

Bayesian analysis of cross-prefectural production function with time varying structure in Japan

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Abstract. A cross-prefectural production function (CPPF) in Japan is constructed in a set of Bayesian models to examine the performance of Japan's post-war economy. The parameters in the model are estimated by using the procedure of a Monte Carlo filter together with the method of maximum likelihood. The estimated results are applied to regional and historical analysis of the Japanese economy.

INTRODUCTION

The state of the economy in a country varies across regions and changes over time. For researchers and policy makers, explaining the determinants of such performance is undoubtedly one of the important issues set for empirical studies. In the present paper we examine economic performances of regional economies in Japan using a methodology of Bayesian statistics.

In an empirical study of macroeconomics, there are generally two approaches to analyze the sources of economic growth. One is a growth accounting approach and the other a regression approach. As for growth accounting, the most influential paper is that of Solow (1957), while recent major research includes Young (1995), Collins and Bosworth (1996), and Hayashi and Prescott (2002). This approach supposes that the marginal productivity of each factor of production is equal to the price of the relevant factor. And then the growth rate of total factor productivity (TFP) is obtained by deducting the contribution of each productive factor from the growth rate of the output. That is, the conventional growth accounting approach regards the TFP growth rate as a residual. However, such equality between marginal productivity and the factor price may not in practice always be satisfied. On the other hand, the regression approach examines the determinants of economic growth by estimating parameters for the production function or the growth regression model. Barro (1991), Mankiw, Romer, and Weil (1992), and Fukao and Yue (2000), for example, applied this approach. However, because it often happens that there are high correlations among economic variables, the model is subject to the multicollinearity and so it is difficult to obtain stable estimates. Additionally, it is usually assumed that the regression coefficients are constants over all regions. Though in practice such parameters represent regional characteristics and the values may vary from region to region. Furthermore, a common problem for the two traditional approaches is

that TFP trends can not be estimated very well.

The problems mentioned above motivated us to construct a model of cross-prefectural production function (CPPF) in Japan, and show methods for parameter estimation using a Bayesian methodology. The features of our analysis can be summarized as follows: (1) The model has parameters that vary from region to region so that regional characteristics of economic growth can be expressed by their values. (2) The TFP trend can be successfully estimated by applying Bayesian analysis using a smoothness priors approach. (3) The explanatory variables include human capital, which is usually not considered in literature regarding regional economies in Japan.¹

The rest of this paper is as follows. We construct the model in the second section, and introduce our procedure for parameter estimation in the third section. Then, the main results are given for regional analysis of the Japanese economy. Finally, we present our conclusions.

MODEL CONSTRUCTION

The CPPF model

We begin by describing the CPPF model. In the following discussion, the symbol t denotes a time variable. Consider the prefecture i 's economy that, given the state of technological knowledge $A_i^*(t)$, uses the physical capital in private sector (hereinafter private capital) $K_i(t)$, the physical capital in public sector (hereinafter public capital) $G_i(t)$, human capital $H_i(t)$ and labor $L_i(t)$ to produce output $Q_i(t)$ at time t . The production function for any prefecture is assumed to have Cobb-Douglas form.² Specifically, prefecture i 's production function is given by

$$Q_i(t) = K_i(t)^{\alpha_i} G_i(t)^{\beta_i} H_i(t)^{\gamma_i} [A_i^*(t)L_i(t)]^{1-\alpha_i-\beta_i-\gamma_i} \quad (i = 1, 2, \dots, m), \quad (1)$$

where m is the number of prefectures. For the parameters α_i , β_i and γ_i , it is assumed that

$$\alpha_i > 0, \beta_i > 0, \gamma_i > 0, \alpha_i + \beta_i + \gamma_i < 1. \quad (2)$$

The term $A_i^*(t)$ is also called the efficiency in labor.³ If we define $\theta_i = 1 - \alpha_i - \beta_i - \gamma_i$, then $A_i^*(t)^{\theta_i}$ denotes TFP for the prefecture i at time t . In Eq.(1), α_i , β_i , γ_i and θ_i represent the elasticity of output with respect to the each input factor of the private capital, public capital, human capital and labor, respectively. Further, we assume that $A_i^*(t)^{\theta_i} = C_i A(t)^{\theta_i}$ and $A(0) = 1$, so that $C_i = A_i^*(0)^{\theta_i}$ holds.

Under logarithmic transformation, the model in Eq.(1) is expressed as follows:

$$y_i(t) = \mu_i + \alpha_i x_{1i}(t) + \beta_i x_{2i}(t) + \gamma_i x_{3i}(t) + \theta_i a(t) \quad (i = 1, 2, \dots, m), \quad (3)$$

¹ One of the exceptions is Fukao and Yue (2000).

² The Cobb-Douglas form has the properties of the well-behaved production function in the neoclassical growth model. See Osumi (1986) for details of the neoclassical growth theory.

³ The term $A_i^*(t)L_i(t)$ is often referred to as the effective amount of labor.

where $y_i(t) = \ln\{Q_i(t)/L_i(t)\}$, $\mu_i = \ln C_i$, $a(t) = \ln A(t)$, $x_{1i}(t) = \ln\{K_i(t)/L_i(t)\}$, $x_{2i}(t) = \ln\{G_i(t)/L_i(t)\}$, $x_{3i}(t) = \ln\{H_i(t)/L_i(t)\}$. In Eq.(3), the parameters $\alpha_i, \beta_i, \gamma_i, \theta_i$ represent the technical and economic structure of the prefecture i ; thus they are called the structural parameters.

When the model is fitted to a set of practical data an error term has to be taken into account, i.e., for $t = 1, 2, \dots, n$, the model in Eq.(3) can be rewritten as follows:

$$y_i(t) = \mu_i + \alpha_i x_{1i}(t) + \beta_i x_{2i}(t) + \gamma_i x_{3i}(t) + \theta_i a(t) + \varepsilon_i(t) \quad (i = 1, 2, \dots, m). \quad (4)$$

Here, $\varepsilon_i(t)$ stands for the error term, which is regarded as a random variable with

$$\varepsilon_i(t) \sim N(0, \sigma^2) \quad (i = 1, 2, \dots, m). \quad (5)$$

Assume also that for $i \neq j$ and $t_1 \neq t_2$ $\varepsilon_i(t_1)$ and $\varepsilon_j(t_2)$ are independent of each other.

It can be seen that there are many parameters in the CPPF model. To make the structure of the model simpler, we treat σ^2 and $\theta_1, \theta_2, \dots, \theta_m$ as unknown constants. We can then estimate them by using the method of maximum likelihood. On the other hand, all other parameters are treated as random variables, so that the consistency and certainty of the estimates can be improved by using Bayesian analysis.

Setting up prior distribution

For the structural parameters, we have only the prior information from the conditions in Eq.(2). Thus, for given $0 < \theta_i < 1$, the prior distribution of $b_i = (\alpha_i, \beta_i, \gamma_i)^\top$ is constructed as follows:

$$p_i(b_i|\theta_i) = \begin{cases} \lambda_i(\theta_i)^{-1}, & b_i \in \mathcal{X}(\theta_i) \\ 0, & \text{otherwise} \end{cases} \quad (i = 1, 2, \dots, m), \quad (6)$$

where $\mathcal{X}(\theta_i) = \{x_1, x_2, x_3 | x_1 > 0, x_2 > 0, x_3 > 0, x_1 + x_2 + x_3 = 1 - \theta_i\}$, and $\lambda_i(\theta_i) = \int_{\mathcal{X}(\theta_i)} dx_1 dx_2 dx_3$. Further, because of the lack of prior information the non-informative priors are used for μ_i as

$$q(\mu_i) \propto 1, \quad -\infty < \mu_i < \infty \quad (i = 1, 2, \dots, m). \quad (7)$$

Based on the assumption that technological change has continuity and smoothness, the smoothness priors approach introduced by Kitagawa and Gersch (1996) is applied to setting up a prior distribution for $a(t)$ ($t = 1, 2, \dots, n$). Concretely, we use a first order stochastic difference equation as follows:

$$a(t) - a(t-1) = v(t), \quad v(t) \sim N(0, \tau^2) \quad (t = 1, 2, \dots, n) \quad (8)$$

with $v(t)$ standing for a stochastic disturbance. In Eq.(8), τ^2 denotes the variance of $v(t)$ that is also treated as an unknown constant. Moreover, we have $a(0) = \ln A(0) = 0$ from the assumption that $A(0) = 1$.

The following assumptions are required: (a) $v(t_1)$ and $v(t_2)$ are independent of each other for $t_1 \neq t_2$. (b) The priors in Eqs.(6), (7) and (8) can be constructed independently for any i and t . (c) $\varepsilon_i(t)$ and $v(t)$ are independent of each other for any i and t .

PARAMETER ESTIMATION

Preparation for estimation

The model in Eq.(4) together with Eq.(5) is expressed by the matrix-vector form as

$$y_i = 1_n \mu_i + X_i b_i + \theta_i a + \varepsilon_i, \quad \varepsilon_i \sim N(0_n, \sigma^2 I_n) \quad (i = 1, 2, \dots, m), \quad (9)$$

where $1_n = (1, 1, \dots, 1)^t$ and $0_n = (0, 0, \dots, 0)^t$ denote two n -dimensional vectors. X_i is an $n \times 3$ matrix with $(x_{1i}(t), x_{2i}(t), x_{3i}(t))$ being the t -th row, and the other symbols are defined respectively as $y_i = (y_i(1), y_i(2), \dots, y_i(n))^t$, $a = (a(1), a(2), \dots, a(n))^t$, $\varepsilon_i = (\varepsilon_i(1), \varepsilon_i(2), \dots, \varepsilon_i(n))^t$. Similarly, the prior in Eq.(8) can be rewritten as:

$$Da = v, \quad v \sim N(0_n, \tau^2 I_n). \quad (10)$$

Here, $v = (v(1), v(2), \dots, v(n))^t$, $D = (d_{ij})$ is an $n \times n$ matrix that the elements are defined by $d_{ij} = 1$ for $j = i$, $d_{ij} = -1$ for $j = i - 1$ and $i > 1$, and $d_{ij} = 0$ otherwise.

By using Householder transformation (see Kitagawa and Gersch, 1996), the model in Eq.(9) can be transformed into the following form:

$$\tilde{y}_i^{(1)} = c_i \mu_i + \tilde{X}_i^{(1)} b_i + \theta_i H_{i1} a + \tilde{\varepsilon}_i^{(1)} \quad (i = 1, 2, \dots, m), \quad (11)$$

$$\tilde{y}_i^{(2)} = \tilde{X}_i^{(2)} b_i + \theta_i H_{i2} a + \tilde{\varepsilon}_i^{(2)} \quad (i = 1, 2, \dots, m), \quad (12)$$

$$\tilde{y}_i^{(3)} = \theta_i H_{i3} a + \tilde{\varepsilon}_i^{(3)} \quad (i = 1, 2, \dots, m), \quad (13)$$

where $c_i = H_{i1} 1_n$, $\tilde{y}_i^{(1)} = H_{i1} y_i$, $\tilde{y}_i^{(2)} = H_{i2} y_i$, $\tilde{y}_i^{(3)} = H_{i3} y_i$, $\tilde{X}_i^{(1)} = H_{i1} X_i$, $\tilde{X}_i^{(2)} = H_{i2} X_i$, $\tilde{\varepsilon}_i^{(1)} = H_{i1} \varepsilon_i$, $\tilde{\varepsilon}_i^{(2)} = H_{i2} \varepsilon_i$, $\tilde{\varepsilon}_i^{(3)} = H_{i3} \varepsilon_i$. An $n \times n$ orthogonal matrix $H_i = [H_{i1}^t \ H_{i2}^t \ H_{i3}^t]^t$ is used here with H_{i1} , H_{i2} , and H_{i3} being $1 \times n$, $3 \times n$, and $(n - 4) \times n$ matrices, respectively. Parameter estimation can be done by utilizing the structure of the models in Eqs.(11), (12), and (13).

Process of estimation

First, the procedure proposed by Akaike (1980) is applied to estimate a , σ^2 and τ^2 . We construct a Bayesian model for a based on the models in Eqs.(13) and (10). Given a set of reasonable values of $\Theta = \{\theta_1, \theta_2, \dots, \theta_m\}$, estimates of a and σ^2 are obtained as follows (see Jiang, 1995):

$$\hat{a}(\Theta) = (W(\Theta)^t W(\Theta))^{-1} W(\Theta)^t z, \quad (14)$$

$$\hat{\sigma}^2(\Theta) = (z - W(\Theta) \hat{a}(\Theta))^t (z - W(\Theta) \hat{a}(\Theta)), \quad (15)$$

and the log-likelihood of $\eta = \sigma/\tau$ is given by

$$\ell(\eta, \Theta) = -\frac{m(n-4)}{2} (\ln\{2\pi\hat{\sigma}^2(\Theta)\} + 1) - \frac{1}{2} \ln\{\det(W(\Theta)^t W(\Theta))\} + \frac{n}{2} \ln \eta^2.$$

Here, $W(\Theta) = [\theta_1 H_{13}^t \ \theta_2 H_{23}^t \ \cdots \ \theta_m H_{m3}^t \ \widehat{\eta}(\Theta) D^t]^t$, $z = (\widetilde{y}_1^{(3)t} \ \widetilde{y}_2^{(3)t} \ \cdots \ \widetilde{y}_m^{(3)t} \ 0_n^t)^t$. So, the estimate $\widehat{\eta}(\Theta)$ of η can be obtained by maximizing $\ell(\eta, \Theta)$ numerically. Thus, the conditional density $h(a|z, \Theta)$ of a given z is obtained based on the fact that

$$a|z \sim N(\widehat{a}(\Theta), \widehat{\sigma}^2(\Theta)(W(\Theta)^t W(\Theta))^{-1}).$$

On the other hand, for a fixed i we can obtain a density $f_i^{(0)}(\widetilde{y}_i^{(2)}|a, b_i, \theta_i, \widehat{\sigma}^2(\Theta))$ for $\widetilde{y}_i^{(2)}$ based on the model in Eq.(12), then a likelihood function of b_i and θ_i is given by

$$f_i^{(1)}(\widetilde{y}_i^{(2)}|b_i, \theta_i, z, \Theta) = \int f_i^{(0)}(\widetilde{y}_i^{(2)}|a, b_i, \theta_i, \widehat{\sigma}^2(\Theta))h(a|z, \Theta)da.$$

Further, given θ_i we can also generate a set of random numbers $\{b_i^{(l)}(\theta_i); l = 1, 2, \dots, N\}$ for b_i based on the prior in Eq.(6). Then, a likelihood function of θ_i is obtained by

$$f_i^{(2)}(\widetilde{y}_i^{(2)}|\theta_i, z, \Theta) = \int f_i^{(1)}(\widetilde{y}_i^{(2)}|b_i, \theta_i, z, \Theta)p_i(b_i|\theta_i)db_i \approx \frac{1}{N} \sum_{l=1}^N f_i^{(1)}(\widetilde{y}_i^{(2)}|b_i^{(l)}(\theta_i), \theta_i, z, \Theta).$$

The estimate $\widehat{\theta}_i(\Theta)$ of θ_i is obtained by maximizing $f_i^{(2)}(\widetilde{y}_i^{(2)}|\theta_i, z, \Theta)$ numerically with respect to θ_i . Moreover, the Monte Carlo filter (Kitagawa, 1996) is applied to obtain the posterior probabilities, $r_i(b_i^{(l)}(\widehat{\theta}_i(\Theta))|\widetilde{y}_i^{(2)}, z, \Theta)$, for $\{b_i^{(l)}(\widehat{\theta}_i(\Theta)); l = 1, 2, \dots, N\}$ based on the likelihood function $f_i^{(1)}(\widetilde{y}_i^{(2)}|b_i, \widehat{\theta}_i(\Theta), z, \Theta)$. Then, we renew the values of Θ with $\{\widehat{\theta}_1(\Theta), \widehat{\theta}_2(\Theta), \dots, \widehat{\theta}_m(\Theta)\}$ and repeat the above process until the values of Θ converge to a set of values, say Θ^* . Hence, we obtain the estimates of a and σ^2 by applying $\Theta = \Theta^*$ to Eqs.(14) and (15), respectively; hence that of τ^2 is given by $\widehat{\tau}^2(\Theta^*) = \widehat{\sigma}^2(\Theta^*)/\widehat{\eta}^2(\Theta^*)$. Finally, the Monte Carlo estimate of b_i is given by

$$\widehat{b}_i = \frac{1}{N} \sum_{l=1}^N b_i^{(l)}(\widehat{\theta}_i(\Theta^*))r_i(b_i^{(l)}(\widehat{\theta}_i(\Theta^*))|\widetilde{y}_i^{(2)}, z, \Theta^*) \quad (i = 1, 2, \dots, m).$$

Incidentally, the estimates of μ_i ($i = 1, 2, \dots, m$) can be obtained based on the model in Eq.(11) and the prior in Eq.(7). The details are omitted for lack of space.

RESULTS AND ANALYSIS

The data used for the parameter estimation were obtained from the Japanese prefectural database constructed by Fukao and Yue (2000).⁴ We use private capital, public capital (industrial infrastructure), human capital and domestic employed persons by prefecture for the variables of the input factors. The output variable is the gross prefectural domestic

⁴ The data is available from the website of professor Kyoji Fukao at Hitotsubashi University, Japan: <http://www.ier.hit-u.ac.jp/~fukao/japanese/data/index.html>.

expenditure. The sample is composed of 46 Japanese prefectures, with data from 1955 to 1995. Okinawa was excluded because of data for some variables were available only from 1972.

As before noted, the sum of the elasticities α_i , β_i , γ_i and θ_i for each prefecture i is unity. The estimates of the elasticities for the 46 prefectures are indicated in Figure 1.

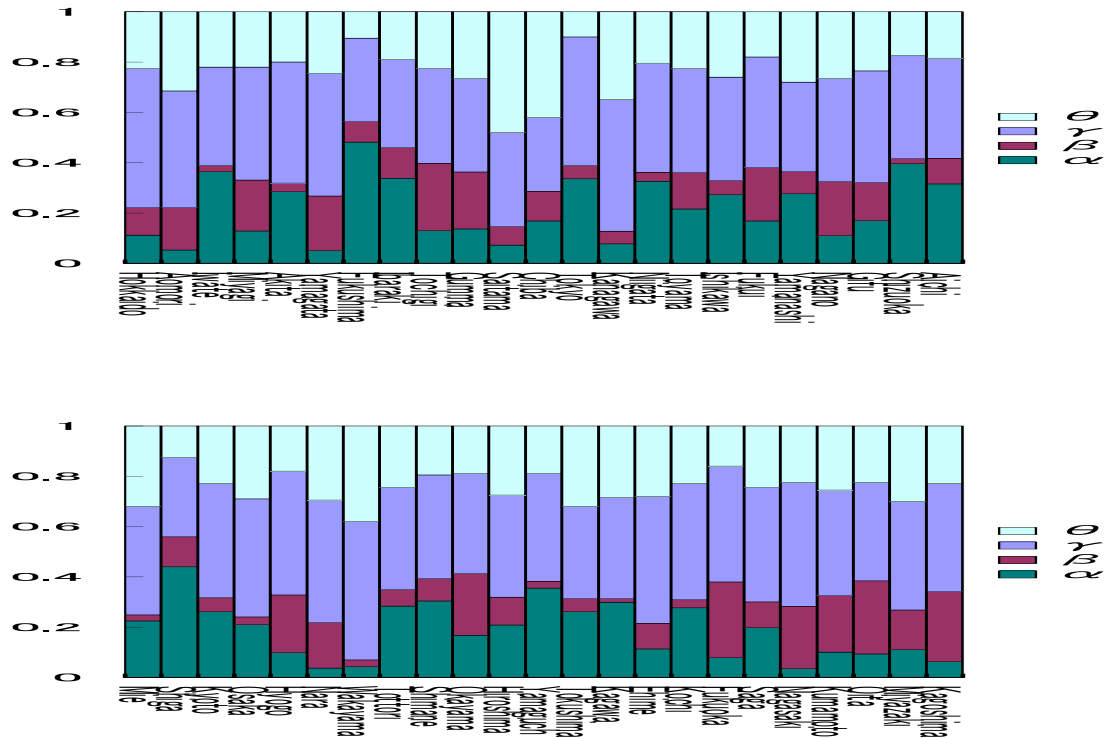


FIGURE 1. The Estimates of Elasticities for 46 Japanese Prefectures

Let $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\theta}$ be the estimates of α , β , γ and θ , respectively. The elasticity associated with each factor denotes an increase of output by percent when the relevant factor increased by 1%. Some descriptive statistics of the estimated elasticity of output with respect to each factor of input are shown in Table 1. For example, the prefectural mean regarding the estimated elasticity of output with respect to human capital is the highest value 0.429, so we find that if human capital increases by 1%, all else being equal, the output rises by 0.429 % on average for the 46 prefectures. Focusing on the variances of $\hat{\alpha}$ and $\hat{\beta}$, we find that the corresponding value of physical capital, 0.014, is twice that of the corresponding value of public capital, 0.007, for the variance of the estimated elasticity of output with respect to the relevant factor. This suggests that the prefectural disparity of public capital is relatively small compared to that of the private capital.

Next, we examine the regional characteristics. In this paper, the 46 prefectures are divided into 10 regional classifications as follows: Hokkaido, Tohoku (Aomori, Iwate, Akita, Miyagi, Yamagata, Fukushima), North Kanto (Ibaraki, Tochigi, Gunma, Yamanashi, Nagano), South Kanto (Saitama, Chiba, Tokyo, Kanagawa), Tokai (Shizuoka,

TABLE 1. Descriptive Statistics of the Elasticity

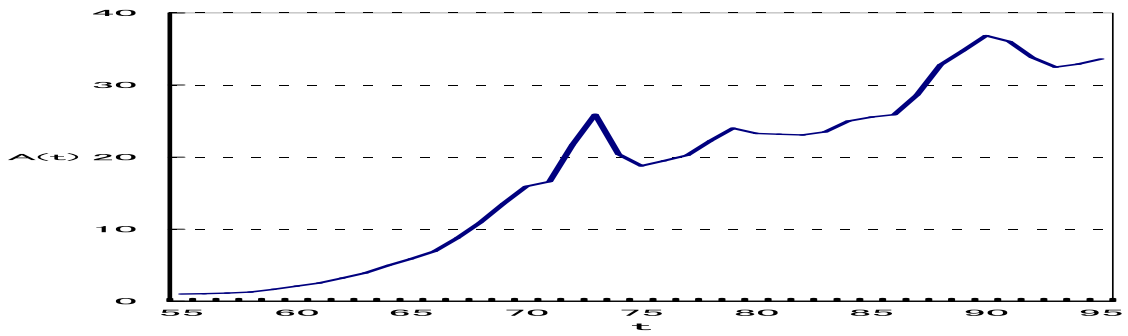
	mean	variance	max	min
$\hat{\alpha}$	0.201	0.014	0.481	0.035
$\hat{\beta}$	0.126	0.007	0.301	0.016
$\hat{\gamma}$	0.429	0.003	0.554	0.295
$\hat{\theta}$	0.244	0.005	0.480	0.100

TABLE 2. Averages of the Elasticity

	$\hat{\alpha}$	$\hat{\beta}$	$\hat{\gamma}$	$\hat{\theta}$
Hokkaido	0.112	0.110	0.554	0.225
Tohoku	0.227	0.121	0.434	0.218
North Kanto	0.198	0.184	0.373	0.245
South Kanto	0.163	0.073	0.426	0.338
Tokai	0.276	0.074	0.421	0.229
Hokuriku	0.246	0.112	0.424	0.218
Kinki	0.182	0.107	0.462	0.250
Chugoku	0.264	0.168	0.410	0.219
Shikoku	0.237	0.050	0.434	0.279
Kyushu	0.097	0.229	0.440	0.234

Gifu, Aichi, Mie), Hokuriku (Niigata, Toyama, Ishikawa, Fukui), Kinki (Shiga, Kyoto, Nara, Wakayama, Osaka, Hyogo), Chugoku (Tottori, Shimane, Okayama, Hiroshima, Yamaguchi), Shikoku (Tokushima, Kagawa, Ehime, Kochi), and Kyushu (Fukuoka, Saga, Nagasaki, Oita, Kumamoto, Miyazaki, Kagoshima). Table 2 shows the averages of $\hat{\alpha}$, $\hat{\beta}$, $\hat{\gamma}$ and $\hat{\theta}$ in each region. From Table 2, for instance, we find the following results. The Tokai region has comparatively high $\hat{\alpha}$ but it is relatively low in Kyushu. $\hat{\beta}$ is high in Kyushu but comparatively low in Shikoku. Hokkaido show relatively high $\hat{\gamma}$ but North Kanto has low in comparison. $\hat{\theta}$ is comparatively high in South Kanto, while it is comparatively low in Tohoku.

We turn now to analyzing the dynamics of estimated $A(t)$ that denotes a component part of TFP, that is, $C_i A(t)^{\theta_i}$. Let $\hat{A}(t) = \exp(\hat{a}(t))$ be estimates of $A(t)$. Figure 2 shows the trend for $\hat{A}(t)$ from 1955 to 1995. Because θ_i and C_i are not changed with time, each prefecture's TFP changes along with the movement of $A(t)$ in the same direction. Therefore, Figure 2 suggests that each prefecture's TFP contributed remarkably to its economic growth from 1955 to 1973 and from 1987 to 1991. But, that it had the negative effect on economic growth from 1974 to 1976, and from 1991 to 1993 in particular. As

**FIGURE 2.** The Estimated Trend for $\hat{A}(t)$

has been well documented, 1973 was the year of the first oil crisis; it is an interesting that there was an evident change in the trend for $\hat{A}(t)$ immediately after that event.

CONCLUSIONS

In the preceding section we examined the performance of Japan's regional economies using a Bayesian methodology. Consequently, we confirmed that there was a sharp contrast among prefectures in terms of the elasticity of output with respect to the productive factors. Additionally, we found that each prefecture's TFP showed a general upward trend between 1955 to 1995, while there was a negative effect on economic growth from 1974 to 1976 and from 1991 to 1993 in particular.

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