Entropic Dynamics:  
from Entropy and Information Geometry  
to Quantum Mechanics

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MaxEnt 2014, Amboise
J. A. Wheeler (1983):

**Law without Law:**
“The only thing harder to understand than a law of statistical origin would be a law that is not of statistical origin, for then there would be no way for it—or its progenitor principles—to come into being.”

**Two tests:**
“No test of these views looks like being someday doable, nor more interesting and more instructive, than a derivation of the structure of quantum theory...”
“No prediction lends itself to a more critical test than this, that every law of physics, pushed to the extreme, will be found statistical and approximate, not mathematically perfect and precise.”
The subject matter:

The goal is to predict the positions of particles $x$.

(Or other configurational variables, such as fields.)

Positions have definite but unknown values.
Dynamics: Change happens.

The first goal: find $P(x' | x)$.

(Config. space + probability $\rightarrow$ non-locality.)
Entropic Dynamics

Maximize the entropy

\[ S[P, Q] = -\int dx' P(x' | x) \log \frac{P(x' | x)}{Q(x' | x)} \]

Constraints:

short steps: \[ \langle \Delta x \cdot \Delta x \rangle = \kappa \]

some directionality: \[ \langle \Delta x \rangle \cdot \nabla \phi = \kappa' \]
The result:

\[
P(x' | x) = \frac{1}{\zeta} \exp\left[ -\frac{1}{2} \alpha \Delta x^2 + \Delta \Delta \nabla \phi \right]
\]

Displacement: \[ \Delta x = \langle \Delta x \rangle + \Delta w \]

Expected drift: \[ \langle \Delta x \rangle = \frac{1}{\alpha} \nabla \phi \leftarrow O(\alpha^{-1}) \]

Fluctuations: \[ \langle \Delta w^a \rangle = 0 \]
\[ \langle \Delta w^a \Delta w^b \rangle = \frac{1}{\alpha} \delta^{ab} \leftarrow O(\alpha^{-1/2}) \]
Entropic Time

(1) Introduce the notion of an instant,

\[
\rho(x', t') = \int dx \, P(x' | x) \rho(x, t)
\]

(2) Instants are ordered: the Arrow of Entropic Time

(3) Define duration so that the dynamics looks simple.

Fluctuations:

\[
\alpha_n(x, t) = \frac{C_n}{\Delta t} = \frac{m_n}{\eta \Delta t}
\]

\[
\langle \Delta w_n^a \Delta w_n^b \rangle = \frac{1}{\alpha_n} \delta^{ab} = \frac{\eta}{m_n} \Delta t \delta^{ab}
\]

clocks

mass
Entropic dynamics:

\[ \rho(x', t') = \int dx \, P(x' \mid x) \rho(x, t) \]

Fokker-Planck equation: \[ \partial_t \rho = -\nabla \cdot (\rho \, v) \]

\[ m v = \nabla \Phi \]

\[ \frac{\Phi}{\eta} = \phi - \log \rho^{1/2}(x, t) \]

Problem: this is just standard diffusion, not QM!
Solution: allow $\phi$ to be dynamic.

$\phi(x, t)$

$\phi$ dynamics?
ϕ dynamics?

Define $H[\rho, \Phi]$ so that

$$\frac{\delta H}{\delta \Phi} = \partial_t \rho = -\frac{1}{m} \nabla \cdot (\rho \nabla \Phi)$$

$$H[\rho, \Phi] = \int d^3x \ rho \frac{1}{2m} \nabla \Phi \cdot \nabla \Phi + F[\rho]$$
Impose conservation of $H$:

$$\frac{dH[\rho, \Phi]}{dt} = \int dx \left[ \frac{\delta H}{\delta \Phi} \partial_t \Phi + \frac{\delta H}{\delta \rho} \partial_t \rho \right] = 0$$

$$= \partial_t \rho$$

$$\frac{dH}{dt} = \int dx \left[ \partial_t \Phi + \frac{\delta H}{\delta \rho} \right] \partial_t \rho = 0$$

$$\Rightarrow \partial_t \rho = \frac{\delta H}{\delta \Phi} \quad \text{and} \quad \partial_t \Phi = -\frac{\delta H}{\delta \rho}$$

FP eq. \hspace{1cm} HJ eq.
Choosing $F[\rho]$: information geometry

$$g_{ab} = \int dx \rho(x|\theta) \frac{\partial \log \rho(x|\theta)}{\partial \theta^a} \frac{\partial \log \rho(x|\theta)}{\partial \theta^b}$$

Two tensors:

$$P(x'|x) \Rightarrow \gamma_{ab} = m\delta_{ab} \quad \text{(Jacobi)}$$

$$\rho(x) \Rightarrow I_{ab} = \int dx \frac{1}{\rho(x)} \frac{\partial \rho(x)}{\partial x^a} \frac{\partial \rho(x)}{\partial x^b} \quad \text{(Fisher)}$$

Natural choice: $F[\rho] = \xi \gamma^{ab} I_{ab} + \int dx \rho(x)V(x)$
Combine $\rho$ and $\Phi$ into $\Psi_k = \rho^{1/2} \exp(ik \frac{\Phi}{\eta})$

$$\hat{\partial}_t \Psi_k = -\frac{\eta^2}{2k^2 m} \nabla^2 \Psi_k + V \Psi_k + \left(\frac{\eta^2}{2k^2} - 4\xi\right) \frac{\nabla^2 |\Psi_k|}{m |\Psi_k|} \Psi_k$$

Natural choice: $\hat{k} = \left(\frac{\eta^2}{8\xi}\right)^{1/2}$ and set $\frac{\eta}{k} = \Box$

to get QM: $\Rightarrow \quad i\Box \partial_t \Psi = -\frac{\Box^2}{2m} \nabla^2 \Psi + V \Psi$

$\Rightarrow \quad \Psi = \rho^{1/2} \exp(i \frac{\Phi}{\Box}), \quad \xi = \frac{\Box^2}{8}$
Conclusions:

• ED derives Laws of Physics from entropic inference.

• The ED of non-dissipative diffusion leads to Hamiltonians and to Quantum Theory.

• Information geometry is crucial: it explains the configuration space metric and the quantum potential.

• Position is “real”. Other observables are not.

• The $t$ in the Laws of Physics is “entropic” time.