Entropic Dynamics: from Entropy and Information Geometry to Hamiltonians and Quantum Mechanics

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Abstract. Entropic Dynamics is a framework in which quantum theory is derived as an application of entropic methods of inference. There is no underlying action principle. Instead, the dynamics is driven by entropy subject to the appropriate constraints. In this paper we show how a Hamiltonian dynamics arises as a type of non-dissipative entropic dynamics. We also show that the particular form of the "quantum potential" that leads to the Schrödinger equation follows naturally from information geometry.

INTRODUCTION

In the standard view quantum (QT) is a type of mechanics and it is natural to postulate that its dynamical laws are given by an action principle. In contrast, Entropic Dynamics (ED) views quantum theory as an application of entropic methods of inference and there is no underlying action principle. The dynamics is generated by continuously maximizing an entropy as constrained by the appropriate relevant information — it is through these constraints that the "physics" is introduced. [1][2] The ED approach allows a fresh perspective on familiar notions such as time and mass and on long-standing conceptual difficulties, such as indeterminism and the problem of measurement.

The early formulations of ED involved assumptions that were justified only by their pragmatic success — they led to the right answers. For example, use was made of auxiliary variables the physical interpretation of which remained obscure and there were further assumptions about the configuration space metric and the form of the quantum potential. In [2] it was shown that the auxiliary variables were in fact unnecessary and could be eliminated.

In this paper the derivation of QT as a form of entropic dynamics is further strengthened by establishing its relation to information geometry and to Hamiltonian dynamics. We show that a non-dissipative entropic dynamics naturally leads to a Hamiltonian formalism including an action principle. The metric of the N-particle configuration space does not need to be postulated; we derive it from information geometry and show that it coincides with the mass tensor. Finally, the particular form of Hamiltonian that leads to QT requires a so-called "quantum potential" which, we show, is a natural construct

within information geometry.[1]

ENTROPIC DYNAMICS

In order to formulate QT as an example of entropic inference[2] we must identify the microstates that are the subject of our inference, we must identify prior probabilities, and we must identify those constraints that represent the information that is relevant to our problem. First the microstates: We consider N particles living in flat Euclidean space $\mathscr{X}$ with metric δ_{ab}. The particles have definite positions x_n^a and it is their unknown values that we wish to infer.[3] (The index $n = 1 \ldots N$ denotes the particle and $a = 1, 2, 3$ the spatial coordinate.) For N particles the configuration space $\mathscr{X}_N = \mathscr{X} \times \ldots \times \mathscr{X}$.

The basic dynamical assumption is that motion is continuous, that is, large displacements are possible but only as a result of the accumulation of many small steps. We do not explain why motion happens but, given the information that it does, our task is to venture a guess about what to expect. Thus, we first consider a single short step and later we determine how the accumulation of short steps yields a large displacement.

The first goal is to find the transition probability density $P(x'|x)$ for a single short step from a given initial $x \in \mathscr{X}_N$ to an unknown $x' \in \mathscr{X}_N$. The starting point is a prior transition probability $Q(x'|x)$ that expresses our a priori knowledge about which x' to expect *before* any information about the expected step is taken into account. Next, the physically relevant information about the step is expressed in the form of constraints that $P(x'|x)$ must satisfy — this is the stage in which the physics is introduced. Finally, the method of maximum entropy is used to update from the prior probability $Q(x'|x)$ to the desired posterior probability $P(x'|x)$. More specifically, to find $P(x'|x)$ we maximize the (relative) entropy,

$$\mathscr{S}[P, Q] = -\int d^{3N}x' \, P(x'|x) \log \frac{P(x'|x)}{Q(x'|x)} \; . \tag{1}$$

subject to the physically relevant constraints.

We adopt a prior $Q(x'|x)$ that represents a state of extreme ignorance: knowledge of the initial position x tells us nothing about x'. Such ignorance is expressed by assuming that $Q(x'|x)d^{3N}x'$ is proportional to the volume element in $\mathscr{X}_N$. Since $\mathscr{X}_N$ is flat and the proportionality constant has no effect on the entropy maximization we can set $Q(x'|x) = 1$.

Next we introduce some information about the motion. The first piece of information is that motion is continuous—it occurs as a succession of infinitesimally short steps. Each individual particle n will take a short step from x_n^a to $x_n'^a = x_n^a + \Delta x_n^a$ and we require

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[1] Additional references to entropic dynamics and to other information-based approaches to quantum theory including the relation to information geometry are given in [1][2][3].

[2] For an overview of Bayesian and entropic inference and further references see [4].

[3] In this work ED is developed as a model for the quantum mechanics of particles. The same framework can be deployed to construct models for the quantum mechanics of fields, in which case it is the fields that are objectively "real" and have well-defined albeit unknown values.[5]

that the expected squared displacement,

$$\langle \Delta x_n^a \Delta x_n^b \rangle \delta_{ab} = \kappa_n , \qquad (n = 1 \ldots N) \tag{2}$$

be some small value κ_n. For infinitesimally short steps we will eventually take the limit $\kappa_n \to 0$. To reflect the translational symmetry of $\mathscr{X}$ we will assume each κ_n to be independent of x. However, in order to account for differences among non-identical particles we allow κ_n to depend on the particle index n. The constraint (2) leads to a completely isotropic diffusion. Directionality is introduced by assuming the existence of a "potential" $\phi(x)$ and imposing a constraint on the expected displacement $\langle \Delta x \rangle$ along the gradient of ϕ,[4]

$$\langle \Delta x^A \rangle \partial_A \phi = \sum_{n=1}^{N} \langle \Delta x_n^a \rangle \frac{\partial \phi}{\partial x_n^a} = \kappa' , \tag{3}$$

where $\partial_A = \partial/\partial x^A = \partial/\partial x_n^a$ (capitalized indices such as $A = (n,a)$ denote both the particle index and its spatial coordinate). κ' is another small but for now unspecified position-independent constant.

Varying $P(x'|x)$ to maximize $\mathscr{S}[P,Q]$ in (1) subject to the $N+2$ constraints (2), (3) and normalization gives

$$P(x'|x) = \frac{1}{\zeta} \exp[-\sum_n (\frac{1}{2} \alpha_n \Delta x_n^a \Delta x_n^b \delta_{ab} - \alpha' \Delta x_n^a \frac{\partial \phi}{\partial x_n^a})] , \tag{4}$$

where $\zeta = \zeta(x, \alpha_n, \alpha')$ is a normalization constant and α_n and α' are Lagrange multipliers. Since both the function ϕ and the constant κ' are so far unspecified we can, without loss of generality, absorb α' into ϕ which amounts to setting $\alpha' = 1$. The distribution $P(x'|x)$ is Gaussian and is conveniently rewritten as

$$P(x'|x) = \frac{1}{Z} \exp[-\frac{1}{2} \sum_n \alpha_n \delta_{ab}(\Delta x_n^a - \langle \Delta x_n^a \rangle)(\Delta x_n^b - \langle \Delta x_n^b \rangle)] , \tag{5}$$

where Z is a new normalization constant. A generic displacement $\Delta x_n^a = x_n'^a - x_n^a$ can be expressed as an expected drift plus a fluctuation, $\Delta x_n^a = \langle \Delta x_n^a \rangle + \Delta w_n^a$, where

$$\langle \Delta x_n^a \rangle = \frac{1}{\alpha_n} \delta^{ab} \frac{\partial \phi}{\partial x_n^b} , \tag{6}$$

$$\langle \Delta w_n^a \rangle = 0 \quad \text{and} \quad \langle \Delta w_n^a \Delta w_n^b \rangle = \frac{1}{\alpha_n} \delta^{ab} . \tag{7}$$

For very short steps, as $\alpha \to \infty$, the fluctuations become dominant: the drift is $\Delta \bar{x}_n \sim \alpha_n^{-1}$ while $\Delta w_n \sim \alpha_n^{-1/2}$. This implies that, as in Brownian motion, the trajectory is continuous but not differentiable. In the ED approach a particle has a definite position but its velocity, the tangent to the trajectory, is completely undefined.

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[4] Elsewhere, in the context of particles with spin, we will see that the potential $\phi(x)$ can be given a natural geometric interpretation as an angular variable. Its integral over any closed loop is $\oint d\phi = 2\pi n$ where n is an integer.

ENTROPIC TIME

The foundation of all notions of time is dynamics. In ED time is introduced as a book-keeping device to keep track to the accumulation of small changes. As discussed in [1][4] this involves introducing a notion of instants that are ordered, and defining the interval or duration between them. The idea is that if $\rho(x,t)$ refers to a probability distribution at a given instant, which we label t, then entropic time is constructed by defining the *next* instant, labelled t', in terms of a new distribution

$$\rho(x',t') = \int d^3x\, P(x'|x)\rho(x,t) \,, \tag{8}$$

where the transition probability for infinitesimally short steps is $P(x'|x)$ in eq.(5). The iteration of this process defines the dynamics: entropic time is constructed instant by instant: $\rho_{t'}$ is constructed from ρ_t, $\rho_{t''}$ is constructed from $\rho_{t'}$, and so on.

Having introduced the notion of successive instants we now have to specify the interval Δt between them. This amounts to specifying the multipliers $\alpha_n(x,t)$ in terms of Δt.

Time is defined so that motion looks simple. For large α_n the dynamics is dominated by the fluctuations Δw_n. In order that the fluctuations $\langle \Delta w_n^a \Delta w_n^b \rangle$ reflect the symmetry of translations in space and time — a Newtonian time that flows "equably everywhere and everywhen" — we choose α_n to be independent of x and t,

$$\alpha_n = \frac{m_n}{\eta \Delta t} \,. \tag{9}$$

The m_n are particle-specific constants, which will eventually be identified as particle masses, and η is a particle-independent constant that fixes the units of the m_ns relative to the units of time and will eventually (after regraduation) be identified as $\hbar$.

THE INFORMATION METRIC OF CONFIGURATION SPACE

To each point $x \in \mathscr{X}_N$ we can associate a probability distribution $P(x'|x)$. Thus, the configuration space $\mathscr{X}_N$ is a statistical manifold. Up to an arbitrary global scale factor its geometry is uniquely determined by the information metric,

$$\gamma_{AB} = C \int d^{3N}x'\, P(x'|x) \frac{\partial \log P(x'|x)}{\partial x^A} \frac{\partial \log P(x'|x)}{\partial x^B} \,, \tag{10}$$

where C is an arbitrary positive constant. (See *e.g.*, [4].) For short steps ($\alpha_n \to \infty$) a straightforward substitution of eq.(5) using eq.(9) yields

$$\gamma_{AB} = \frac{Cm_n}{\eta \Delta t} \delta_{nn'} \delta_{ab} = \frac{Cm_n}{\eta \Delta t} \delta_{AB} \,. \tag{11}$$

We see that if $\Delta t \to 0$ then $\gamma_{AB} \to \infty$. For smaller Δt the distributions $P(x'|x)$ and $P(x'|x+\Delta x)$ become more sharply peaked and it is easier to distinguish one from the

other which translates into a greater information distance. In order to define a distance that remains meaningful for arbitrarily small Δt it is convenient to choose $C \propto \Delta t$. In what follows the metric tensor will always appear in combinations such as $\gamma_{AB} \Delta t / C$. It is therefore convenient to define the "mass" tensor,

$$m_{AB} = \frac{\eta \Delta t}{C} \gamma_{AB} = m_n \delta_{AB} , \tag{12}$$

and its inverse, the "diffusion" tensor,

$$m^{AB} = \frac{C}{\eta \Delta t} \gamma^{AB} = \frac{1}{m_n} \delta^{AB} . \tag{13}$$

With the choice of the multipliers α_n in (9) the dynamics is indeed simple: $P(x'|x)$ in (5) is a standard Wiener process. The displacement is

$$\Delta x^A = b^A \Delta t + \Delta w^A , \tag{14}$$

where $b^A(x)$ is the drift velocity,

$$\langle \Delta x^A \rangle = b^A \Delta t \quad \text{with} \quad b^A = \frac{\eta}{m_n} \delta^{AB} \partial_B \phi = \eta m^{AB} \partial_B \phi , \tag{15}$$

and the fluctuations Δw^A satisfy,

$$\langle \Delta w^A \rangle = 0 \quad \text{and} \quad \langle \Delta w^A \Delta w^B \rangle = \frac{\eta}{m_n} \delta^{AB} \Delta t = \eta m^{AB} \Delta t . \tag{16}$$

Two remarks are in order: one on the nature of clocks and another on the nature of mass.
On clocks: Time is defined so that motion looks simple. In Newtonian mechanics the prototype of a clock is the free particle and time is defined so that the free particle moves equal distances in equal times. In ED the prototype of a clock is a free particle too — for sufficiently short times all particles are free — and time is defined so that the particle undergoes equal fluctuations in equal times.
On mass: The particle-specific constants m_n will, in due course, be called 'mass' and eq.(16) provides the interpretation: mass is an inverse measure of fluctuations. Thus, up to overall constants the metric of configuration space is the mass tensor and its inverse is the diffusion tensor. In standard QM there are two mysteries: "Why quantum fluctuations?" and "What is mass?". ED offers some progress in this matter: we do not have two mysteries but just one. Fluctuations and mass are two sides of the same coin.

ACCUMULATING CHANGES: THE FOKKER-PLANCK EQUATION

Equation (8) is an integral equation for the evolution of $\rho(x,t)$. As is well known (see *e.g.*, [4]) it can be written in differential form as a Fokker-Planck (FP) equation,

$$\partial_t \rho = -\partial_A \left(b^A \rho \right) + \frac{1}{2} \eta m^{AB} \partial_A \partial_B \rho . \tag{17}$$

which can be rewritten as a continuity equation,

$$\partial_t \rho = -\partial_A \left(\rho v^A \right) . \tag{18}$$

where v^A is the velocity of the probability flow or *current velocity*,

$$v^A = b^A + u^A \quad \text{and} \quad u^A = -\eta m^{AB} \partial_B \log \rho^{1/2} \tag{19}$$

is the *osmotic velocity*, which represents the tendency for probability to flow down the density gradient. Since both b^A and u^A are gradients, it follows that the current velocity is a gradient too,

$$v^A = m^{AB} \partial_B \Phi \quad \text{where} \quad \frac{\Phi}{\eta} = \phi - \log \rho^{1/2} . \tag{20}$$

The FP equation

$$\partial_t \rho = -\partial_A \left(\rho m^{AB} \partial_B \Phi \right) , \tag{21}$$

can be conveniently rewritten in the alternative form

$$\partial_t \rho = \frac{\delta H}{\delta \Phi} , \tag{22}$$

for some suitably chosen functional $H[\rho, \Phi]$. It is easy to check that the appropriate functional H is

$$H[\rho, \Phi] = \int dx \frac{1}{2} \rho m^{AB} \partial_A \Phi \partial_B \Phi + F[\rho] , \tag{23}$$

where $F[\rho]$ is some unspecified functional of ρ. In what follows we will assume that $F = F[\rho]$ rather than the more general $F[\rho;t]$. It is worth emphasizing that eqs.(18), (21), and (22) do not reflect new dynamical principles but are merely different ways to rewrite the very same entropic dynamics already expressed by the FP eq.(17).

   With these results ED reaches a certain level of completion: We figured out what small changes to expect and time was introduced to keep track of how these small changes accumulate; the net result is a standard diffusion and not quantum mechanics.

NON-DISSIPATIVE DIFFUSION

In order to construct a complex wave function in addition to ρ we require a second independent degree of freedom that will be identified with the phase of the wave function. The problem is that the externally prescribed potential ϕ is not an independent degree of freedom. The solution is to change the constraint by promoting the potential ϕ, or equivalently Φ in eq.(20), to a fully dynamical degree of freedom. This is achieved by readjusting the potential ϕ at each time step in response to the evolving ρ. The appropriate constraint arises from imposing that the potential ϕ be updated in such a way that a certain functional, that we will later call "energy", remains constant. Thus the dynamics consists in the coupled non-dissipative evolution of $\rho(x,t)$ and $\Phi(x,t)$.

In the standard approaches to dynamics the conservation of energy is derived from an action principle plus symmetry under time translations. This approach is not open to us because we do not have access to an action principle. In order to define equations of joint evolution for ρ and Φ we must identify the relevant constraints. Accordingly, the logic of our derivation runs in the opposite direction: we first identify the conservation of an energy and the invariance of the expression for energy under time translations as the pieces of information that are relevant to our inferences and then we derive Hamilton's equations and its associated action principle.

The ensemble Hamiltonian. For the quantum systems that interest us, the energy functional that codifies the correct constraint is of the form (23). We therefore impose that, irrespective of the initial conditions, the potential ϕ will be updated in such a way that the functional $H[\rho,\Phi]$ in (23) is always conserved,

$$\frac{dH}{dt} = \int dx \left[\frac{\delta H}{\delta \Phi} \partial_t \Phi + \frac{\delta H}{\delta \rho} \partial_t \rho \right] = 0 . \tag{24}$$

Using eq.(22) we get

$$\frac{dH}{dt} = \int dx \left[\partial_t \Phi + \frac{\delta H}{\delta \rho} \right] \partial_t \rho = 0 . \tag{25}$$

We require that $dH/dt = 0$ hold for arbitrary choices of the initial values of ρ and Φ. Using eq.(21) we see that this amounts to imposing $dH/dt = 0$ for arbitrary choices of $\partial_t \rho$. Therefore the factor in brackets in eq.(25) must vanish at the initial t_0. But t_0 is arbitrary — any time t can be taken as the initial time for evolution into the future. Therefore the requirement that H be conserved for arbitrary initial conditions amounts to imposing that

$$\partial_t \Phi = -\frac{\delta H}{\delta \rho} \tag{26}$$

for all values of t. At this point we recognize that eqs.(22) and (26) have the form of a canonically conjugate pair of Hamilton's equations with the conserved functional $H[\rho,\Phi]$ in (23) playing the role of the Hamiltonian.

Remark: Note that one can start talking about a Hamiltonian only *after* a considerable amount of the ED formalism is in place. In particular, first one must introduce the notion of time, and only then one can show that a suitable choice of constraints leads to a Hamiltonian dynamics.

The action, Poisson brackets, etc.. The field ρ is a generalized coordinate and Φ is its canonical momentum. Eq.(26) leads to a generalized Hamilton-Jacobi equation,

$$\partial_t \Phi = -\frac{1}{2} m^{AB} \partial_A \Phi \partial_B \Phi - \frac{\delta F}{\delta \rho} . \tag{27}$$

It is easy to check that Hamilton's equations, (22) and (26), can be derived from an action principle

$$\delta A = 0 \quad \text{where} \quad A[\rho,\Phi] = \int dt \left(\int dx \Phi \dot{\rho} - H[\rho,\Phi] \right) . \tag{28}$$

The time evolution of any arbitrary function $f[\rho, \Phi]$ is given by a Poisson bracket,

$$\frac{d}{dt}f[\rho,\Phi] = \int dx \left[\frac{\delta f}{\delta \rho} \frac{\delta H}{\delta \Phi} - \frac{\delta f}{\delta \Phi} \frac{\delta H}{\delta \rho} \right] = \{f, H\} , \tag{29}$$

so that H is the generator of time evolution. Similarly one can check that $P_A = \int dx \rho \partial_A \Phi$ is a kind of momentum — it is the generator of translations in configuration space.

A Schrödinger-like equation. Given ρ and Φ we can always combine them into a single complex function,

$$\Psi_k = \rho^{1/2} \exp(ik\Phi/\eta) , \tag{30}$$

where k is some arbitrary positive constant the choice of which will be discussed below. The two coupled equations (22) and (26) can then be written as a single complex Schrödinger-like equation,

$$i\frac{\eta}{k}\partial_t \Psi_k = -\frac{1}{2}\frac{\eta^2}{k^2}m^{AB}\partial_A\partial_B\Psi_k + \frac{1}{2}\frac{\eta^2}{k^2}m^{AB}\frac{\partial_A\partial_B|\Psi_k|}{|\Psi_k|}\Psi_k + \frac{\delta F}{\delta \rho}\Psi_k . \tag{31}$$

INFORMATION GEOMETRY AGAIN: THE SCHRÖDINGER EQUATION

Next we discuss the choice of the functional $F[\rho]$. Let us first recall the definition of the Fisher information matrix. Consider the family of distributions $\rho(x|\theta)$ that are generated from a distribution $\rho(x)$ by pure translations by a vector θ^A, $\rho(x|\theta) = \rho(x - \theta)$. The extent to which $\rho(x|\theta)$ can be distinguished from the slightly displaced $\rho(x|\theta + d\theta)$ or, equivalently, the information distance between θ^A and $\theta^A + d\theta^A$, is given by $d\ell^2 = g_{AB}d\theta^A d\theta^B$ where

$$g_{AB}(\theta) = \int d^{3N}x \frac{1}{\rho(x-\theta)} \frac{\partial \rho(x-\theta)}{\partial \theta^A} \frac{\partial \rho(x-\theta)}{\partial \theta^B} . \tag{32}$$

Changing variables $x - \theta \to x$ yields the Fisher information matrix,

$$g_{AB}(\theta) = \int d^{3N}x \frac{1}{\rho(x)} \frac{\partial \rho(x)}{\partial x^A} \frac{\partial \rho(x)}{\partial x^B} = I_{AB}[\rho] . \tag{33}$$

The functional $F[\rho]$. The simplest choice of functional $F[\rho]$ is linear in ρ, $F[\rho] = \int d^{3N}x \rho V$, where $V(x)$ is some function that will be recognized as the familiar scalar potential. Since ED aims to derive the laws of physics from a framework for inference it is natural to expect that the Hamiltonian might also contain terms that are of a purely informational nature. We have identified two such tensors: one is the information metric of configuration space $\gamma_{AB} \propto m_{AB}$, the other is $I_{AB}[\rho]$. The simplest nontrivial scalar that can be constructed from them is the trace $m^{AB}I_{AB}$. This suggests

$$F[\rho] = \xi m^{AB}I_{AB}[\rho] + \int d^{3N}x \rho V , \tag{34}$$

where $\xi > 0$ is a constant that regulates the realtive strength of the two contributions. From eq.(33) we see that $m^{AB} I_{AB}$ is a contribution to the energy such that those states that are more smoothly spread out tend to have lower energy.[5]

Substituting eq.(34) into (31) gives a non-linear Schrödinger equation,

$$i\frac{\eta}{k}\partial_t \Psi_k = -\frac{\eta^2}{2k^2}m^{AB}\partial_A\partial_B\Psi_k + \left(\frac{\eta^2}{2k^2} - 4\xi\right)m^{AB}\frac{\partial_A\partial_B|\Psi_k|}{|\Psi_k|}\Psi_k + V\Psi_k\ . \quad (35)$$

Regraduation.   We can now return to the choice of the arbitrary constant k in Ψ_k, eq.(30). Since the physics is fully described by ρ and Φ the different choices of k lead to different descriptions of the same theory and among all these equivalent descriptions it is possible to pick one that is singled out by being extremely convenient — a process usually known as 'regraduation'.[6] The optimal choice of k, which we denote with a hat,

$$\hat{k} = (\frac{\eta^2}{8\xi})^{1/2}\ , \quad (36)$$

is such that the non-linear term in eq.(35) drops out. We then identify the optimal regraduated $\eta/\hat{k}$ with Planck's constant $\hbar$,

$$\frac{\eta}{\hat{k}} = (8\xi)^{1/2} = \hbar\ , \quad (37)$$

and eq.(35) becomes the linear Schrödinger equation,

$$i\hbar\partial_t\Psi = -\frac{\hbar^2}{2}m^{AB}\partial_A\partial_B\Psi + V\Psi = \sum_n \frac{-\hbar^2}{2m_n}\nabla_n^2\Psi + V\Psi\ , \quad (38)$$

where the wave function is $\Psi = \rho e^{i\Phi/\hbar}$. The constant $\xi = \hbar^2/8$ in eq.(34) turns out to be crucial: it defines the value of what we call Planck's constant and sets the scale that separates quantum from classical regimes.

Discussion.   We conclude that for any positive value of the constant ξ it is always possible to regraduate Ψ_k to a physically equivalent but more convenient description where the Schrödinger equation is linear. From this entropic perspective the linear superposition principle and the complex Hilbert spaces are important because they are extremely convenient but not because they are fundamental. Note also that the linearity of quantum mechanics is quite robust: once we adopt a non-dissipative Hamiltonian

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[5] The term $m^{AB} I_{AB}$ is sometimes called the "quantum" or the "osmotic" potential but, given its epistemic nature, we should refrain from interpreting it as being either a "potential" or a "kinetic" energy. The relation between the quantum potential and the Fisher information was pointed out in [6]. The case $\xi < 0$ leads to instabilities and is therefore excluded; the case $\xi = 0$ leads to a qualitatively different theory and will be discussed elsewhere.

[6] Other notable examples of regraduation include the Kelvin choice of absolute temperature, the Cox derivation of the sum and product rule for probabilities, and the derivation of the sum and product rules for quantum amplitudes.

diffusion, and the information-inspired quantum potential, any value of $\xi > 0$ leads to a linear quantum theory.

The question of whether the Fokker-Planck and the generalized Hamilton-Jacobi equations, eqs.(22) and (26), are fully equivalent to the Schrödinger equation was first raised by Wallstrom in the context of Nelson's stochastic mechanics and concerns the single- or multi-valuedness of phases and wave functions. [7] Wallstrom objected that stochastic mechanics will lead to phases Φ and wave functions Ψ that are either both multi-valued or both single-valued. Both alternatives are unsatisfactory: quantum mechanics forbids multi-valued wave functions, while single-valued phases can exclude physically relevant states (*e.g.*, states with non-zero angular momentum). We will not discuss the Wallstrom's objection in any detail except to note that it does not arise in the ED approach described here once particle spin is incorporated into the formalism (a similar result is valid for the hydrodynamical formalism, as was shown by Takabayasi [8]). Indeed, earlier we briefly mentioned that the potential $\phi(\vec{x})$ is to be interpreted as an angle. Then integrating the phase $d\Phi$ over a closed path gives

$$\oint \vec{\nabla}\Phi \cdot d\vec{\ell} = \oint \vec{\nabla}\phi \cdot d\vec{\ell} = 2\pi n \tag{39}$$

where n is an integer. This is precisely the quantization condition that leads to full equivalence between ED and the Schrödinger equation because it guarantees that wave functions will remain single-valued even for multi-valued phases.

REFERENCES

1. A. Caticha, J. Phys. A: Math. Theor. **44**, 225303 (2011); arXiv.org/abs/1005.2357.
2. A.Caticha, J. Phys.: Conf. Ser. **504**, 012009 (2014); arXiv:1403.3822.
3. M. Reginatto, "From information to quanta: a derivation of the geometric formulation of quantum theory from information geometry", arXiv:1312.0429.
4. A. Caticha, *Entropic Inference and the Foundations of Physics* (USP Press, São Paulo, Brazil 2012); online at http://www.albany.edu/physics/ACaticha-EIFP-book.pdf.
5. S. Ipek and A. Caticha, "Entropic Quantization of Scalar Fields", in these proceedings (2014).
6. M. Reginatto, Phys. Rev. **A 58**, 1775 (1998).
7. T. C. Wallstrom, Found. Phys. Lett. **2**, 113 (1989); Phys. Rev. **A49**, 1613 (1994).
8. T. Takabayasi, Prog. Theor. Phys. **70**, 1 (1983).