Bayesian Approaches in Acoustic Source Localization and Imaging

Ning CHU[1], Ali MOHAMMAD-DJAFARI[1], Nicolas GAC[1] and José PICHERAL[2]

[1]Laboratoire des Signaux et Systèmes (CNRS-SUPELEC-Univ. Paris–Sud) FRANCE

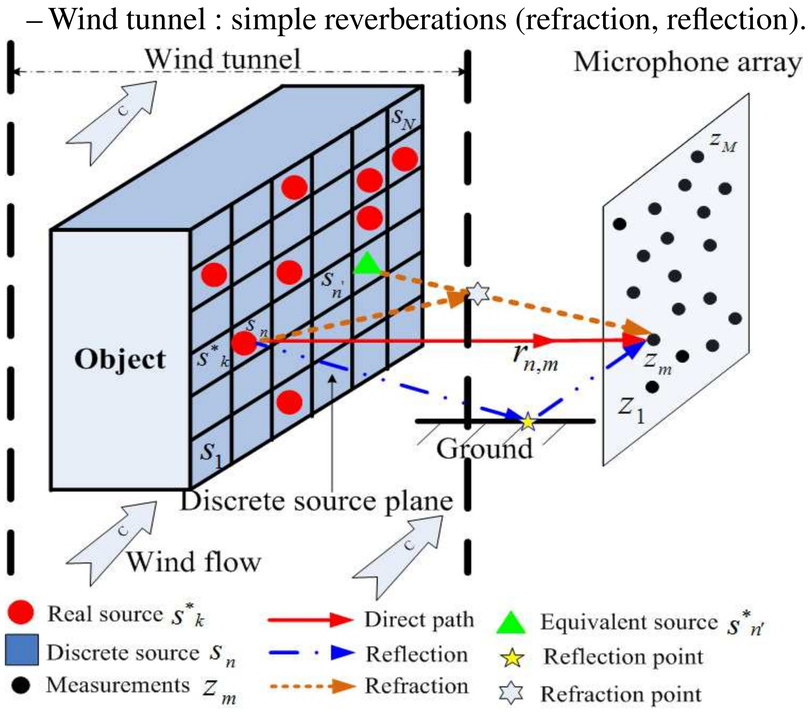
[2] Dept. Signal et Systèmes Electroniques, SUPELEC, FRANCE

Abstract

– **Background :** Acoustic source localization and power reconstruction from limited noisy measurements on microphones.
– **Challenges :** Ill-posed inverse problem, low spatial resolution at low frequencies, time-consuming for large imaging.
– **Motivation :** Super resolution, robustness to noises, large dynamic range, efficient imaging on vehicle surface in wind tunnel.
– **Contribution :** Beamforming, deconvolution, Bayesian approach with sparsity prior, Variational Bayesian Approximation.

1. Forward models of acoustic propagation

◇ **Assumptions :**
– Sources : monopole, uncorrelated ;
– Background noises : i.i.d AGWN ($\mathbf{e} \sim \mathcal{N}(0, \sigma^2)$) ;
– Sensors : omni-directional, unit gain ;
– Wind tunnel : simple reverberations (refraction, reflection).

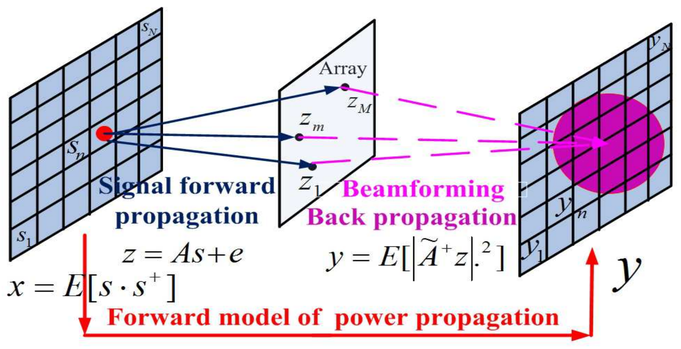


◇ **Discrete signal propagation model :**
Signal measurements $\mathbf{z}$ at M sensors for N dicretized sources $\mathbf{s}$ at positions $\mathbf{P}$ in frequency f [1, 2] :

$$\mathbf{z}(f) = \mathbf{A}(\mathbf{P}, f)\, \mathbf{s}(f) + \boldsymbol{e}(f)$$

– $\mathbf{A}(\mathbf{P}, f) = [\mathbf{a}_{m,n}(\boldsymbol{p}_n, f)]_{M\times N}$: discrete propagation matrix ;
– $\mathbf{a}_{m,n}(\boldsymbol{p}_n, f) = \left\{\frac{1}{r_{n',m}} e^{-j2\pi f \tau_{n',m}} + \rho \frac{1}{r_{-n',m}} e^{-j2\pi f \tau_{-n',m}}\right\}_M^T$
– $\mathbf{s} = [0, s_1^*, 0, \cdots, s_K^*, 0, \cdots]^T$: Spatially K-sparsity ;
– Linear for $\mathbf{s}$, but under-determined for $M \leq N$.

◇ **Discrete power propagation model :**



–

$$\mathbf{y} = \mathbf{C}\,\mathbf{x} + \sigma_e^2\, \mathbf{1}_a$$

$\mathbf{C} = (|\tilde{\mathbf{A}}^\dagger\, \mathbf{A}|.^2)^{\,a}$: PSF matrix, $\mathbf{C}$: $N \times N$:
– $\tilde{\mathbf{A}} = \{\tilde{\mathbf{a}}_n\}_N$: Beamforming steering matrix, $\tilde{\mathbf{A}}$: $M \times N$;
– $\tilde{\mathbf{a}}_n = \frac{\mathbf{a}_n}{||\mathbf{a}_n||^2}$: Beamforming steering vector of $\mathbf{A}$: $M \times N$.
– σ_e^2 : Power of measurement errors $\mathbf{e}$ (i.i.d white noise).
– **Linear**, **determined** for source **powers** $\mathbf{x}$.

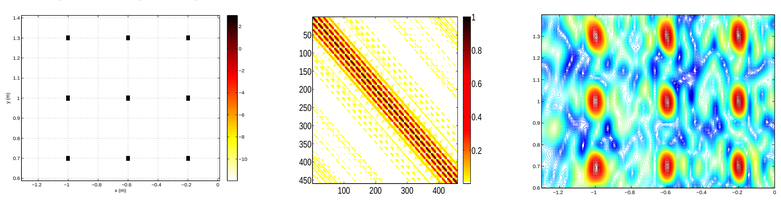
a. $|\mathbf{A}|.^2 = [|a_{i,j}|.^2]_{N\times N}$, for any matrix $\mathbf{A} = [a_{i,j}]_{N\times N}$

2. State-of-the-art methods

◇ **Beamforming [3]** : Simple, fast, but low resolution.

$$\mathbf{y} = \mathbb{E}[|\tilde{\mathbf{A}}^\dagger\, \mathbf{z}|.^2]$$

– $y_n = \sum_{q=1}^N |\tilde{\mathbf{a}}_n^\dagger\, \mathbf{a}_q|^2\, x_q + \sigma_e^2 \frac{1}{||\mathbf{a}_n||^2}$: Beamforming power at grid n ;
– **Low spatial resolution (30cm) at low frequency (2500Hz).**
– $c_{n,q} = |\tilde{\mathbf{a}}_n^\dagger\, \mathbf{a}_q|^2$, $c_{n,q} \in \mathbf{C}$: **spatially variant PSF.**



Source powers $\mathbf{x}$. | **Variant** PSF : $\mathbf{C} = [c_{n,q}]_{N\times N}$. | Beamforming powers $\mathbf{y}$

◇ **Deconvolution and regularization [4, 1]** : High resolution

$$\begin{cases} \mathbf{y} = \mathbf{C}\,\mathbf{x} + \sigma_e^2\, \mathbf{1}_a \\ \hat{\mathbf{x}} = \arg\min_{\mathbf{x}} \left\{||\mathbf{y} - \mathbf{C}\,\mathbf{x}||_2^2 + \alpha\, \mathcal{F}(\mathbf{x})\right\} \\ \text{s.t.} \mathbf{x} \geq 0, \ \textbf{sparsity}; \end{cases}$$

– $\mathcal{F}(\mathbf{x}) = ||\mathbf{D}\,\mathbf{x}||_2$: Tikhonov regularization, non-sparsity ;
– $\mathcal{F}(\mathbf{x}) = ||\mathbf{w}\,\mathbf{x}||_l$ with $0 \leq l \leq 1$, weight vector $\mathbf{w}$:
  – $l = 0$: **Real sparsity**, but hard to solve ;
  – $l = 1$: **Sparsity**, **well solved** by LASSO algorithm ;
– α : to be tuned carefully ;

3. Proposed Joint Bayesian Maximum A Posteriori (JMAP) approach[2]

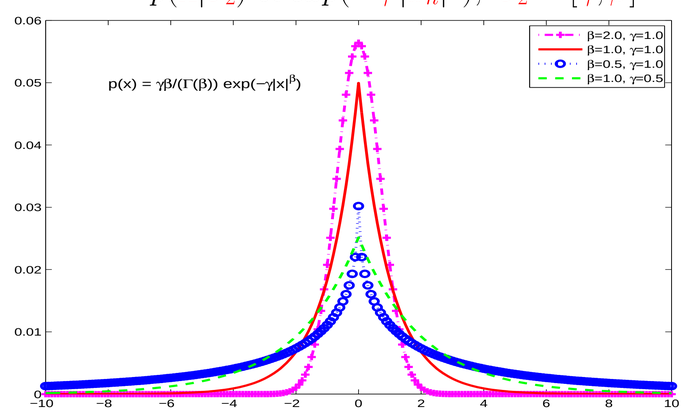
◇ **Bayesian Principle** :
to jointly infer $(\mathbf{x}, \boldsymbol{\theta})$ from measurements $\mathbf{y}$ using useful prior models to obtain the most probably sparse solution of $\mathbf{x}$.

◇ **Bayesian framework of JMAP estimation** :

$$\begin{cases} \mathbf{y} = \mathbf{C}\,\mathbf{x} + \sigma_e^2 \mathbf{1}_N + \boldsymbol{\xi} \\ (\hat{\mathbf{x}}, \hat{\boldsymbol{\theta}}) = \arg\max_{(\mathbf{x},\boldsymbol{\theta})} \{p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y})\} \text{ : Maximizing posterior} \\ p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) \propto \underbrace{p(\mathbf{y}|\mathbf{x}, \boldsymbol{\theta}_1)}_{Likelihood}\ \underbrace{p(\mathbf{x}|\boldsymbol{\theta}_2)}_{Prior}\ \underbrace{p(\boldsymbol{\theta})}_{Hyperprior} \text{ : Full Bayes laws} \\ \boldsymbol{\theta} = [\boldsymbol{\theta}_1, \boldsymbol{\theta}_2],\ \boldsymbol{\theta}_1 = [\sigma_e^2, \sigma_\xi^2] \text{ : Hyper-parameters} \end{cases}$$

– Robust forward model [5] :
random uncertainty $\boldsymbol{\xi} \sim \mathcal{N}(0, \sigma_\xi^2)$ caused by unknown acoustic multi-propagation (reflection, refraction etc).
– Sparsity enforcing prior from Generalized Gaussian family

$$p(\mathbf{x}|\boldsymbol{\theta}_2) \propto exp(-\gamma\, |x_n|^\beta),\ \ \boldsymbol{\theta}_2 = [\gamma, \beta]$$



$\beta = 1$ and $0 \leq \gamma \leq 1$ control heavy tail,sharp peak of sparse distribution, and enforce sparsity and dynamic range.
– $p(\boldsymbol{\theta})$: Positive priors using Jeffrey priors :
$p(\gamma) \sim \frac{1}{\gamma}$, $p(\sigma_\xi^2) \sim \frac{1}{\sigma_\xi^2}$, $p(\sigma_e^2) \sim \frac{1}{\sigma_e^2}$, $\boldsymbol{\theta} = [\gamma, \sigma_e^2, \sigma_\xi^2]$

◇ **Alternate optimization for JMAP** :

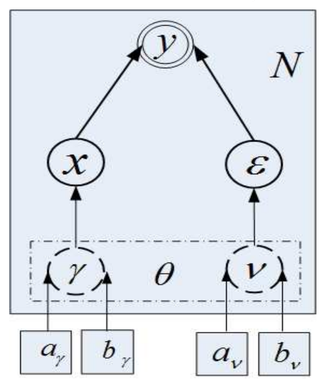
$$\mathcal{J}(\mathbf{x}, \boldsymbol{\theta}) = \underbrace{\frac{\|\mathbf{y} - \mathbf{C}\,\mathbf{x} - \sigma_e^2\, \mathbf{1}_a\|^2}{2\sigma_\xi^2}}_{Likelihood: data\ fitting} + \underbrace{\gamma\, \|\mathbf{x}\|_{\beta=1}}_{Sparse\ prior} + \underbrace{\frac{N}{2}\, \ln \sigma_\xi^2 - N\, \ln \gamma}_{Hyperprior}$$

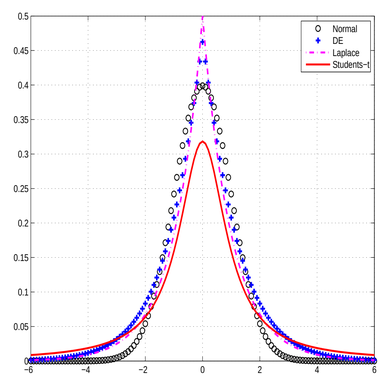
– Prior models $p(\mathbf{x}|\boldsymbol{\theta}_2)$, $p(\boldsymbol{\theta})$ perform as regularization.
– Non-quadratic optimization on joint estimation of $\mathbf{x}$, $\boldsymbol{\theta}$;
– Huge costs $O(N^2)$ due to $\mathbf{C}\,\mathbf{x}$ of $N \times N$ dimension.

4.Proposed Variational Bayesian Approximation (VBA) [6]

$$\begin{cases} \mathbf{y} = \mathbf{H}\,\mathbf{x} + \boldsymbol{\epsilon} \\ (\hat{\mathbf{x}}, \hat{\boldsymbol{\theta}}) = \arg\max_{(\mathbf{x},\boldsymbol{\theta})} \{p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y}) \propto \hat{q}(\mathbf{x}, \boldsymbol{\theta})\} \\ \hat{q}(\mathbf{x}, \boldsymbol{\theta}) = \underbrace{\arg\min_{q(\mathbf{x},\boldsymbol{\theta})} \left\{\int_{(\mathbf{x},\boldsymbol{\theta})} q(\mathbf{x}, \boldsymbol{\theta})\, \ln \frac{q(\mathbf{x}, \boldsymbol{\theta})}{p(\mathbf{x}, \boldsymbol{\theta}|\mathbf{y})}\, d(\mathbf{x}, \boldsymbol{\theta})\right\}}_{\text{Minimizing K-L divergence}}, \\ \hat{q}(\mathbf{x}, \boldsymbol{\theta}) = \hat{q}_1(\mathbf{x})\, \hat{q}_2(\boldsymbol{\gamma})\, \hat{q}_3(\boldsymbol{\nu}), \end{cases}$$

◇ Invariant Convolution model [7] : parallelization on GPU
$\mathbf{C}\,\mathbf{x} \approx \mathbf{H}\,\mathbf{x} = \mathbf{h} * \mathbf{x}$, much faster





Graphic model of hierarchical framework ; Dash circle means hidden variable

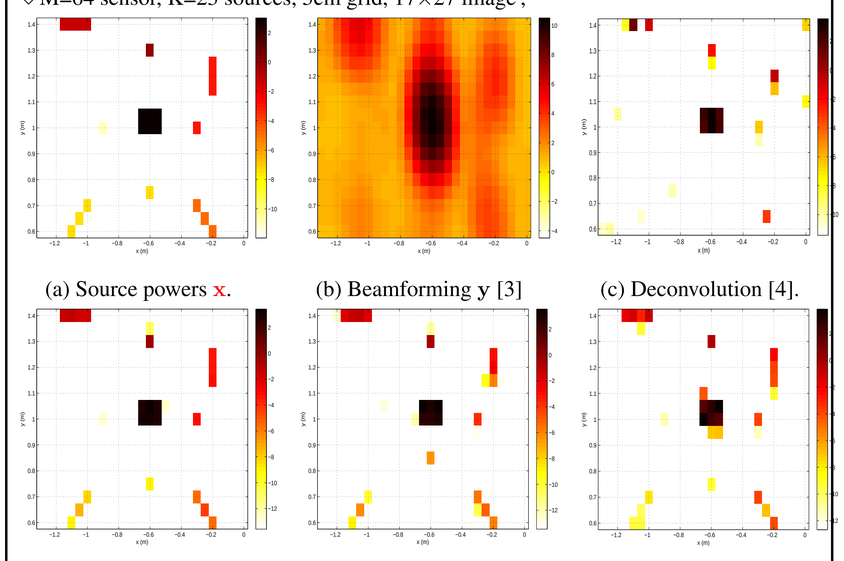
Students-t priors $St(\cdot)$ of hidden variables, $St(\mathbf{x}) = \int_\gamma \mathcal{N}(\mathbf{x}|\mathbf{0}, \boldsymbol{\Sigma}_\gamma^{-1}) \prod_n^N \mathcal{G}(\gamma_n|a_\gamma, b_\gamma) d\gamma$

◇ Sparse sources $\mathbf{x} \sim St(\mathbf{x})$ are mixture scaled variables by hidden variables which indicate source variances.
◇ Model errors $\boldsymbol{\epsilon} \sim St(\boldsymbol{\epsilon})$ are spatially variant noises.
◇ Analytical solutions using Conjugate priors :

$$\begin{cases} \hat{q}_1(\mathbf{x}) = \mathcal{N}(\mathbf{x}|\hat{\boldsymbol{\mu}}_x, \hat{\boldsymbol{\Sigma}}_x) \\ \hat{q}_2(\boldsymbol{\gamma}) = \prod_{n=1}^N \mathcal{G}(\gamma_n|\hat{a}_\gamma, \hat{b}_\gamma^n), \quad \hat{q}_3(\boldsymbol{\nu}) = \prod_{n=1}^N \mathcal{G}(\nu_n|\hat{a}_\nu, \hat{b}_\nu^n) \end{cases},$$

5.Simulation at 2500Hz and SNR=0dB

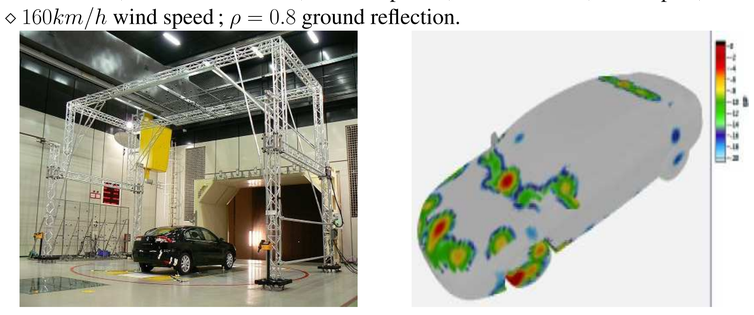
◇ Efficient imaging for monopole and extended sources
◇ M=64 sensor, K=23 sources, 5cm grid, 17×27 image ;



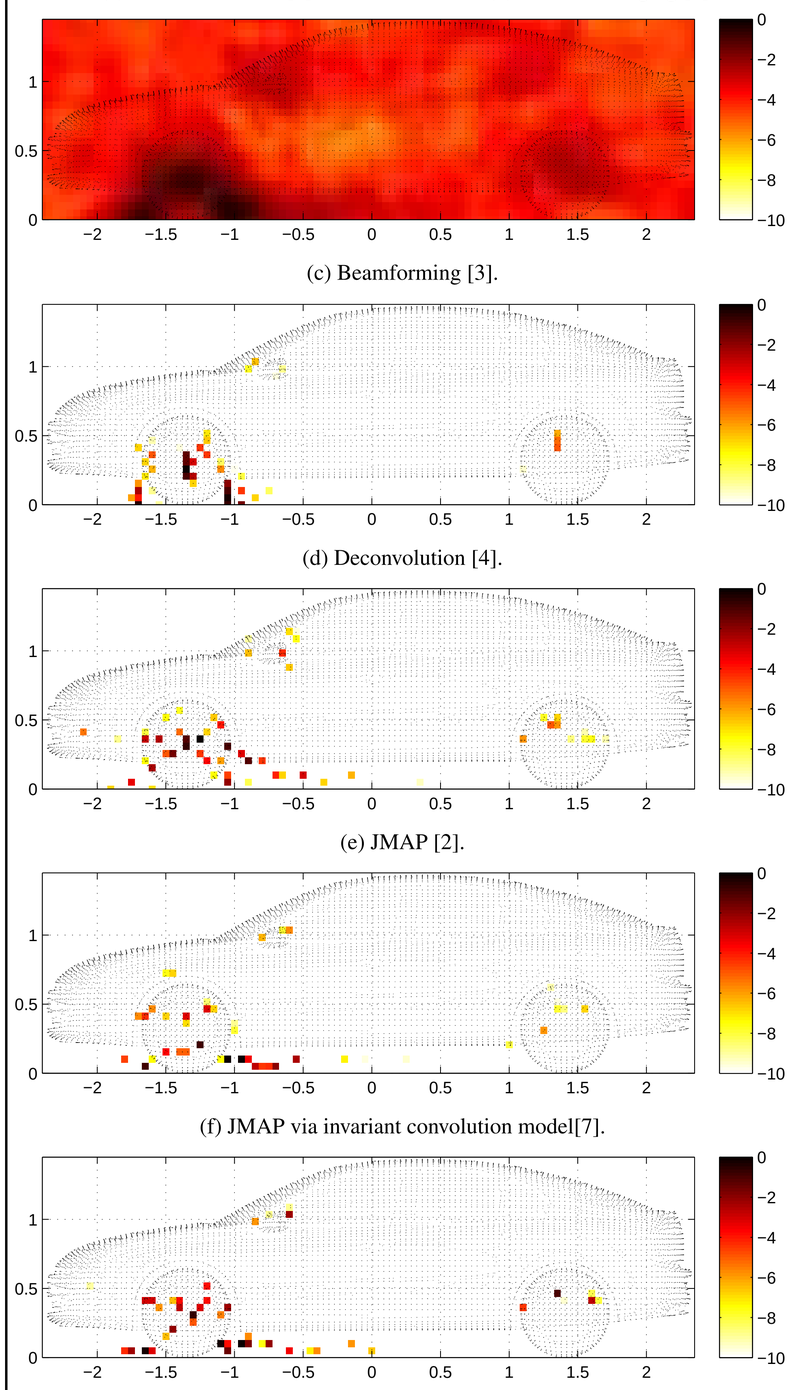
(a) Source powers $\mathbf{x}$. (b) Beamforming $\mathbf{y}$ [3] (c) Deconvolution [4].
(d) JMAP [2]. (e) JMAP + conv[7]. (f) VBA+ convolution[6].

6. Wind tunnel experiments at 2500Hz

◇ Mapping acoustic powers on vehicle ;
◇ 64 sensors ; 5cm discretization ; 31×101 pixels ; 4.5m distance ; 10dB span ;
◇ $160km/h$ wind speed ; $\rho = 0.8$ ground reflection.



(a) Wind tunnel S2A[8] (b) Acoustic imaging [8]



(c) Beamforming [3].
(d) Deconvolution [4].
(e) JMAP [2].
(f) JMAP via invariant convolution model[7].
(g) VBA via invariant convolution model[6].

Table 1. Computational time in imaging 31×101 pixels, based on CPU :3.33GHz.

Methods	BF	Deconv	JMAP	JMAP+conv	VBA+conv
Time (s)	1	10	1012	180	578

7. Perspectives

◇ Forward model of full-wave acoustic propagation ;
◇ Sparsity priors on correlated sources ;
◇ Real-time realization using Graphical Processor Unit (GPU).

References

[1] T. Yardibi, J. Li, P. Stoica, and L.N. Cattafesta III, "Sparsity constrained deconvolution approaches for acoustic source mapping," *The Journal of the Acoustical Society of America*, vol. 123, no. 5, pp. 2631–42, 2008.

[2] N. Chu, A. Mohammad-Djafari, and J. Picheral, "Robust Bayesian super-resolution approach via sparsity enforcing a priori for near-field aeroacoustic source imaging," *Journal of Sound and Vibration*, vol. 332, no. 18, pp. 4369–4389, 2013.

[3] J.C. Chen, K. Yao, and R.E. Hudson, "Source localization and beamforming," *Signal Processing Magazine, IEEE*, vol. 19, no. 2, pp. 30–39, 2002.

[4] T.F. Brooks and W.M. Humphreys, "A Deconvolution Approach for the Mapping of Acoustic Sources (DAMAS) determined from phased microphone arrays," *Journal of Sound and Vibration*, vol. 294, no. 4-5, pp. 856–879, 2006.

[5] N. Chu, A. Mohammad-Djafari J. Picheral, and N. Gac, "A robust super-resolution approach with sparsity constraint in acoustic imaging," *Applied Acoustics*, vol. 76, pp. 197–208, 2014.

[6] Ning Chu, Ali Mohammad-Djafari, José Picheral, and Nicolas Gac, "An efficient variational Bayesian inference approach via student's-t priors for acoustic imaging in colored noises," 2013, vol. 19, p. 055031, JASA.

[7] N. Chu, N. Gac, A. Mohammad-Djafari, and J. Picheral, "2d convolution model using (in)variant kernels for fast acoustic imaging," in *Berlin Beamforming Conference 2014 (BeBeC2014)*, Berlin, Germany, Feb.19-20,2014, p. No.5.

[8] Arnaud Menoret, Nathalie Gorilliot, and Jean-Luc Adam, "Acoustic imaging in wind tunnel S2A," in *10th Acoustics conference (ACOUSTICS2010)*, Lyon, France, 2010.