Phase estimation for signals with low S/N-ratio

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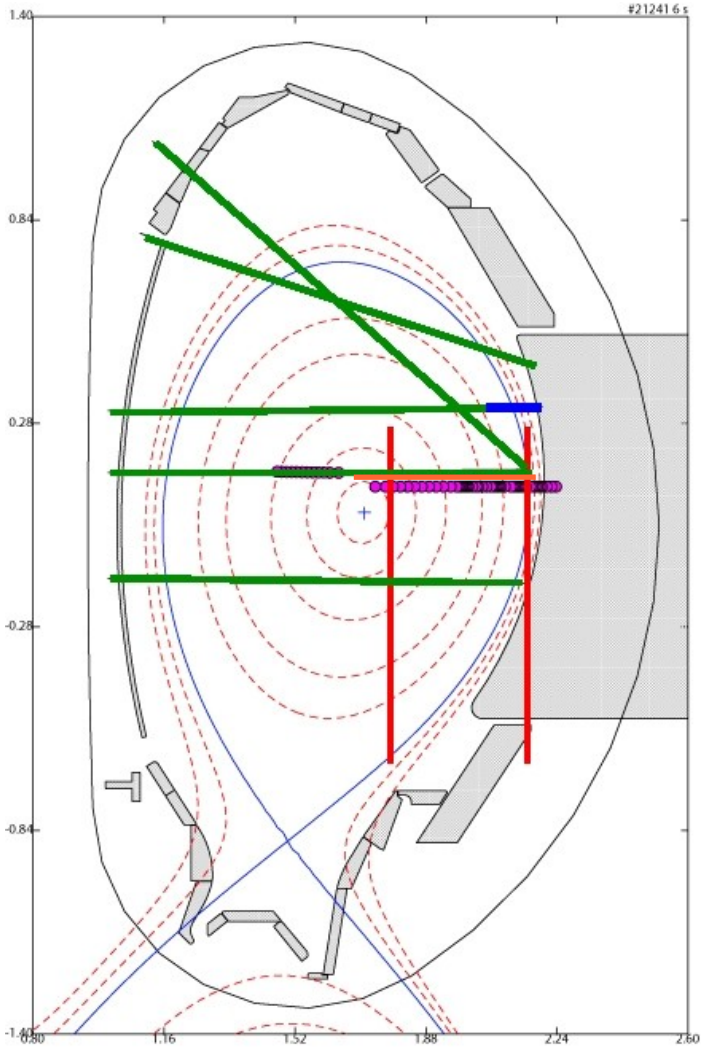
with contributions by A. Mlynek

MaxEnt 2014, Amboise

Introduction

- **Phase shift estimation**

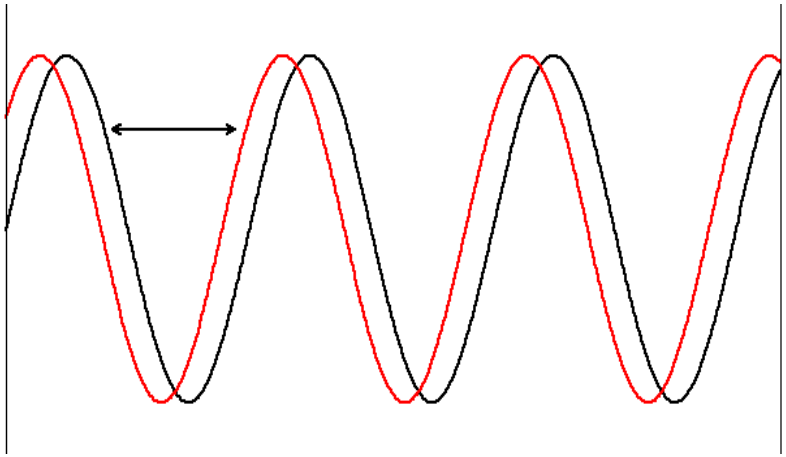
Interferometry for plasma electron density

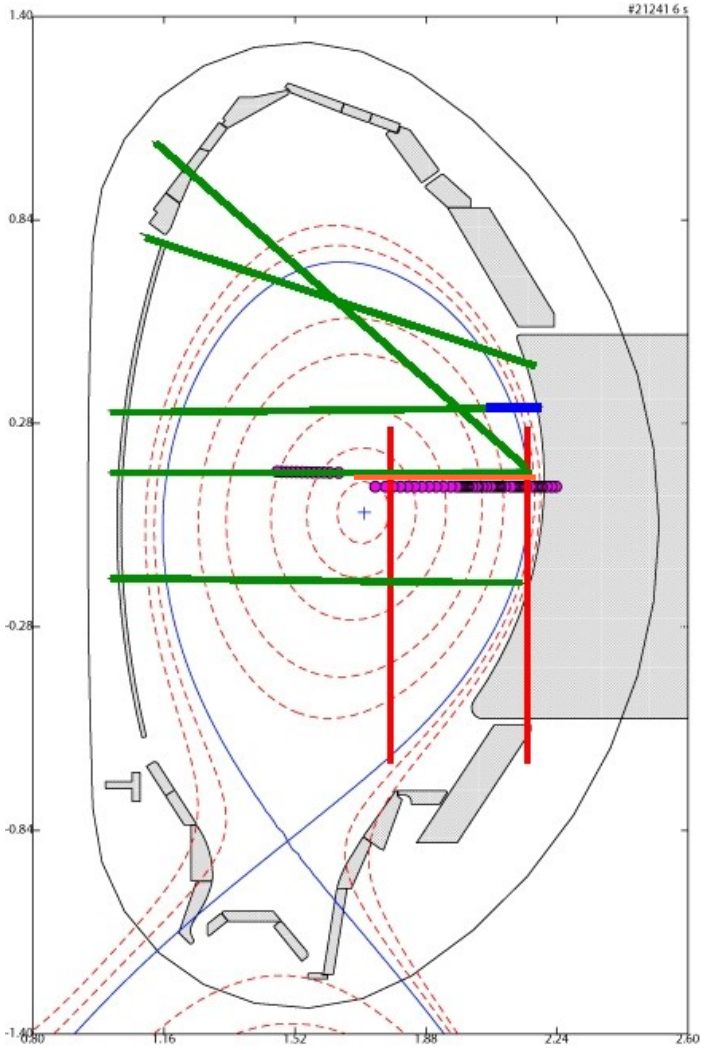


Introduction

➢ Phase shift estimation

Interferometry for plasma electron density





- Appeared to be a relatively simple problem...

Introduction

➢ Underlying Physics

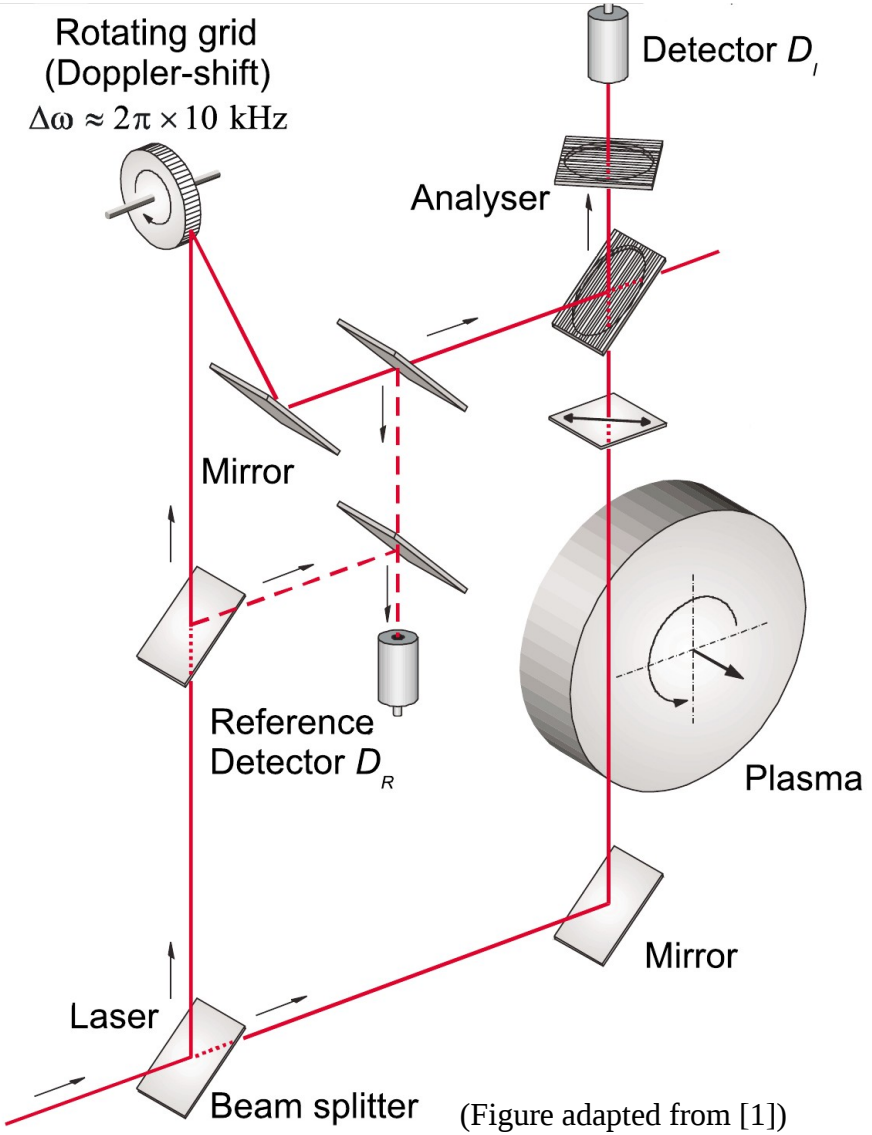
- Plasmas refractive index: **n<1**

$$\phi = \lambda_0 r_e \int_0^{L_{geom}} dx n_e(x)$$

- Measurement based on Mach-Zehnder interferometer
- **Beat** frequency:

Shift signal to measurable Δω:

$$\sin(\omega t) + \sin((\omega + \Delta\omega)t) = 2\sin\left(\frac{2\omega + \Delta\omega}{2}t\right)\cos\left(\frac{\Delta\omega}{2}t\right)$$

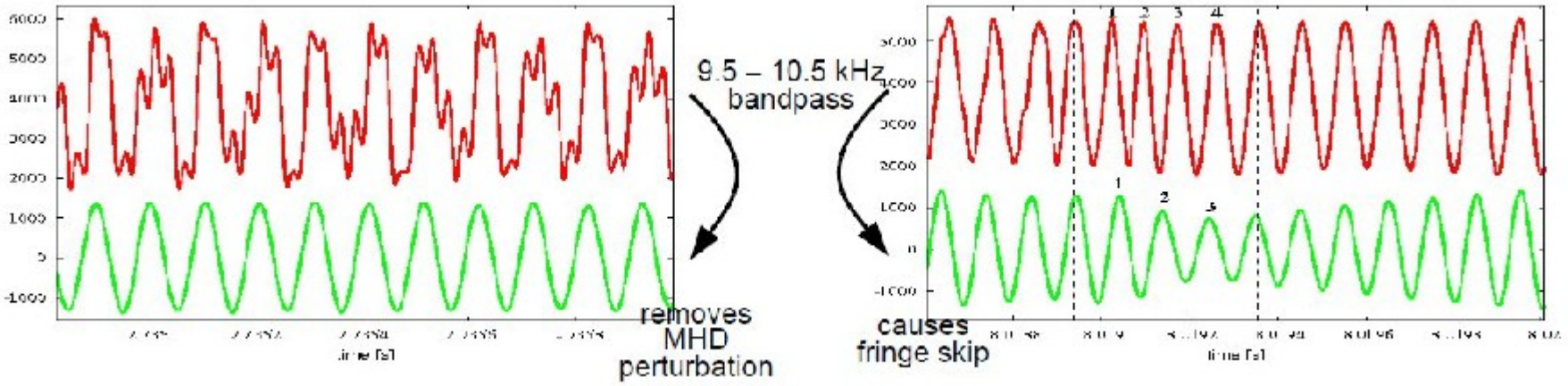


(Figure adapted from [1])

Introduction

➢ Present evaluation

- Band-pass-filter around beat-frequency
- Followed by nonlinear fit to constant frequency



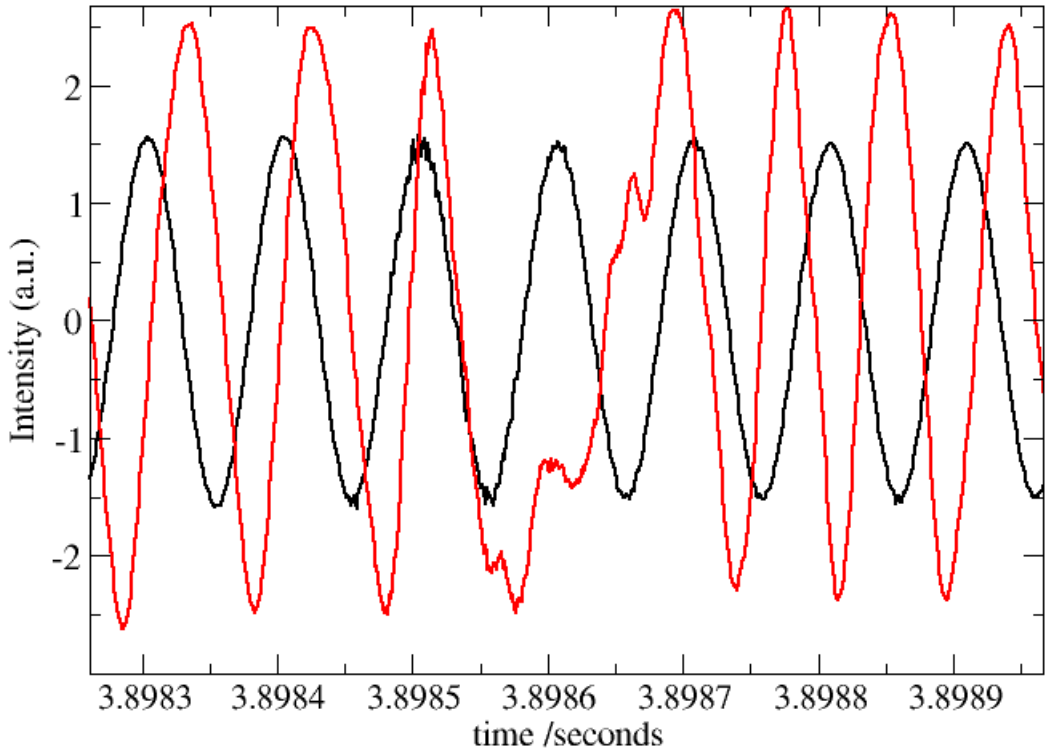
⇨ Hard to predict fringe losses

- Better algorithm desirable...

Signal

➢ Signal properties

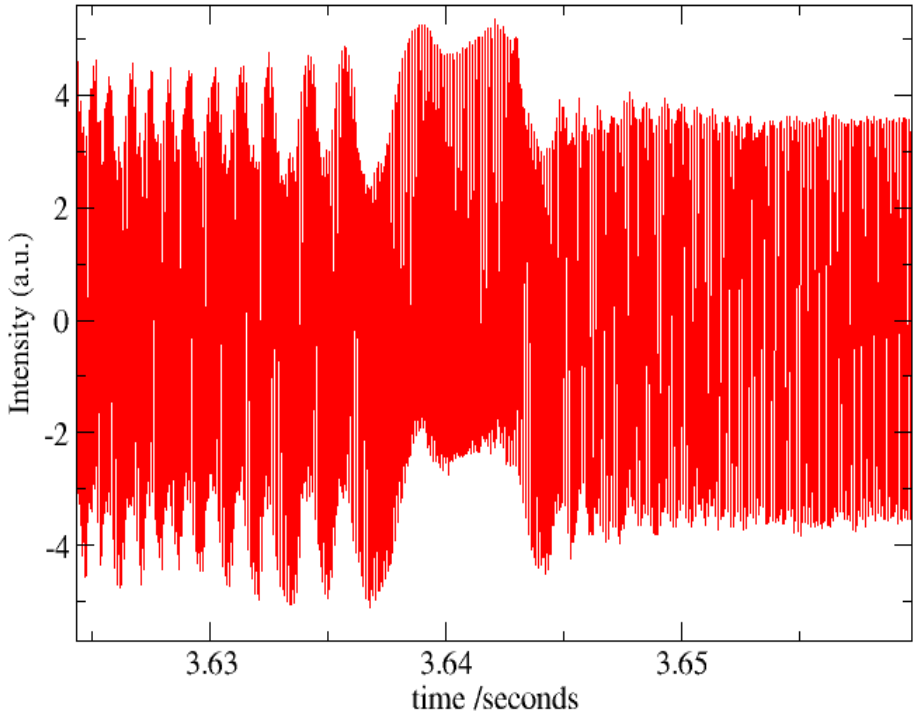
- Signal distortions by plasma disturbances (edge localized modes (ELMs))



Signal

➢ Signal properties

• Interference by MHD-modes (varying frequency)



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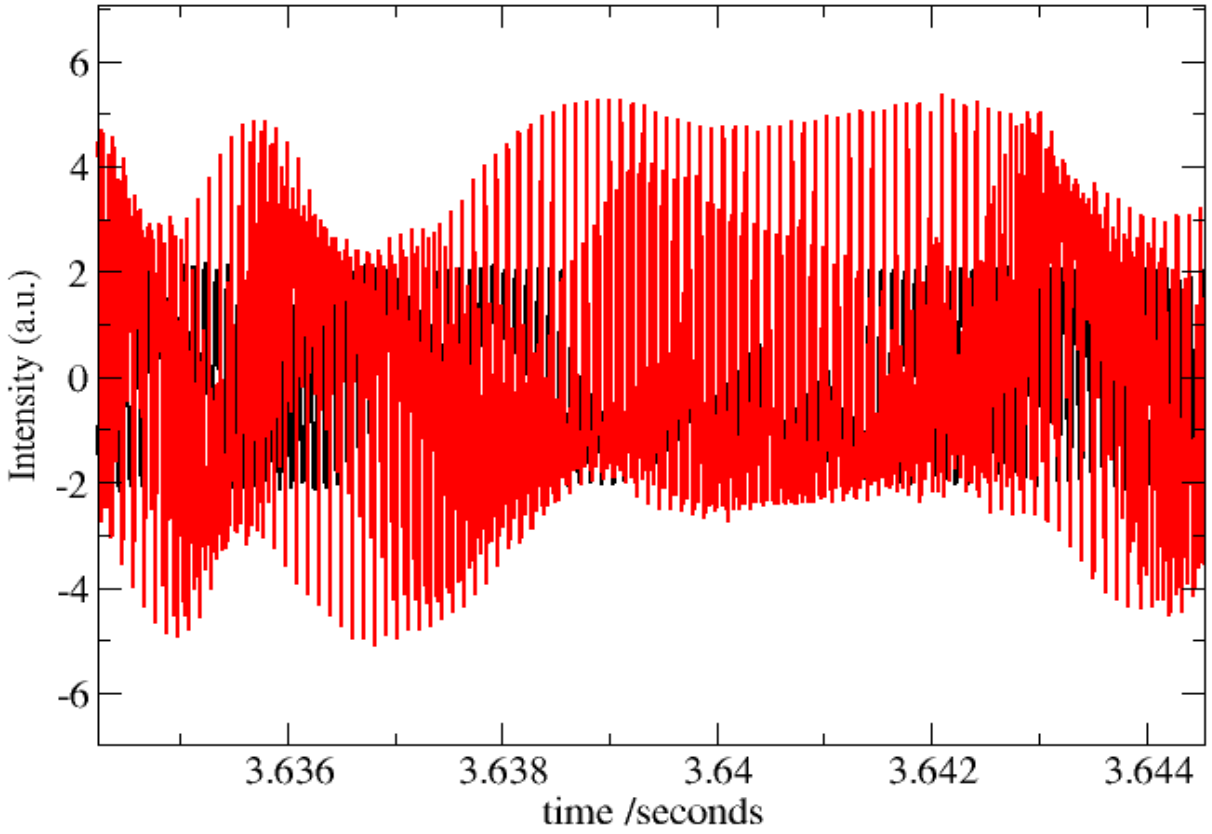

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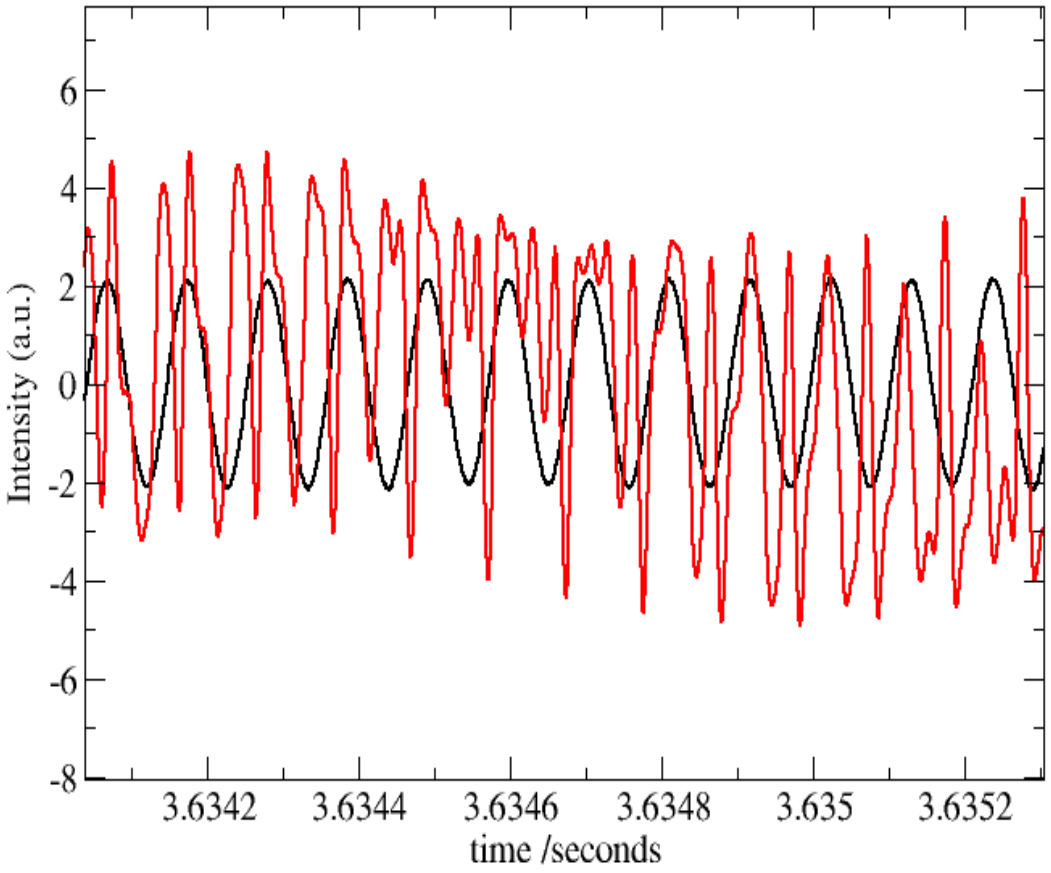

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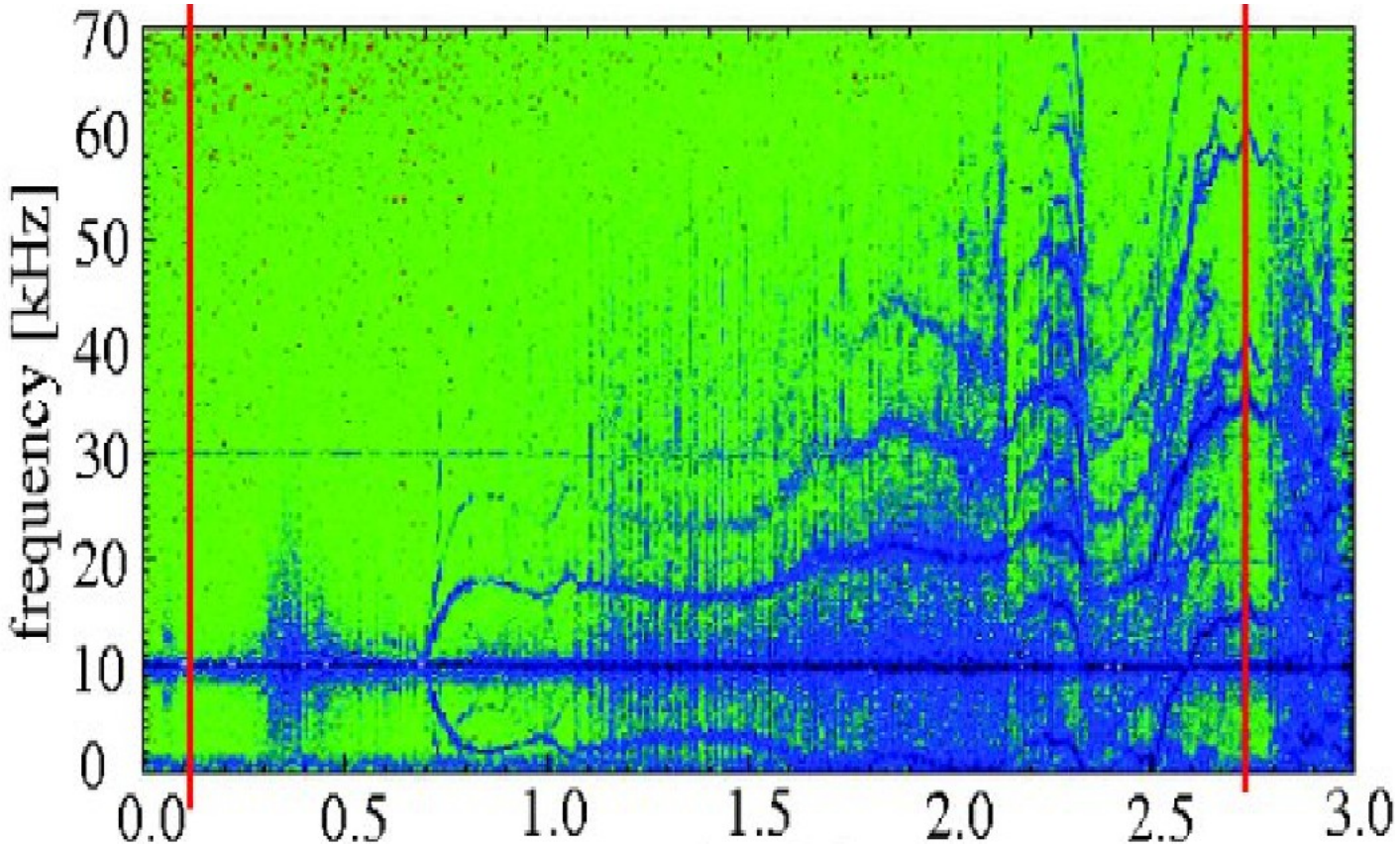
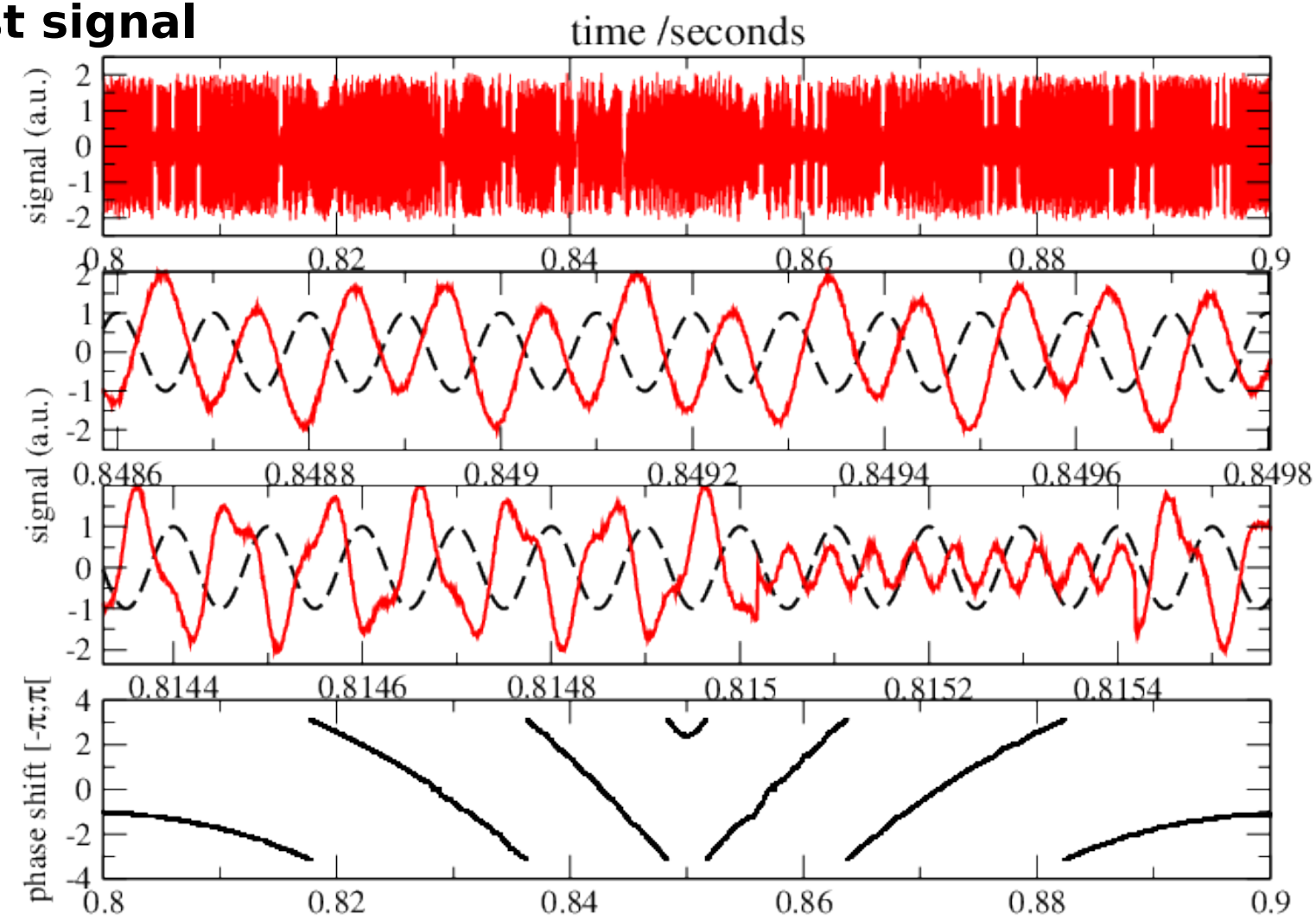


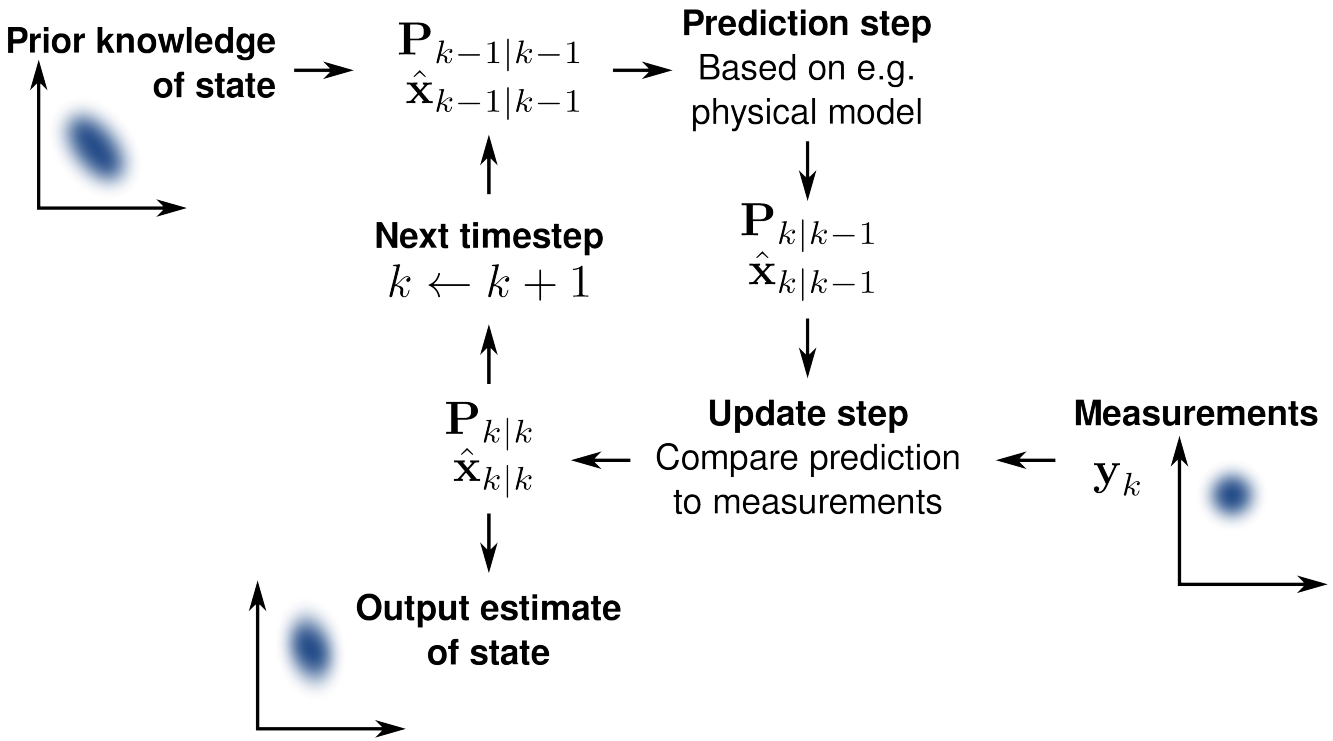
Signal

- **Test signal**



➢ Approach I: Kalman filter

- Follow signal development (real-time):



- Correct parameter settings are a challenge...

➢ Approach II: via frequency estimation

- Frequency change provides phase shift

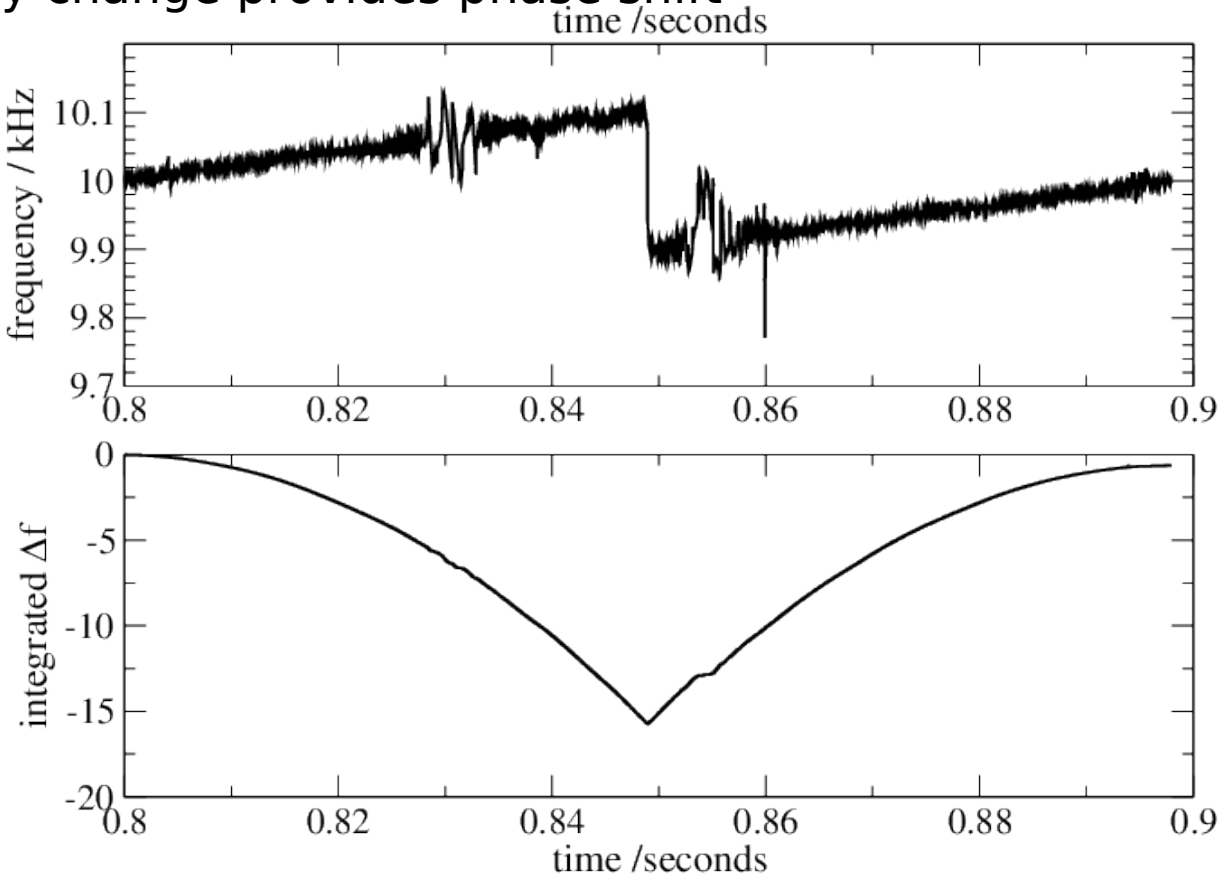
$$\cos\left(\omega_0 t + \phi\left(t\right)\right) = \cos\left(\omega_0 t + \phi_0 + \Delta\phi t + \ldots\right)$$
$$= \cos\left(\left(\underbrace{\omega_0 t + \Delta\phi}_{\omega}\right) t + \phi_0 + \ldots\right)$$

- Benefit: Integrated signal (→ robust)
- FFT-based approaches (Lomb-Scargle periodigram etc.) are not optimal
- → Bretthorst algorithm (Bretthorst, 1988 [2], 2001)

Analysis

➢ Approach II: via frequency estimation

• Frequency change provides phase shift



• Successful on simulated data...

Analysis

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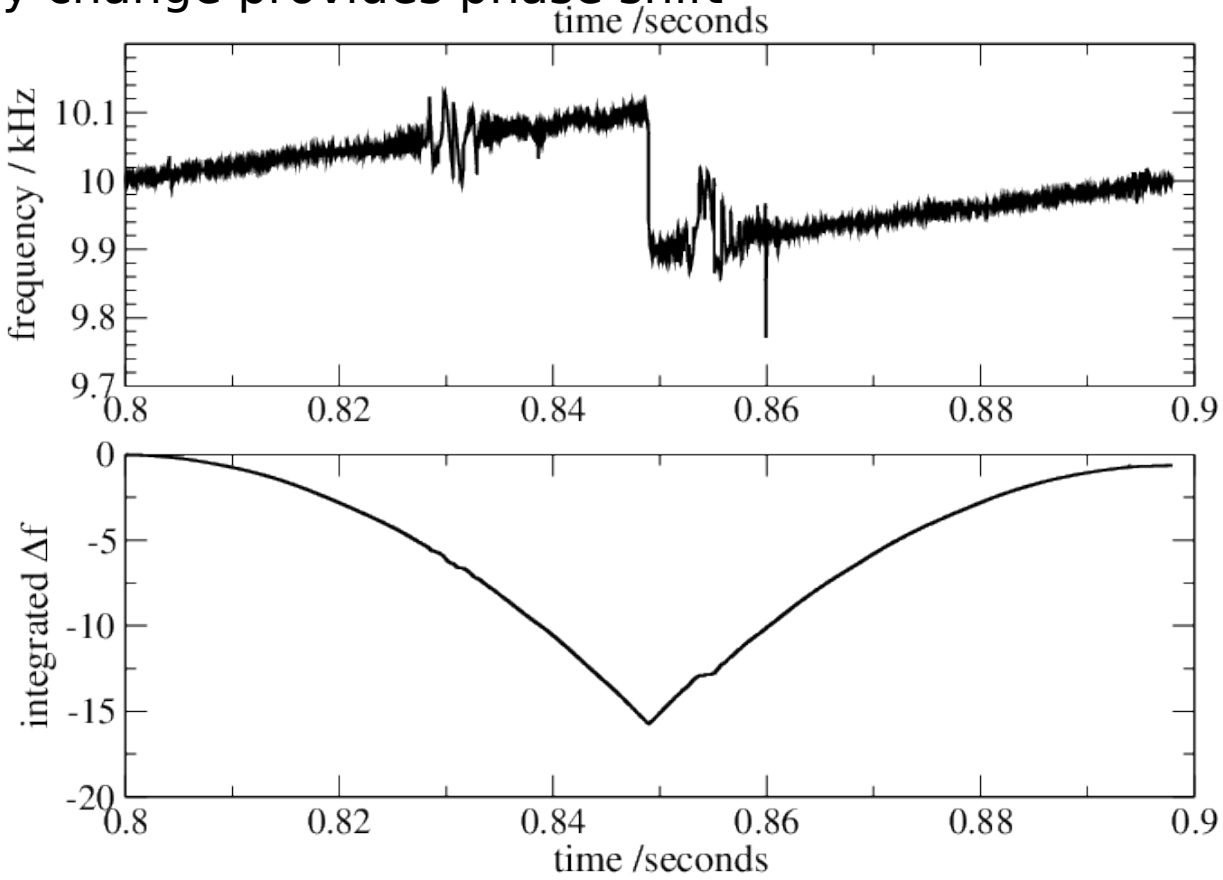


- Successful on simulated data...failed on measured data

Analysis

➢ Approach II: via frequency estimation

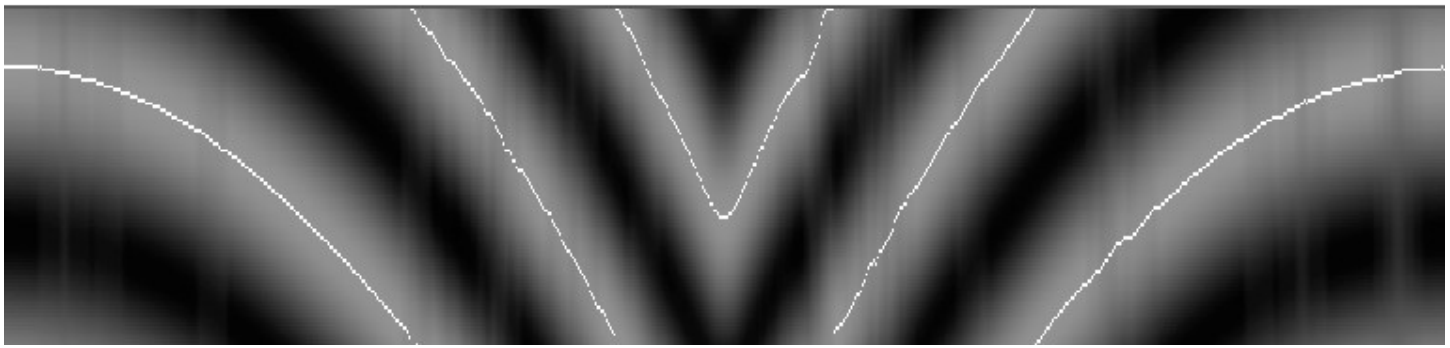
- Frequency change provides phase shift
- Unstable reference (beat) frequency…pressurized air driven wheel.

$$s(t) = \cos\left(\omega(t) + \phi(t)\right)$$

- Integration of small difference of large quantities: errors accumulate

➢ Approach III

• Cross correlation:



• Cross correlation **does not** provide probabilities...

• Normalized probability distribution via complete set of shifts and L1-Norm

$$\Delta d\left(t \mid \tau_i\right) = d_{\mathrm{ref}}\left(t\right) - d_{\mathrm{probe}}\left(t + \tau_i\right)$$

•...provides mean & uncertainty of **wrapped** phase as function of time

Analysis

➢ Approach III

- Phase unwrapping?

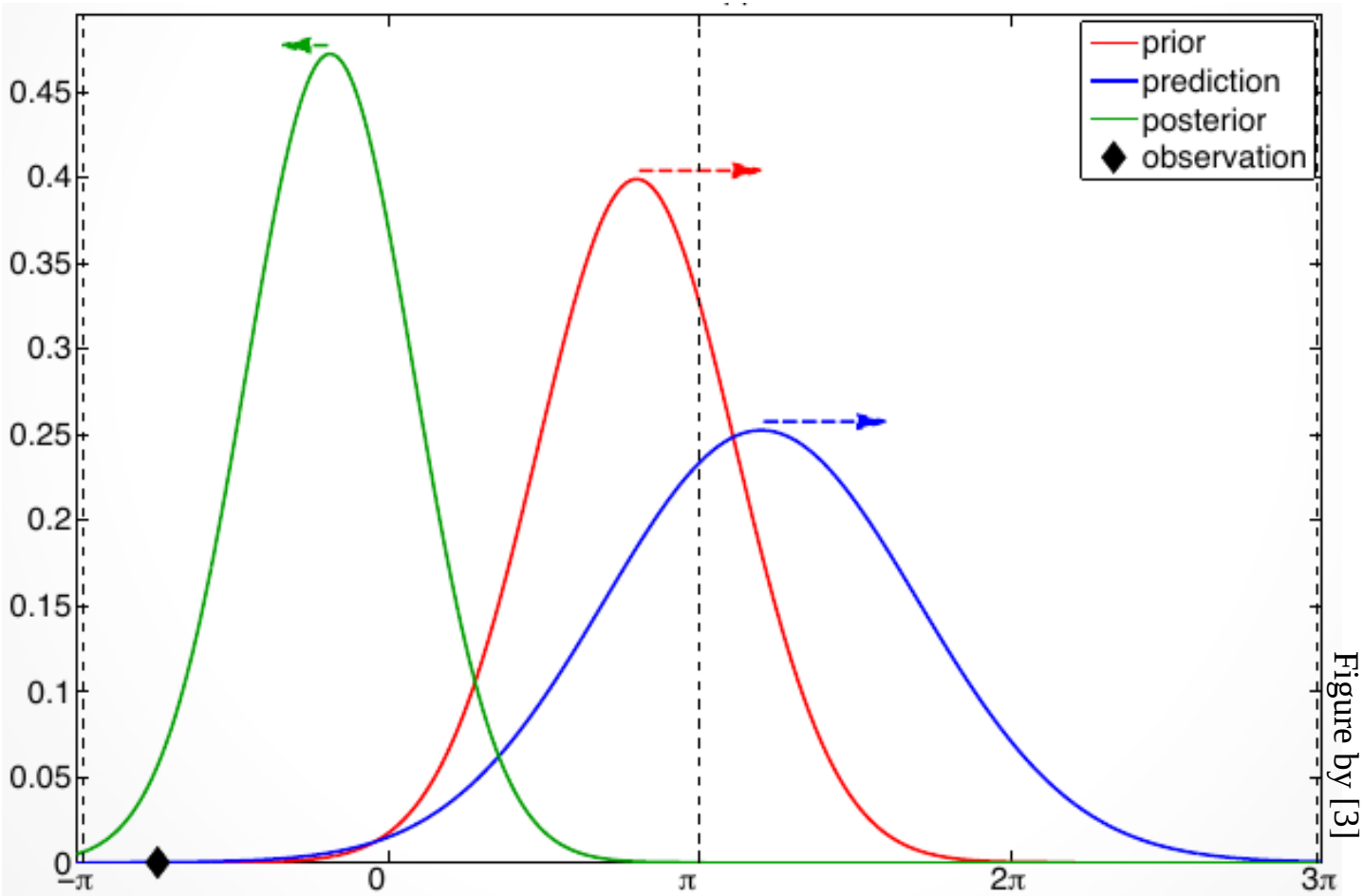


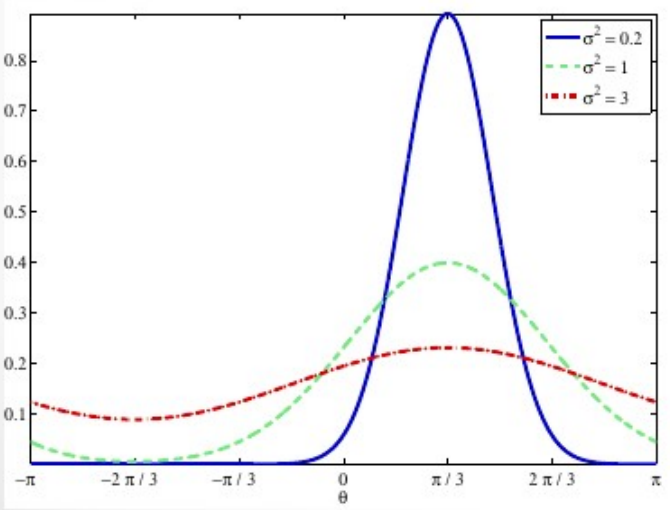
Figure by [3]

Analysis

➢ Approach III

- Phase unwrapping?
- Phase angle: estimation of trajectory on circle/cylinder
- Wrapped Gaussian distribution (not von-Mises distribution)

$$\mathcal{WN}\left(\theta\,;\,\mu,\sigma^2\right) = \sum_{l=-\infty}^{\infty} \frac{1}{\sqrt{2\pi\sigma^2}} e^{-\frac{(\theta-(\mu+2\pi l))^2}{2\sigma^2}} \quad , \quad \theta \in \mathbb{S}^1$$



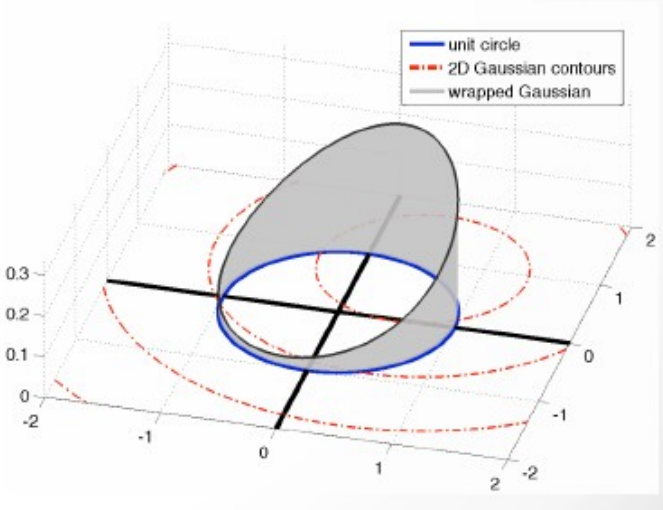
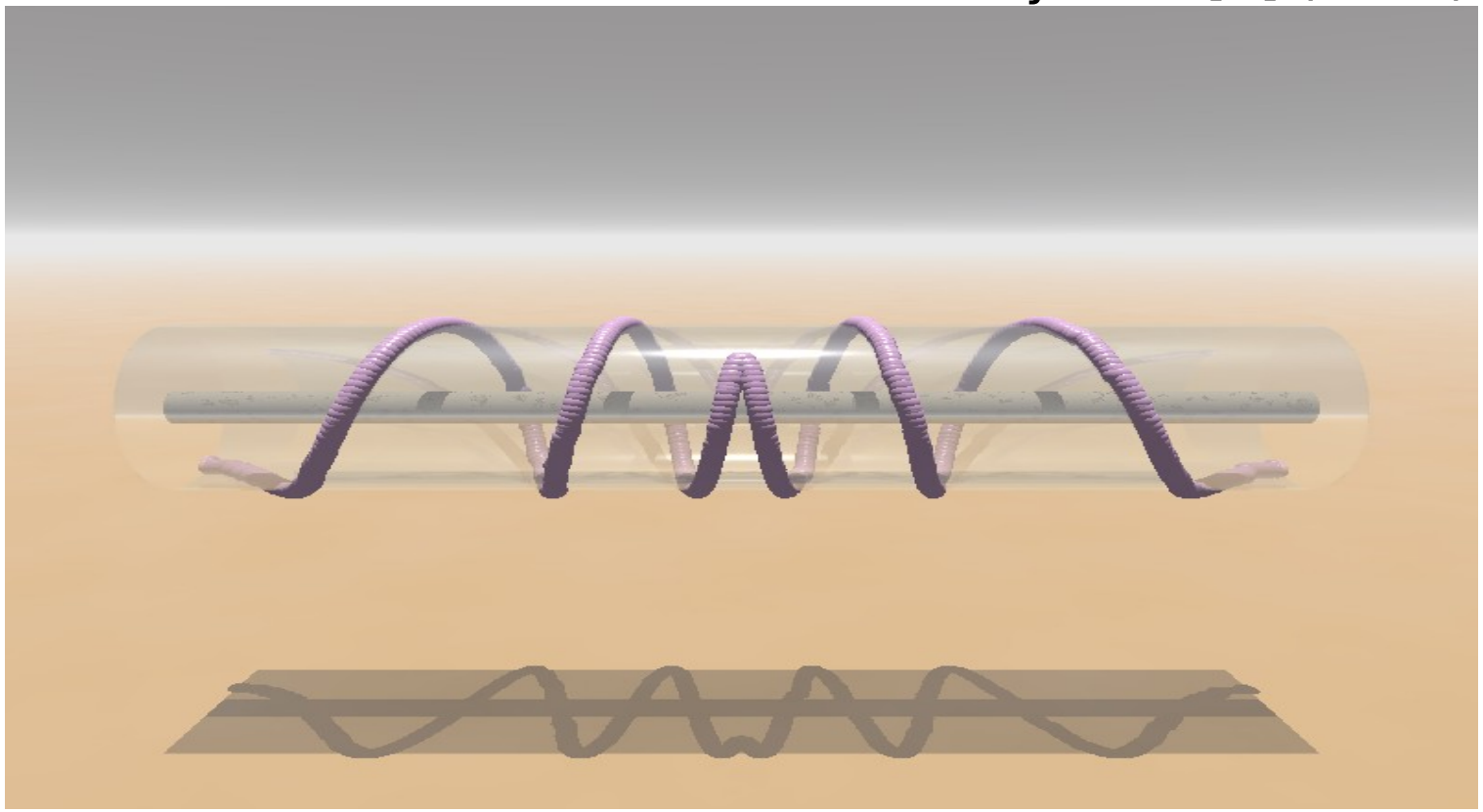


Figure by [3]

Result

- **Processing using a wrapped Kalman Smoother**
- Kalman smoother based on Kalman filter by Traa [3] (2013)



Conclusion & Outlook

- Several approaches to phase estimation tested
- Most promising approach based on two-step evaluation.
- However: Experimental improvements indicated
- Expansive computations can be parallelized
- Next: Extensive testing on shot-database
- Open/To do:
  - pdf of cross-correlation for shifted signals with known auto-correlation function
  - data fusion with CO-interferometer

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Thank you

Acknowledgement & References

- Data, experimental insights and problem setting have been provided by Alexander Mlynek (IPP). Thanks!

- References:

[1] G. Bertschinger et al. Rev.Sci. Instr. 79, 10E709 (2008)

[2] L. Bretthorst, Bayesian Spectrum Estimation, Springer (1988)

[3] J. Traa, IEEE Signal Processing 20(12), p. 1257 (2013) + 2 figures from presentation

[4] A. Mlynek et al, Rev.Sci. Instr. 85, 11D408 (2014)