Optimum Operating Conditions for Two-Phase Flows in Pore Networks: Conceptual /Numerical Justification Based on the MEP principle (43)

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Abstract

The mechanistic model DeProF considers steady-state two-phase flow in porous media as a composition of three flow patterns: connected-oil pathway flow, ganglion dynamics and drop traffic flow. The key difference is the degree of disconnection of the non-wetting phase which affects the relative magnitude of the rate of energy dissipation caused by capillary effects, compared to that caused by viscous stresses. An appropriate mesoscopic scale analysis leads to the determination of all the internal flow arrangements of the basic flow patterns that are compatible to the externally imposed flow conditions. The observed macroscopic flow is an average over the canonical ensemble of the flow arrangements.

Extensive DeProF simulations revealed that there exist a continuous line [a locus, r*(Ca)] in the domain of the process operational variables -the capillary number, Ca, and the oil-water flowrate ratio, r- on which the efficiency of the process (oil produced per kW dissipated in pumps) attains local maxima. Such maxima have been experimentally identified.

Subsequently, the existence of the locally optimum operating conditions could be rationally justified by the following conceptual inference. Steady state two-phase flow in porous media is an off-equilibrium process. The rate of global entropy production (a measure of the process spontaneity) is the sum of two components: the rate of mechanical energy dissipation at constant temperature (thermal entropy), Q/T, and a Boltzmann-type statistical-entropy production component, kDeProF lnΦ, directly related to the number of different physically admissible internal flow arrangements, Φ, associated with every flow condition (configurational entropy). By applying the MEP principle we may infer that optimum operation of the process is met on a locus of conditions whereby the process total entropy production rate takes maximum values.

To reduce the falsifiability of that inference, one needs to provide numerical evidence. To do so, it is necessary to deliver: (a) an efficient analytical/numerical scheme, to evaluate the number Φ of the different flow arrangements; (b) an expression for the constant kDeProF in the Boltzmann-entropy expression.

Combinatorial considerations provided the analytical background to evaluate the number of different micro-arrangements of the flow per physically admissible solution. The limiting procedure based on Stirling’s approximation has been applied to downscale the excessively large computational effort associated with the numerical handling of operations between large factorial numbers. Still, an appropriate application of the Boltzmann principle needs to be implemented, to deliver an expression for the constant kDeProF pertaining to the sought process.
Scope

Immiscible, two-phase flow in porous media is the core process in many industrial applications (recovery of hydrocarbons, soil remediation, protection of aquifers, etc.)

A variety of phenomena take place across scales from below the pore scale to the fracture or field scale (hierarchical system).

An inherent characteristic of ss2φfpm is the existence of optimum operating conditions (OOC) in terms of efficiency.

Scope is to provide a theoretical justification -of the existence of OOC- based on statistical thermodynamics, i.e. the maximum entropy production (MEP) principle.

Outline

The examined process
Phenomenology & essentials

The DeProF model essentials (in brief)
True-to-mechanism modeling essentials

Predictions of the DeProF model for SS 2φ-f-pm
dp/dz, Rel-Perm, intrinsic flow arrangement /variables

Current issues, Open Problems, Ideas & Challenges in 2φfpm
Retrospective examination of rel-perm. diagrams
Universal Operational Efficiency Map for SS2φfpm Processes
Normative Characterization of 2φ flows in pm
Justification of phenomenology based on Statistical Thermodynamics Principles
Implementation of different rheology (compressible flows, emulsions, USS, other)
Technical Applications (reconsideration of API/RP40 standard, field geometries etc.)
The examined process:

**Immiscible, stationary (a.k.a. “steady-state”) two-phase flow in porous media (or networks)**

Applications

- Oil & H/C industry (upstream/downstream),
- soil remediation processes,
- Reactors, PEM fuel cells etc.
- Phenomenology & essentials

`tbd`
Enhanced Oil Recovery (EOR)
Secondary & Tertiary oil displacement in reservoirs to recover trapped oil (~50% of original oil in place)
Use of displacing media: CO₂, water + liquid polymers (and combinations), nitrogen, foams, in-place combustion gas etc.

Applications of 2φFPM

Soil remediation

Problem

Remedy

Typical DNAPL migration processes
[from Kamon et al. Engineering Geology 76 (2003)]

In-situ soil flushing process
[from Khan et al. J Environ Management 71 (2004)]
**Statement of the SS2φFPM Problem**

Stationary Two-phase Flow in Porous Media

**Flow Regimes during Steady-State Two-Phase Flow in Porous Media. Experimental Study (1).**

(Avraam & Payatakes, JFM, 293, 207-236, 1995)

- Large Ganglion Dynamics (LGD)
- Small Ganglion Dynamics (SGD)
- Drop Traffic Flow (DTF)
- Connected Pathway Flow (CPF)
The **DeProF** model essentials

Evolution of **DeProF** theory for steady-state 2phase flow in porous media (SS2φFPM)

Time- & scale-wise evolution of research leading to the development of the **DeProF** theory

| Time period | Theory/macro-model | Mechanistic model | Network simulation | Coalescence
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### Experimental works

- Motion of solitary oil ganglia in p.m. (Shalaby et al., 1987)
- S2φF flow in planar & non-planar models (Arana et al., 1994)
- Flow regimes & coalescence (S2φFPM) (Arana et al., 1995)

### Pore dynamics

- Network models for 2φFPM (Pond et al., 1989)
- S2φFPM based on pore dynamics (Arana et al., 1994)
- Pore-to-mesoscopic scales

### Mesoscopic scales

- Conceptual justification of the existence of OOC
- From mesoscopic to statistical thermodynamics scales

### Predictions of optimum operating conditions (OOC)

- SS2φFPM
- Valavanides & Payatakes, 2003
- Pore-to-macroscopic scales

### Reveal of latest experimental evidence on the existence of OOC

- For SS2φFPM
- Valavanides, 2013

#### Study scales

- Microscale
- Mesoscopic
- Macroscopic
- Statistical thermodynamics scales

#### Pore scales

- Network/core
- Field

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**Theoretical and semi-analytical models**
**DeProF**

**A True-to-mechanism Theoretical Model**
For Stationary Two-Phase Flow in Porous Media (SS2pFPM)

Modeling essentials
- Decomposition into 3 Prototype Flows: Connected-oil Pathway Flow & Disconnected Oil Flow = (Ganglion Dynamics + Drop Traffic Flow)
- Physicochemical characteristics of oil/water/p.m.
- Dynamic wettability (contact angles)
- Mobilization and stranding probabilities for disconnected oil
- Accounting of unit cell Conductivities for all flow configurations
- Implementation of Effective medium theory
- Hierarchical modeling / Scale-up pore-to-"core"-to-field scales
- Physically admissible solutions & ergodicity principles

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**The “DeProF” model**

\[
\begin{align*}
\text{Process} & \quad \gamma_i, \ \xi_{pm} \\
\text{(Pore network)} & \quad \mu_{w}, \mu_{o} \gamma_{ow}, \theta_{A}, \theta_{R} \\
\text{(Oil & water)} & \quad \hat{q}^o, \ \hat{q}^w \\
\text{("Pumps")}&
\end{align*}
\]

\[
\text{System param.} \quad \xi_{pm}, \ \kappa, \ \theta_{A}, \ \theta_{R}
\]

\[
\text{Operational param.} \quad \gamma, \ \theta, \ Ca_r
\]

\[
\text{DeProF} \quad \text{mech/stic model algorithm}
\]

\[
\text{RESULT} \quad \text{The macroscopic rheological state equation:}
\]

\[
\begin{align*}
x & = \xi \left( \gamma, \ \theta, \ \theta_{A}, \ \theta_{R}, \ \xi_{pm} \right)
\end{align*}
\]

Interstitial physical characteristics of SS 2p flow in pm
- \( S^\nu, \ \beta, \ \alpha \) Flow arrangement variables (FAV)
- \( n^{o/w}, n^{o/o} \) Oil flow rates in CPF & DOF (GD)
- \( U^{o/w/o} \) Flowrate of o/w interfaces
- \( \phi_{ow} \) Coefficient of oil fragmentation,
- \( \phi_{o/o} \) Flowrate of o/w interface through DTF
- \( \omega \) Ganglion size distribution
- \( \eta_r \) Energy utilization factor

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MaxEnt 2014, Amboise, France 92514 M.S. Valavanides "Optimum Operating Conds. for 2ph Flow in Pore Network – MEP Justification" 13 / 30
Decomposition into Prototype Flows

“DeProF”

Externally imposed system parameters: \( \{ C_A, r, \kappa, \theta_A^0, \theta_R^0, \phi_{mu} \} \)

The following variables are introduced:

Flow Arrangement Variables (FAV): \( \{ S^*, \beta, \alpha \} \)

Prototype Flow Variables: \( U = \{ U^{\text{CPF}}, U^{\text{GD&DTF}}, U^{\text{DOF}} \} \) \( S = \{ S^{\text{CPF}}, S^{\text{D}}, S^{\text{G}} \} \)

Prototype Flow Interlocking Condition:

\[
\begin{align*}
\frac{\partial \rho}{\partial x} &= \frac{\partial \rho}{\partial y} = \frac{\partial \rho}{\partial z} = \frac{\sigma}{\rho} \frac{C_A}{k} \chi
\end{align*}
\]

DOF modelling - microscopic scale (1)

GD domain reduced cell-conductances:

\[
\begin{align*}
A_{jk}^b &= \frac{\tilde{q}^G_{jk} \mu_{jk}}{\Delta P_{jk}} \left( A_{jk}^b \frac{1}{C_A} \frac{1}{\chi} u_i(x) \right) \quad \text{GT domain} \\
B_{jk}^b &= \frac{1}{\Delta P_{jk}} \left( B_{jk}^b \frac{1}{C_A \chi} u_i(x) \right)
\end{align*}
\]

b : “G”, “E,G” or “X,G”

\( A_{jk}^b \), \( B_{jk}^b \) : reduced effects of bulk phases & interphases

\( u_i(x) \) : reduced ganglion velocity, \( \chi \) : tortuosity of ganglion spine

DTF domain reduced cell-conductances:

\[
\begin{align*}
A_{jk}^D &= \frac{\rho D_{jk}^D}{\rho \Delta P_{jk}} \left( A_{jk}^D \frac{1}{U} \rho \right) \\
B_{jk}^D &= \frac{\sigma m^D}{\rho \Delta P_{jk}} \left( B_{jk}^D \frac{1}{U} \rho \right)
\end{align*}
\]

\( m^D \) : reduced cell-conductances

\( \rho \) : density

\( U \) : velocity

\( \rho \) : density

\( \Delta P_{jk} \) : pressure drop
**DeProf equations**

8 equations for \( I_{\text{max}} + 6 \) unknowns

### Macroscopic scale

- \( u^{\text{CPF}} = \frac{1}{Kn} \) \{ Darcy’s law in CPF \} (1)
- \( (1-\beta)U^{\text{DFG}} = 1 \) \{ Total water mass balance \} (2)
- \( \beta U^{\text{CPF}} + (1-\beta)U^{\text{DFG}} = 1 \) \{ Total oil mass balance \} (3)
- \( \beta \left[ \beta \left[ \frac{1}{2} \text{CPF} + (1-\beta) \text{DFG} \right] + (1-\beta) \text{DFG} \right] = 1 - S^w \) \{ Total oil (or water) mass arrangement condition \} (4)

### Macro-micro scale consistency

- \( \sum_{i=1}^{N} n_i^w n_i^G = 2 N_d \) \{ Oil ganglion volume normalization condition \} (5)
- \( \frac{1}{2} \sum_{i=1}^{N} n_i^G V_i^G = S_i^G \) \{ GD oil saturation in terms of ganglion volume distribution \} (6)
- \( \sum_{i=1}^{N} n_i^w m_i^w n_i^G = 2rU^{\text{DFG}} \) \{ Oil ganglion mass balance in DOF \} (7)

### Effective Medium Theory in DOF

- \( \sum_{i,j} r_i^p \sum_{s} \left[ \frac{k_i(x_i+r_i)-k_i(x_i)}{k_i(x_i+r_i)-k_i(x_i)} \frac{1}{x_i(x_i+r_i)} \right] \) \{ EMT equation for "Equivalent Water" \} (8)

- \( \sum_{i,j} r_i^p \sum_{s} \left[ \frac{k_i(x_i+r_i)-k_i(x_i)}{k_i(x_i+r_i)-k_i(x_i)} \frac{1}{x_i(x_i+r_i)} \right] \) \{ Flow in DOF \} (8)
A sharply decreasing ganglion size distribution is imposed based on experimental arguments\cite{1} & theoretical observations\cite{2,3}.

\[
\begin{align*}
\max_{i} & \quad n_i^G \kappa^{(i-2)}, \quad 3 \leq i \leq I_{\text{max}} \\
0 & \quad , \quad i > I_{\text{max}} 
\end{align*}
\]

\(n_i^G\): variable \(i=0,1\)

\(n_i^C\): variable \(i=0,1\)

\(I_{\text{max}}\): such that \(\Pr(\text{mobilization } i \geq I_{\text{max}}) = 1\)

\(z\): exponent that scans all admissible values in \((0,1)\)

The DeProF model predictions following simulations for 2ph flow in pore networks
The pore networks used in simulations

2D

3D

ganglion tortuosity, \( \gamma_{G}^{3D} = \sqrt{2} \)

\( \gamma_{G}^{2D} = \gamma_{G}^{3D} \rightarrow k_{2D} = k_{3D} \)

ganglion tortuosity, \( \gamma_{G}^{3D} = \sin^{-1} \left( \frac{\sqrt{2}}{2} \right) = 1.732 \)

The Domain of Physically Admissible Solutions (PAS)
(in DeProF theory)

For each set \((\text{Ca}, \tau)\) of system parameter values the 2p flow visits a continuum of physically admissible flow configurations represented by \((S^*, \beta, \omega)\) – the PAS domain (cloud of red balls).

The PAS domain is a canonical ensemble

A unique set of values for \(S^*, \beta\) and \(\omega\) (black balls) are obtained by averaging over the PAS domain.

The volume of the PAS domain (red cloud) is a measure

• of the process number of degrees of freedom and

• of the process contribution to configurational entropy.
Effect of $r=q^*/q^*$
on the PAS domain
(degrees of freedom)

For any fixed value of $C_a$ and as $r$ gradually increases, the PAS domain progressively swells, extends to a max. size and shrinks to zero (just as 2-ph flow cease to be sustainable).

When the PAS domain attains a max. volume, the 2ph flow is as rich as possible in different flow configurations.

A systematic behavior is observed for the other o/w systems examined ($n=0.66$ & 3.35).

**Benchmark: DeProf model (-----) vs experiment (▼)**

Reduced Mechanical Power Dissipation, $W$

Effect of $C_a=\mu_w U_w/\gamma_{ow}$
& $\kappa=\mu_h/\mu_w$

No adjustable parameters
No interpolation

Reduced mechanical power dissipation of the total flow

\[ W = \frac{\tilde{W}}{\tilde{W}_{1\Phi}} = \frac{\tilde{W}k_\mu}{(\gamma_{\text{ow}} \cdot Ca)^2} \]

Reduced pressure gradient

\[ x = \frac{\tilde{\epsilon}}{\epsilon} \frac{k}{\gamma_{\text{ow}} \cdot Ca} \]

Flow arrangement variables (FAV)

Oil saturation, \( S^* \)

Connected-oil saturation, \( \beta \)

“ganglia saturation” of the DOF cells, \( \omega \)

(pm volume fraction occupied by the connected oil)

(ganglion cells over all the DOF region cells)
An important result on SS2φFPM efficiency (efficiency = lt of oil per kWh spent) (as indicated by the DeProF model)
**Energy utilization factor** ($f_{EU} = r/W$)

& **Optimum Operating Conditions (OOC)**

\[ f_{EU} = \frac{\text{flow rate of oil}}{\text{mechanical power supplied to the system}} \]

For any $Ca=\text{const}$, Locus $r^*(Ca)$:

\[ f_{EU}(Ca, r^*) = \max[f_{EU}(Ca, r)] \]

$r^*(Ca) \rightarrow \text{Optimum Operating Conditions (OOC)}$
In what way does process **efficiency** correlates to the process **degrees of freedom**?
Optimum Operating Conds in SS2φFPM

Q: Are they just a theoretical artifact?  
("contrived" by the DeProF model – a “self-fulfilled prophecy")

A1: OOC are latent in $k_{ij}(S_w)$ diagrams  
look up at experimental works! - latent information!  
$A_1 \rightarrow$ Valavanides, Totaj, ImproDeProF Internal project report

A2: OOC are justifiable on statistical thermodynamics (the aSaPP concept)  
$A_2 \rightarrow$ Valavanides, SPE135429 ATCE2010 (2010),  
Daras & Valavanides, MaxEnt2014

OOC in ss2φfpm:  
Experimental Verification  
(Reveal of latent experimental evidence)
Retrospective examination of rel-perm diagrams

- Le, H.Y., Mungan, N. (1973) SPE 4505 Dallas, Texas
- Maloney D., Doggett K., Brinkmeyer, A. (1993) NPER 648

Details of retrospective rel-perm study users.teiath.gr/marval/ArchIII/retrorelerperm.pdf
Free download the SS Rel-Perm Data Transformer users.teiath.gr/marval/ArchIII/relpermutrans.xls
Your lab study is not included? Join the effort in building a rel-perm Data Base! —> marval@teiath.gr
Transformation of relative permeability data reveals operational efficiency aspects of SS rel.perm. diagrams

In stationary conds, oil/water flowrate ratio \((= \lambda, \text{ the mobility ratio})\)

\[
r = \frac{\bar{q}_o}{\bar{q}_w} = \frac{\bar{U}_o}{\bar{U}_w} = \frac{k_{ro}}{k_{rw}} \frac{\bar{\mu}_o}{\bar{\mu}_w} = \frac{1}{\kappa} \frac{k_{ro}}{k_{rw}} \quad (1)
\]

Energy utilization factor ("oil flowrate per kW spent")

\[
f_{EU} = \frac{r}{W} = k_{ro} \frac{1}{\kappa(r + 1)} = k_{rw} \frac{r}{r + 1} = k_{ro} \left( \frac{k_{ro}}{k_{rw} + \kappa} \right)^{-1} \quad (2)
\]

where

\[
k = \frac{\mu_o}{\mu_w} : \text{oil/water viscosity ratio}
\]

\[
W = \bar{W}_k \bar{\mu}_w \left( \frac{1}{\kappa \sigma_{ro}} \right)^2 : \text{reduced mech. power dissipation} \text{ (over equiv. 1ph flow)}
\]

Demarcation of ss2φfpm operational efficiency (asymptotic limits)

As \(Ca \rightarrow \infty \ldots \) the capillary effects <<<< viscous effects, therefore

\[
OE_{\infty} = \frac{\text{Oil flowrate}}{\text{mechanical power}} = \frac{\bar{q}_o}{(\bar{W}_o + \bar{W}_w)\bar{\Lambda} \bar{\Delta} Z} = \frac{(1 - S_{w,\infty})S_{w,\infty}}{(1 - S_{w,\infty} + \kappa S_{w,\infty}) Ca \bar{\gamma}_{ro}} \quad \frac{1}{\bar{\Delta} Z} 
\]

\[
\left. \frac{d(OE_{\infty})}{dS_{w,\infty}} \right|_{S_{w,\infty}} = 0 \quad (\kappa - 1)S_{w,\infty}^2 + 2S_{w,\infty} - 1 = 0 \quad \Rightarrow \quad S_{w,\infty} = \frac{1}{1 + \sqrt{\kappa}}
\]

Therefore,

\[
r_* = \frac{1}{\kappa} \frac{k_{ro}}{k_{rw}} = \frac{1}{\kappa} \frac{S_{w,\infty}}{1 + \sqrt{\kappa}} \quad \Rightarrow \quad r^{*}_{\infty} = \frac{1}{\sqrt{\kappa}}
\]

and

\[
f_{EU,\infty} = \left( \frac{1 + \sqrt{\kappa}}{1} \right)^2
\]
Typical transformation of rel-perm diagrams

\[ \kappa = \frac{C \cdot \log d}{\kappa_1} \]


Universal trend of rel-perm diagrams
DeProF predictions (mechanistic modeling) +
Latent experimental evidence (phenomenology) +
Fractional flow analysis & efficiency demarcation

Universal Operational Efficiency Map
Open Problems

Open Problem 1: Flow characterization

\[
\log r^* = -\log \sqrt{\kappa} + \frac{A}{C_a^{B}}, \quad A, B > 0
\]

For \( \kappa = 1 \), \( A = 0 \), \( r^* = 1 \)

Implement determine:
1. \( \kappa = 1 \rightarrow B \)
2. \( \kappa \neq 1 \rightarrow A \)
3. \( 1/\kappa \rightarrow \text{verify } A \)
Open Problem 2
Detection of the locus \( r^*(Ca) \)
of Optimum Operating Conditions

Determination of local max \( f_{EU} \)
\[
\frac{\partial f_{EU}}{\partial r} \bigg|_{r=r^*}=0 \implies k_{10}^* = C_o(Ca)(r^*+1)
\]
\[
\frac{\partial f_{EU}}{\partial r} \bigg|_{r=r^*} = 0 \implies k_{rw}^* = C_w(Ca)(r^*+1)/r^*
\]

Integration “constants”, are functions of \( Ca \), related as
\[
C_o(Ca) = \kappa C_w(Ca)
\]

To detect OptOpConds [locus \( r^*(Ca) \) of max \( f_{EU} \)],
we need to resolve the functional form \( C_o(Ca) \) [or \( C_w(Ca) \)]

Open Problem 3

OOC in ss2φfpm
Justification on the basis of statistical thermodynamics
(maximum entropy production principle)

Valavanides, SPE135429, ATCE 2010
### Statistical Thermodynamics aspects of ss2φfpm efficiency

**ss2φfpm is a stationary, off equilibrium process**

Provide energy to keep it stationary at fixed operating conditions

The efficiency of a stationary process in dynamic equilibrium, is proportional to its spontaneity*

\[
\text{process efficiency} |_{\text{op constr}} \uparrow \Rightarrow \text{process spontaneity} \uparrow \text{ or } \text{irreversibility} \downarrow
\]

\[
\text{(spontaneity)} = \text{irreversibility}^{-1} = \text{entropy produced globally}
\]

The physical domains in which ss2φfpm takes place (to account for entropy production)

- **{System}**: the porous medium and the two fluids
- **{Surroundings}**: the heat reservoir in which the {System} resides at constant temperature
- **{Universe}**: {System} + {Surroundings}


### Identification of Sources of Entropy

In the sought process (immiscible two-phase flow in porous media), entropy is present in different scales:

**Molecular level** (bulk & interfaces) $\Rightarrow$ **thermal entropy** $\Rightarrow$ \(Q/T\)

\(Q\): energy released to the environment (or “dissipated” as “heat”) at temperature \(T\)

**Core/Field - level** $\Rightarrow$ **configurational entropy** $\Rightarrow$ \(k \ln W\)

\(W\): number of microstates freely & equiprobably attained

\(k\): Boltzmann-type constant for the particular process
Identification of two complementary domains:
1) frontal area perpendicular to macroscopic flow for the connected-oil pathway flow (CPF) / “balls-in-boxes” problem
2) reference volume for the disconnected-oil flow (GD+DTF) / “chains-in-barbs” problem

\[ \text{Configurational Entropy (1)} \]
Identification of Microstates

\[ \text{Estimation of Number of Microstates} \]
Combinatorics

\[ P' = P_{\text{COP}} \times P_{\text{DOF}} = P_{\text{COP}} \times (P_{\text{DOF1}} \times P_{\text{DOF2}}) = \frac{K_{\text{CP}}!}{N_{\text{COP}}!(K_{\text{CP}} - N_{\text{COP}})!} \times \left( \frac{(N_{\text{DTF}} + N_{\text{C}})!}{N_{\text{DTF}}!N_{\text{C}}!} \times \prod_{i=1}^{\text{max}} (N_{i}!) \right) \]

\[ N_{\text{COP}}, \text{balls—in—K}_{\text{CP}} \text{boxes} \quad N_{\text{i}} \text{chains—in—(N}_{\text{DTF}} + N_{\text{C}}) \text{barbs} \]

Configurational Entropy (3)

Estimation of Number of Microstates

Application of Stirling’s approximation limiting procedure

\[ P' = P_{\text{COP}} \times P_{\text{COG}} = \frac{K_{\text{CP}}!}{N_{\text{COP}}!(K_{\text{CP}} - N_{\text{COP}})!} \times \frac{(N_{\text{DTF}} + N_c)!}{N_{\text{DTF}}!} \prod_{i=1}^{N_{\text{W}}} N_i! \]

\[ \ln P' = \ln(K_{\text{CP}}!) - \ln(N_{\text{COP}}!) - \ln((K_{\text{CP}} - N_{\text{COP}})!) \]

\[ + \ln([N_{\text{DTF}} + N_c]! - \ln(N_{\text{DTF}}!) - \sum_{j=1}^{\text{max}} \ln(N_j!) \]

\[ \ln(n!) \geq n \ln(n) - n \]

\[ \ln P' = K_{\text{CP}} \ln K_{\text{CP}} - N_{\text{COP}} \ln N_{\text{COP}} - (K_{\text{CP}} - N_{\text{COP}}) \ln(K_{\text{CP}} - N_{\text{COP}}) \]

\[ + (N_{\text{DTF}} + N_c) \ln(N_{\text{DTF}} + N_c) - N_{\text{DTF}} \ln N_{\text{DTF}} - \sum_{i=1}^{\text{max}} N_i \ln(N_i) \]

Configurational Entropy (4)

Separation between extensive & intensive contributions

\[ \ln P' = K_{\text{CP}} \left[ -\beta \ln \beta - (1-\beta) \ln(1-\beta) \right] \]

\[ + M(1-\beta) \left[ -(1-\omega) \ln(1-\omega) - \omega \ln \omega - \sum_{i=1}^{\text{max}} n_i^G \ln n_i^G \right] \]

If no CPF (\( \beta=0 \))

\[ \ln P' = M(1-\beta) \left[ -(1-\omega) \ln(1-\omega) - \omega \ln \omega - \sum_{i=1}^{\text{max}} n_i^G \ln n_i^G \right] \]

And if disconnected oil in singlets

\[ \ln P' = M \left[ -(1-\omega) \ln(1-\omega) - \omega \ln \omega \right] \]
**Configurational Entropy (5)**

Estimation of Boltzmann-type constant pending problem!

\[ S_{SYS} = k_{\text{DeProF}} \ln P' \]

\( k_{\text{DeProF}} \): Boltzmann’s constant for the particular process

---

**The aSaPP concept (MEP)**

\[ \frac{1}{T} + k_D \times \ln(A_o N_{PAS}) = C_f \times \ln(W) \]

\( S_{SUR} \): Entropy released to the Surroundings

\( S_{SYS} \): Entropy produced within the System

\( S_{UNIV} \): Total Entropy produced in the Universe

\( W \): reduced rate of mechanical energy dissipation = heat dumped to the surroundings

\( T \): absolute temperature

\( k_D \): bridge from meso-to-macroscopic physics (similar to Boltzmann’s const) - not yet estimated!

\( N_{PAS} \): number of physically admissible solutions (internal flow arrangements at mesoscopic scale)

\( A_o \): correlation factor between number of DeProF estimated \((N_{PAS})\) and actual number of flow arrangements

\( f_{EU} \): Energy utilization factor (oil flow rate per kW of power dissipated in pumps)

\( C_f \): correlation coefficient
CONCLUSIONS

➢ Two-phase flow in p.m. is “burdened”:

• with oil disconnection and capillarity effects that restrain or inhibit -to a certain extent- the superficial transport of o & w
• the bulk phase viscosities of oil & water.

➢ Process engineers can take advantage of the natural intrinsic characteristics of 2φ flow in p.m., namely the multitude of internal flows that act as -potentially beneficial- degrees of freedom against the imposed macroscopic constraints.

➢ Process engineers must always judge where to set the balance between capillarity or viscosity (order of magnitude benefit in process efficiency)

CONCLUSIONS (cont.)

Metaphorically speaking,

the process designer may trade with the “Daemon” (a.k.a. Nature) -avid for chaos in any form, an amount of configurational chaos (created from the multitude of intrinsic flow arrangements) in exchange for microscopic chaos (dissipating mechanical energy into heat).
The DeProf theory for steady state two-phase flow in porous media:

- evolved from coordinated research efforts (implementing experimental/empirical observations, true-to-mechanism hierarchical modeling & numerical simulations at multiple scales and physical interpretation & experimental verification of simulation predictions)

- is consistent with the pre-existing theory based on Darcy’s fractional flow formulation (nevertheless, it shows conventional wisdom to be not precisely correct, e.g. water saturation is not an independent variable in the description of steady-state two-phase flow in porous media)

- shows remarkable specificity (model predictions are consistent with available empirical knowledge; the modelling provision for a plurality of physically admissible mesoscopic flow configurations, appropriately supported by ergodicity considerations, explains the experimental observation of intrinsically unsteady but time averaged steady-state flow regimes)

- is tentative and dynamic in allowing for changes as new facts are discovered

Suggestions

Project Management:
Optimization of project /portfolio scheduling
(Valavanides, 2014, Procedia-SBS 119)

Transport:
Efficiency optimization of multi modal transport systems
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Thank you!

Hoping, I didn’t shoot the messenger!

Dewar, R. (2009) “Maximum Entropy Production as an Inference Algorithm that Translates Physical Assumptions into Macroscopic Predictions: Don’t Shoot the Messenger” Entropy 11