

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Outline

Escort distribution

- Escort distributions originates from multifractals.
- Actually the singular spectrum $f(\alpha)$ and the Rényi dimension are linked by a Legendre transform

A. Chhabra and R.V. Jensen, «Direct determination of the f(alpha) singularity spectrum,» Physical Review Letters, vol. 62, Mar. 1989, or D. Harte, Multifractals: Theory and Applications, Chapman & Hall/CRC, 2001.

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subject to $\alpha = \sum P_i \log p_i$ and normalization, that leads to

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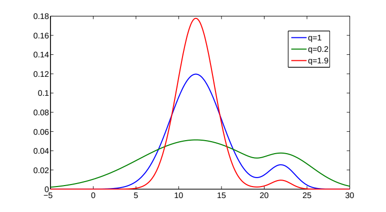

$$P_i = \frac{p_i^q}{\sum p_i^q}$$

It can be shown that

$$D(U||P) \geq D(U||p) \text{ if } q > 1$$

$$D(U||P) \leq D(U||p) \text{ if } q < 1$$

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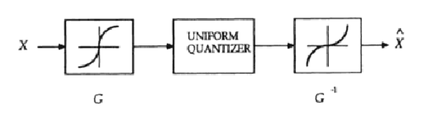
Applications - Quantization

In a nonuniform companding quantization



$$\text{Distorsion} \qquad D = \sum_i \int_{x_i}^{x_i+1} (x-y_i)^p f_X(x) dx$$

in the HR limit, the density of points that minimizes D is

$$\lambda(x) = \frac{f_X(x)^q}{\int f_X(x)^q dx} \propto G'(x) \quad \text{with} \quad q = \frac{1}{p+1}.$$

- Possible trade-off between entropic quantization $\lambda(x) = cte$ and density distorsion (work in progress).

Allen Gersho and Robert M. Gray, "Vector Quantization and Signal Compression", 1992

And also source coding...

- 1948, Shannon's source coding theorem
- en 1965, L.L. Campbell the first *operational* characterization of Rényi entropy...

Escort-distribution : $P_i = p_i^q / \sum_i p_i^q$

	length	bound	opt. length
Shannon	$\bar{L} = \sum_i p_i l_i$	$H_1(p)$	$l_i = -\log_D(p_i)$
Campbell	$C_\beta = \frac{1}{\beta} \log_D \sum_{i=1}^N p_i D^{\beta l_i}$	$H_{1/(\beta+1)}(p)$	$l_i = -\log_D(P_i)$
Gen. mean	$M_q = \sum_{i=1}^N P_i l_i$	$H_1(P)$	$l_i = -\log_D(P_i)$

- Generalized mean: Obvious way to obtain the Campbell's codes using a standard coder!

J.-F. Bercher, "Source Coding with Escort Distributions and Rényi Entropy Bounds," Physics Letters A, vol. 373, pp. 3235-3238, august 2009.

Rényi-Tsallis entropy maximization

Proposition

The probability distribution that maximizes the Tsallis entropy

$$H_q[f] = \frac{1}{q-1}\left(1 - \int f(x)^q dx\right)$$

subject to the q-variance constraint $\sigma_q^2 = \frac{\int x^2 f(x)^q dx}{\int f(x)^q dx}$ is the q-gaussian distribution defined by

$$p_q(x) = \frac{1}{Z_q(\beta)}\left(1-(1-q)\beta x^2\right)_+^{\frac{1}{1-q}}, \quad q \neq 1$$

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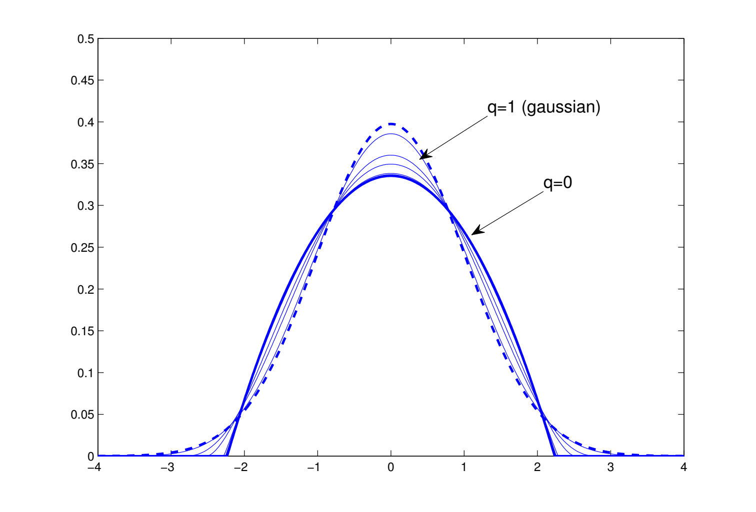
Proof: Compute the Rényi information divergence $D_q(p||p_q)$ with p admissible, use the q-variance constraint and check that

$$D_q(p||p_q) = H_q(p_q) - H_q(p) \geq 0.$$

q-gaussians

Example of q-gaussians for $q \leq 1$



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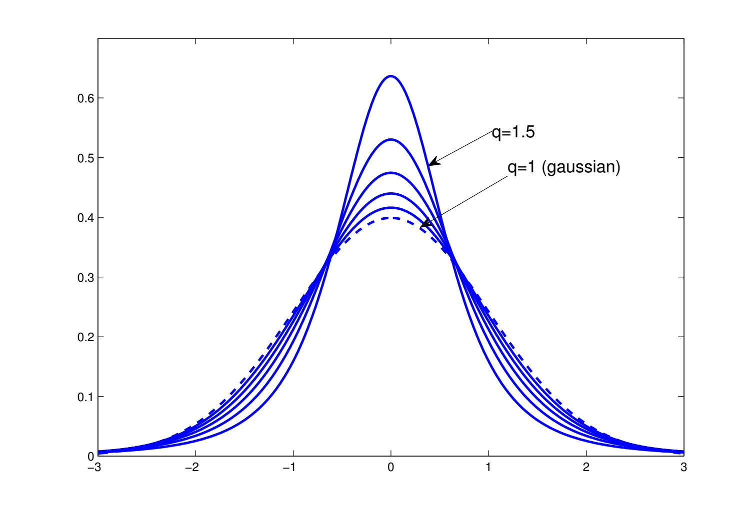
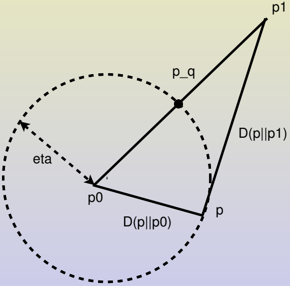


The *escort* path

- The problem

$$\text{Min}_p D(p||p_1) \text{ s.t. } D(p||p_0) = \eta$$



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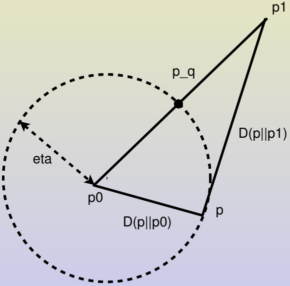
[Some similarities with Carlos' setting]



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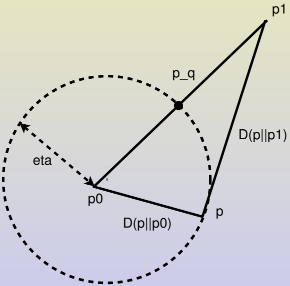
$$\mathrm{Min}_p D(p||p_1) \text{ s.t. } D(p||p_0) = \eta$$

leads to $p_q = p_0^{1-q} p_1^q / N_q$ with $N_q = \int p_0^{1-q} p_1^q$.



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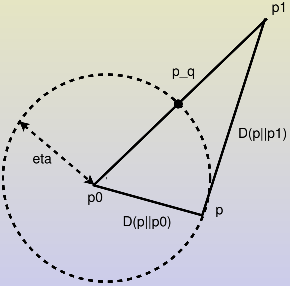

- p_q describe a path from p_0 $(q = 0)$ to p_1 $(q = 1)$.

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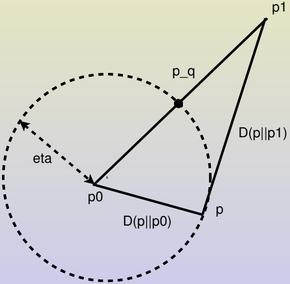
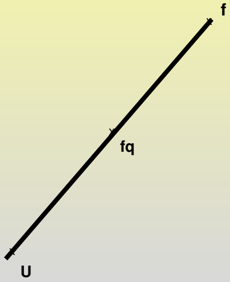


- p_q describe a path from p_0 $(q=0)$ to p_1 $(q=1)$.
- $D(p_q||p_1) = A + B.D_q(p_1||p_0)$

Selecting f...

On the setting $U \leftrightarrow f_q = \frac{f^q}{\int f^q} \leftrightarrow f$,



How to select f/f_q ?

- if $\sigma_q^2 = E_{f_q}\left[X^2\right]$ is fixed?
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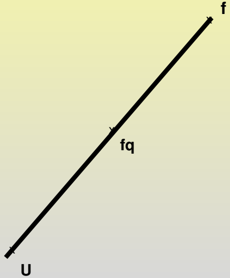

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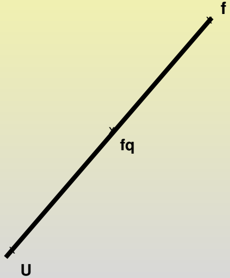

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And what about the evolution of such optimum distributions when q varies?

We know that $\max_f H_q[f]$ s.t variance constraint $\Rightarrow$ a q-gaussian.

Selecting f - Fisher minimization...

- Fisher minimization
  - We use the Fisher information of the distribution (translation families)
  - important applications in statistical physics, [Frieden et al]
  - information theoretic grounds
- Paths with minimum length – thermodynamic length/divergence [Weinhold-Crooks]= flux of Fisher information along the curve
- What about the most general distribution in the Fisher sense?

First problem: distributions with fixed q-variance

$$\begin{cases} & \inf_f \int f(x) \left(\frac{f'(x)}{f(x)}\right)^2 dx = I[f], \\ s.t. & \sigma_q^2 = \frac{\int x^2 f(x)^q dx}{\int f(x)^q dx} \end{cases}$$

Fisher min with q-variance constraint

- Even if $I[f]$ convex, the constraint is not and uniqueness can not be guaranteed
- The formulation of the constraint is awkward

$$\begin{cases} & \inf_f I[f], \\ s.t. & \sigma_q^2 = \frac{\int x^2 f(x)^q dx}{\int f(x)^q dx}, \qquad = \\ s.t. & f(x) \geq 0, \int f(x) dx = 1, \end{cases}$$

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- The initial constraint still present since $\sigma_q^2 = V_q / N_q$.

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- A two steps procedure

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- Expect a parametric solution on positive functions; the normalized version is the solution on the subset $\int f(x) dx = 1$

A simple transformation

- Nonlinear equality constraints into linear equality constraints...
- $f_q(x) = f(x)^q \rightarrow$ substitute $f(x)^q$ by $f_q(x)$

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with, for the Fisher information

$$I[f] = I\left[f_q^{\bar{q}}\right] = \bar{q}^2 \int f_q(x)^{\bar{q}} \left(\frac{f_q'(x)}{f_q(x)}\right)^2 dx = I_{\bar{q}}\left[f_q\right]. \qquad \bar{q} = 1/q$$

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–> The transformation of the q-gaussian with $f_q(x) = f(x)^q$ satisfies the new Euler-Lagrange equation. Minimum? Global?

A pseudo-convex formulation

- The $\bar{q}$-Fisher information $I_{\bar{q}}\left[f_q\right] = I\left[f_q^{\bar{q}}\right]$ is the composition of a convex function $I[.]$ and of the monotone function $f^{\bar{q}}$.
- Intuitively, $I_{\bar{q}}[.]$ is either monotone or unimodal - its minimization subject to linear constraints shall lead to a unique solution.

Definition

A function $f(x)$ defined on an open set Γ is said pseudo-convex at x_0 if it is differentiable at x_0 and

$$f(x) - f(x_0) < 0 \Rightarrow f'(x_0)(x - x_0) < 0 \ \forall x \in \Gamma.$$

If this is true for all $x_0 \in \Gamma$, the function is said pseudo-convex on Γ [Mangasarian 1987]

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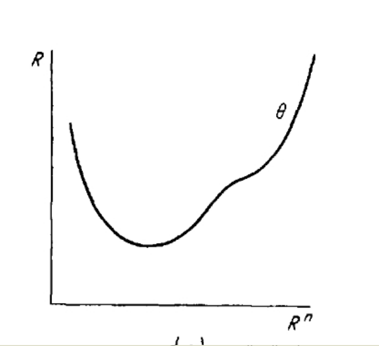
Example –



- Can be extended to the functional case –> Pseudo convex functional

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Result

If $H[p] = \int h(p(x))\,dx$ where h is the composition of a strictly convex function and a monotone function, then $H[p]$ is pseudo-convex.

- Can be extended to the functional case –> Pseudo convex functional

Result

If $H[p] = \int h(p(x))\,dx$ where h is the composition of a strictly convex function and a monotone function, then $H[p]$ is pseudo-convex.

Result

If $H[p] = \int h(p(x))dx$ is a pseudo-convex functional and if $p_0(x)$ is an admissible stationary point of the Lagrangian associated with

$$min_{p\in\Gamma_p} H[p] \text{ subject to } \int a_i(x)p(x)dx = c_i, i = 1 \dots n.$$

then $p_0(x)$ is a global minimum.

So we get finally the result:

Theorem

For $q \in [0,5)$, the probability density function that minimizes the Fisher information under q-variance constraint is the q-gaussian distribution

$$f_*(x) = \frac{1}{Z_q(\beta)} \left(1-(1-q)\beta x^2\right)_+^{\frac{2}{1-q}}, \;\; q \neq 1$$

where the q-variance is given by

$$\sigma_q^2 = \frac{1}{\beta\,(q+3)}.$$

Generalized Cramér-Rao inequality

- For any distribution $f(x)$ with the same q-variance as the q-gaussian $f^{*}(x)$ we have $I[f] \geq I[f_{*}]$.
- We can derive the expression of $I[f_{*}]$ in term of q and the q-variance. This leads us to the following inequality:

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Corollary

(Cramér-Rao inequality). *For any distribution f with q-variance σ_q^2, we have*

$$I[f] \geq I[f_*] = \frac{2(5-q)}{(q+1)(q+3)} \frac{1}{\sigma_q^2},$$

with equality if f is the q-gaussian f_*.

Result on Escort-Fisher

Let

$$p(x)=\frac{f(x)^q}{\int f(x)^q dx} \quad f(x)=\frac{p(x)^{\bar{q}}}{\int p(x)^{\bar{q}} dx}.$$

Then, we obtain an Escort-Fisher of order $\bar{q}$

$$I[f]=I_{\bar{q}}[p]=\bar{q}^2\int\frac{p(x)^{\bar{q}}}{\int p(x)^{\bar{q}}dx}\left(\frac{p'(x)}{p(x)}\right)^2 dx$$

and

$$\begin{cases} & \inf_f I[f], \\ s.t. & \sigma_q^2=\frac{\int x^2 f(x)^q dx}{\int f(x)^q dx} \\ and & f\geq 0 \quad \int f(x)dx=1 \end{cases} = \begin{cases} & \inf_p I_{\bar{q}}[p], \\ s.t & \sigma_q^2=\int x^2 p(x)dx \\ \text{and} & p\geq 0 \quad \int p(x)dx=1. \end{cases}$$

- New problem with standard variance constraint.
- Same global minimum value and $p_*$is the escort of order q of f_*

If we swap q and $\bar{q} = 1/q$, a similar result follows

Theorem

For $q > \frac{1}{5}$, the probability density function that minimizes the Escort-Fisher information of order q under a standard variance constraint is the q-gaussian distribution

$$p_*(x) = \frac{1}{Z_{\bar{q}}(\beta)} \left(1 - \frac{(q-1)}{q} \beta x^2\right)_+^{\frac{2}{q-1}}$$

Corollary

(Cramér-Rao inequality for escort-Fisher). *For any distribution p with a given (standard) variance σ^2, the escort-Fisher information of order $q > \frac{1}{5}$ satisfies*

$$I_q[p] \geq \frac{2q(5q-1)}{(q+1)(3q+1)} \frac{1}{\sigma^2}.$$

Numerical results

- We use a parametric model of probability densities
- Transform the variational problems into optimizations over (a finite number of) parameters.
- We work with the model of densities:

$$p(x) = u(x)^2 = \frac{A}{\left(\sum_{k=1}^{p} a_k x^k\right)^2}.$$

- Rationale: Weierstrass approximation theorem,
- Implementation of the variationals Fisher minimization problems achieved using the active-set algorithm provided in the `fmincon` script of the the Matlab software.
- Initial condition: coefficients of the polynomial that fits the inverse of a normal density on the interval I. Order $p = 10$,

Comparison of numerical/analytical results

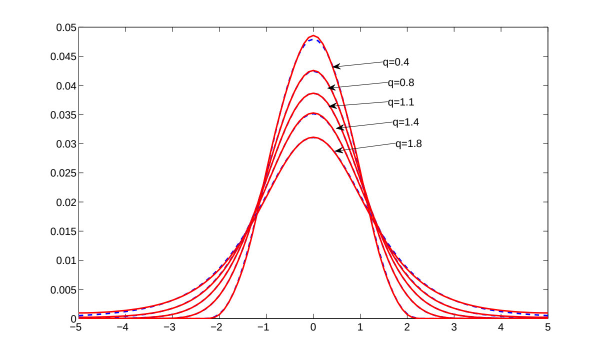

Figure: Comparison of minimum Fisher information distributions (plain lines) and q-gaussians (dashed lines) for several values of q. For values of $q < 1$, the distributions have compact support, and heavy tails for $q > 1$. The comparison shows a remarkable agreement of experimental results with analytical derivations.

Cramér-Rao planes

- We randomly generates densities f according to the model.
- Compute $I[f]$, $I_q[f]$, σ^2, σ_q^2 and populate the plane with the variance-Fisher coordinates.

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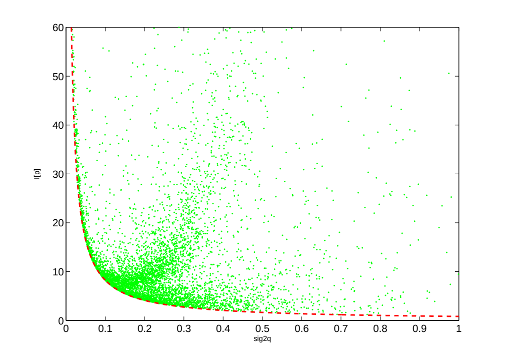


Figure: Cramér-Rao $(\sigma_q^2, I[p])$ information plane, with $q = 1.2$,

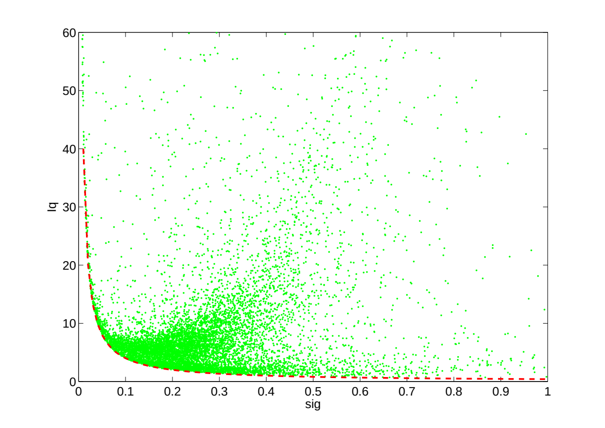
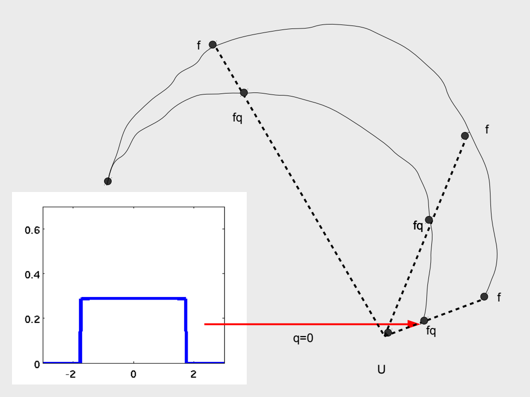


Figure: Cramér-Rao $(\sigma^2, I_q[p])$ information plane, with $q = 0.5$, populated with the coordinates of randomly generated densities p. The analytical bound in dashed line is satisfied.

Minimum Fisher Transition with constant σ_q^2

From $q = 0$ to $q = 1$, the minimum Fisher transition is obtained for q-gaussians with $\sigma_q^2 = 1/\beta(3+q)$ and the thermodynamic divergence $\int_0^1 I[f_*]dq$ is $8 * ln(3) - 10 * ln(2) \approx 1.86$



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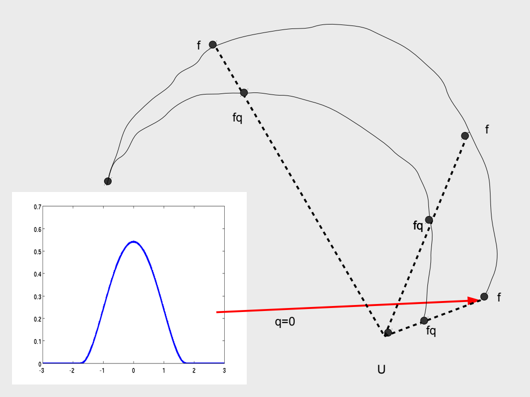

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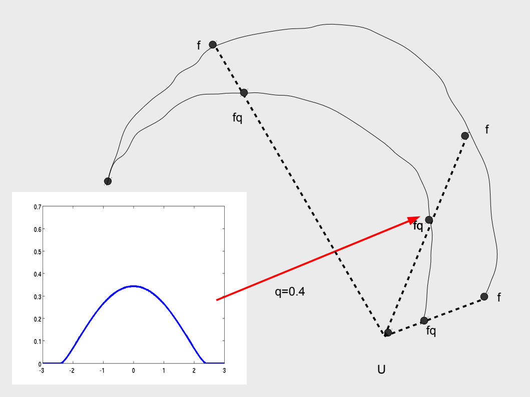

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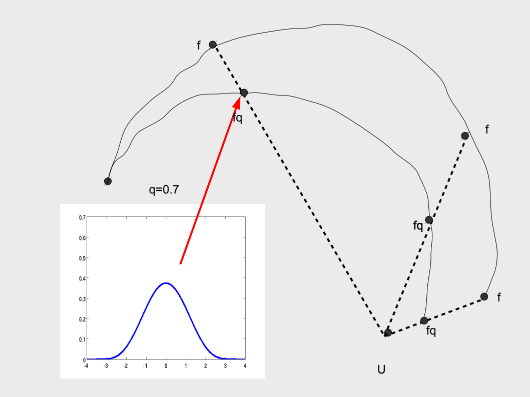

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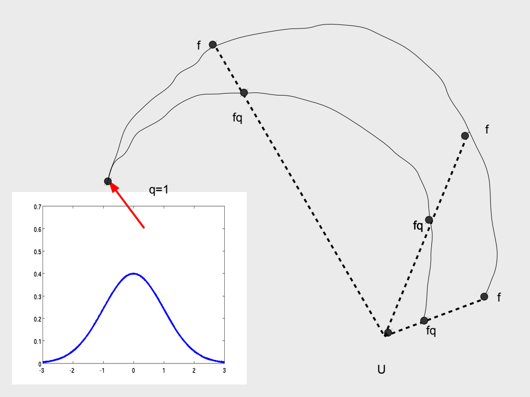

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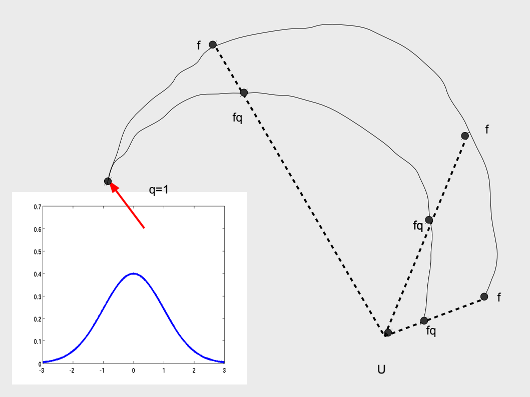
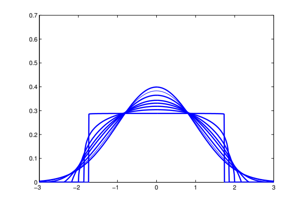
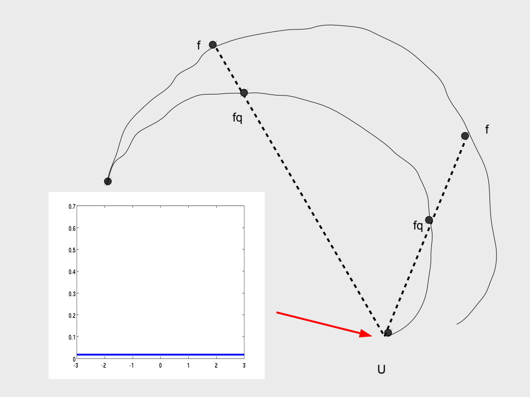




Minimum escort-Fisher Transition with constant σ^2

From $q = 0$ to $q = 1$, the minimum Fisher transition is obtained for q-gaussians with $\sigma^2 = 1/\beta(3 + 1/q)$ and the thermodynamic divergence $\int_{1/5}^{1} I_q[p_*]dq$ is ≈ 0.42



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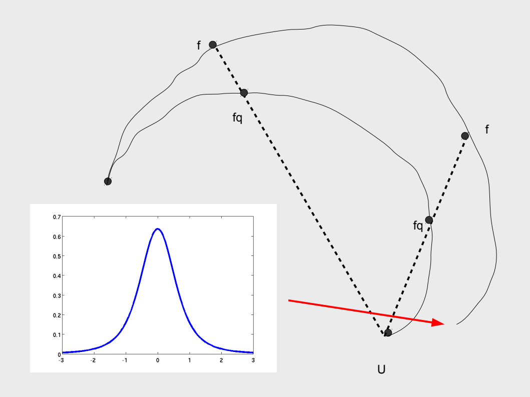

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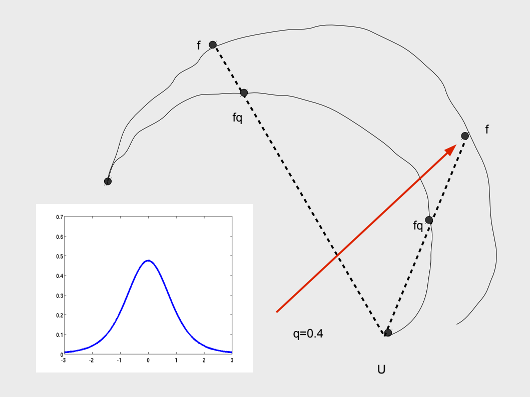

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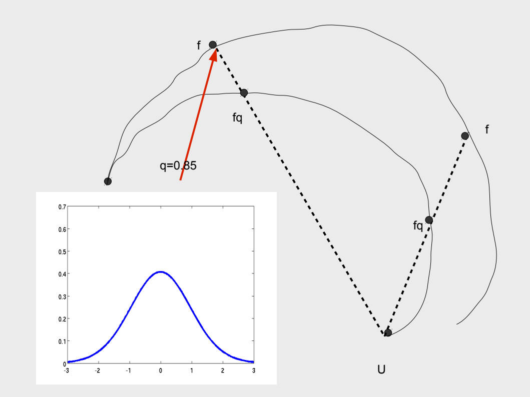

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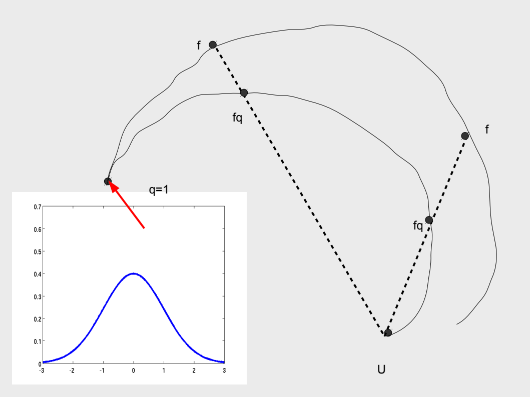

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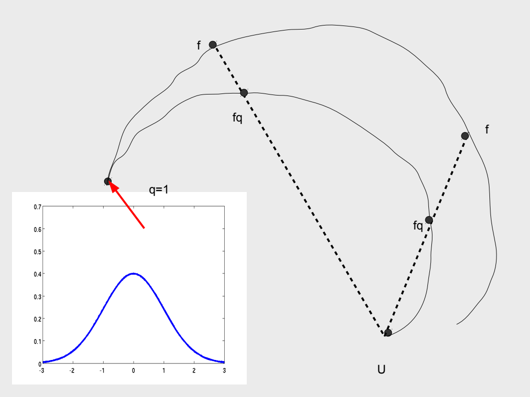

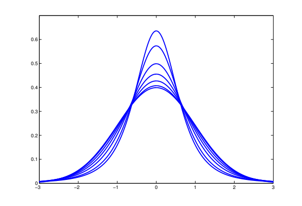

Conclusions and future directions

- Alternative information measures
- Role of escort distributions - Quantization, coding...
- Transition between states - Escort path
- Minimum Fisher
  - A new characterization of q-gaussians
  - Valid Legendre structure
  - Generalized Cramér-Rao planes –> new tools
- General moments instead of q-variance
- Multivariate case.
- Numerical results. New inequalities?