Nonparametric Bayesian Estimation of Censored Counter Intensity from the Indicator Data

Éric Barat[*], Thomas Dautremer[*] and Thomas Trigano[†]

[*]*CEA Saclay, DETECS, Electronics and Signal Processing Laboratory, Gif-sur-Yvette, France*
[†]*Department of Statistics, Hebrew University, Jerusalem, Israel*

Abstract. The nonparametric Bayesian estimation of non homogeneous Poisson process intensity in presence of Type-I or Type-II dead times is addressed in the framework of multiplicative intensity counting processes. In addition to the counting process, the idle/dead time (on/off) process is observed. Inference is based on the partial likelihood either for non-informative (Type-I) or for informative censoring (Type-II). A Pólya tree process with suitable partition construction is proposed as nonparametric prior for the normalized multiplicative intensity. Performances are illustrated on both types of censored counters.

Keywords: Bayesian nonparametrics, Type-I / Type-II counters, multiplicative intensity, Pólya tree process.
PACS: 02.50.Ey, 02.50.-r

INTRODUCTION

The purpose of physical counting devices is to analyze particles randomly emitted and recorded by a detector. In the framework of nuclear science, the user is interested in an estimation of the time-varying intensity of the underlying point process (assumed to be a Poisson process) modeling arrival times of particles. Inference on intensity from counts may however bring out some difficulties. Indeed, electrical pulses resulting from the interaction between the particles and the detector have finite duration and may overlap. Thus, some arrivals may not be recorded. This censoring period is referred to as dead-time. It might be noticed that dead-time can also be produce by other independent phenomena, like disrupted measure or temporal breakdown. Several experimental areas in nuclear physics [1], astrophysics (e.g. γ bursts [2, 3]) and biology [4] are faced to the problem of censored counters. Further physical description of particles counters can be found in [5] and theoretical results for counter processes in [6, 1, 7].

To model the observed process, the general framework of the multiplicative intensity point processes can be used (see [8, 9]) . Based on this model and for a large number of applications, [10] shows that a beta process prior is relevant and conducts to tractable *a posteriori* distributions. His proposed prior, though adapted for the cumulative hazard rate, cannot be used for the direct estimation of the intensity of the input Poisson process.

Our motivation in this contribution is therefore to address the problem of estimating the intensity of the input Poisson process given a sample path of an indicator data function Y, that is a function whose value is 0 during dead-time and 1 otherwise. We propose a nonparametric Bayesian approach for this purpose, based on Pólya tree processes methodology. The paper is organized as follows: first, assumptions and results

of [10] are recalled. A description of Pólya tree processes, as well as a description of the methodology used are provided. Finally, applications are presented in the case of so-called Type-I and Type-II counters, which give promising results.

ASSUMPTIONS AND THEORETICAL RESULTS

We recall in this section the assumptions and theoretical results that can be found in [10]. Let A denote the point process associated the arrival times of particles in the detector. We assume that

(H-1) A is a non-homogeneous Poisson process on the positive half-line with intensity λ. We also assume that $\|\lambda\|_\infty$ is finite on the positive half-line.

Let N be the point process associated to the recorded particles. We denote by S_i the arrival time of the i-th recorded particle and by C_i its corresponding censoring duration. Let Y be an observed associated indicator process, such that for all nonnegative t:

$$Y(t) \stackrel{\text{def}}{=} I(t \geq S_{N(t-)} + C_{N(t-)}) . \tag{1}$$

Processes A, N and Y can be related as follows:

$$N(t) = \int_0^t Y(s) \, dA(s) . \tag{2}$$

We further assume that

(H-2) N has multiplicative intensity (see Aalen [8]), that is

$$\lim_{\Delta t \downarrow 0} \frac{1}{\Delta t} \mathbb{E}(N(\Delta t + t) - N(t)|\mathcal{F}_t) = Y(t+)\lambda(t+) \tag{3}$$

where $\mathcal{F}_t \stackrel{\text{def}}{=} \sigma\{\mathcal{N}_t \cup \mathcal{Y}_t\}$ and $\{\mathcal{N}_t\}_{t \geq 0}$ (resp. $\{\mathcal{Y}_t\}_{t \geq 0}$) is the filtration generated by N (resp. Y), that is for all t, $\mathcal{N}_t \stackrel{\text{def}}{=} \sigma\{N(u), 0 \leq u \leq t\}$, $\mathcal{Y}_t \stackrel{\text{def}}{=} \sigma\{Y(u), 0 \leq u \leq t\}$.

A requirement on Y is that it is predictable, that is left-continuous with right limit and adapted to $\{\mathcal{F}_t\}_{t \geq 0}$. By construction the process Y defined by (1) meets this requirement. The problem of intensity estimation can be therefore considered as follows: given a sample path of Y over $[0, T]$, we wish to estimate λ. We first proceed to express the observed data likelihood $\mathcal{L}(\lambda)$ using product integral notation [9, Section II.6]. Let $\Delta N(t) \stackrel{\text{def}}{=} N(t) - N(t-)$ and $\Delta Y(t) \stackrel{\text{def}}{=} Y(t+) - Y(t)$.

$$
\begin{aligned}
\mathcal{L}(\lambda) &= \prod_{[0,T]} \Pr(\Delta N(t), \Delta Y(t)|\mathcal{F}_{t-}, \lambda) \\
&= \prod_{[0,T]} \Pr(\Delta N(t)|\mathcal{F}_{t-}, \lambda) \prod_{[0,T]} \Pr(\Delta Y(t)|\mathcal{F}_{t-}, \Delta N(t), \lambda)
\end{aligned} \tag{4}
$$

Since we assumed (H-2), we use the results of [8, Section 3.3] can be used, which leads to:

$$\mathcal{L}_P(\lambda) \stackrel{\text{def}}{=} \prod_{[0,T]} \Pr(\Delta N(t)|\mathcal{F}_{t-}, \lambda) \propto \prod_{i=1}^{N(T)} \lambda(S_i) \exp\left(-\int_0^T \lambda(u)Y(u)du\right). \tag{5}$$

On the other hand, noting that by definition of $Y(t)$, $\Pr(\Delta Y(t) = -1|\Delta N(t) = 1, \mathcal{F}_{t-}, \lambda) = 1$ and $\Pr(\Delta Y(t) = 0|\Delta N(t) = 0, Y(t-) = 1, \lambda) = 1$, it comes that

$$\begin{aligned}
\mathcal{L}_C(\lambda) &\stackrel{\text{def}}{=} \prod_{[0,T]} \Pr(\Delta Y(t)|\mathcal{F}_{t-}, \Delta N(t), \lambda) \\
&= \prod_{[0,T]} \Pr(t \leq S_{N(t)} + C_{N(t)}|\mathcal{F}_{t-}, \Delta N(t), \lambda)^{I(Y(t+)=0,Y(t)=0)} \\
&\quad \times (1 - \Pr(t \leq S_{N(t)} + C_{N(t)}|\mathcal{F}_{t-}, \Delta N(t), \lambda))^{I(Y(t+)=1,Y(t)=0)} \\
&= \prod_{i=1}^{N(T)} \Pr(C_i|S_1, \ldots, S_i, C_1, \ldots, C_{i-1}, \lambda). \tag{6}
\end{aligned}$$

In the case of Type-I counters the sequence $\{C_i\}_{i\geq 1}$ is i.i.d. and independent of $\{S_i\}_{i\geq 1}$ and of λ [6, 10]. $\mathcal{L}_C(\lambda)$ is then non-informative for the estimation of λ and the partial likelihood $\mathcal{L}_P(\lambda)$ preserves the form of the likelihood $\mathcal{L}(\lambda)$ (see [9, Sections II.7, III.2]). For Type-II counters, independence does not hold anymore in $\mathcal{L}_C(\lambda)$ which appears thus informative. Since inference from $\mathcal{L}_C(\lambda)$ may exhibit great complexity, we choose to infer only from the partial likelihood $\mathcal{L}_P(\lambda)$. This corresponds to exact inference for Type-I counters and to an approximation for Type-II counters.

Though giving a theoretical framework, it is not easy to estimate λ directly from $\mathcal{L}_P(\lambda)$ expression. Remark that for all i, $Y(S_i) = 1$, hence equation (5) can be transformed to obtain a more tractable expression. Specifically, given a sample $\mathbf{S} = \{S_1, \ldots, S_{N(T)}\}$, we get $\mathcal{L}_P(\lambda) \propto \mathcal{L}_{N(T)}(K)\mathcal{L}_{\mathbf{S}}(\rho)$, where

$$K \stackrel{\text{def}}{=} \int_0^T \lambda(u)Y(u)\,du$$

$$\rho(t) \stackrel{\text{def}}{=} \frac{Y(t) \cdot \lambda(t)}{K}$$

$$\mathcal{L}_{N(T)}(K) \stackrel{\text{def}}{=} K^{N(T)} e^{-K} \tag{7}$$

$$\mathcal{L}_{\mathbf{S}}(\rho) \stackrel{\text{def}}{=} \prod_{i=1}^{N(T)} \rho(S_i) \tag{8}$$

This reparametrization leads to a separable likelihood. Here ρ corresponds to the normalized intensity of N and will be considered as an intermediate infinite-dimensional parameter for inference on λ provided an estimate of K is known. Next section provides methods of inference for these functions.

METHODOLOGY

The partial likelihood depends on the parameter K and on the function ρ. We propose a standard Bayesian approach for the estimation of K, and a nonparametric Bayesian method based on the Pólya tree framework for ρ, which are detailed below.

From (7), the likelihood $\mathcal{L}(K)$ is proportional to a Poisson distribution whose conjugate prior is known to be a Gamma distribution. We further denote by $\Gamma(\mu, \nu)$ such a distribution, with shape parameter μ and scale parameter ν are chosen by the user. Thus the posterior distribution of K, denoted by $K|N(T)$, is given by

$$K|N(T) \sim \Gamma(\mu + N(T), \nu + 1) \tag{9}$$

We now focus on the nonparametric estimation of ρ. Based on the expression of $\mathcal{L}(\rho)$ in (7), the idea is to consider the estimation of the normalized multiplicative intensity ρ as a probability measure estimation problem. Indeed, we recognize the likelihood of an exchangeable sequence which admits a De Finetti measure (see [11, Chap. 2.6]). As mentioned in [12], Pólya tree processes correspond to the De Finetti measure in a generalized Pólya urn scheme. This makes this random measure prior a good candidate for a nonparametric Bayesian estimation of ρ. We give now a definition and some properties of Pólya-trees.

Let $E = \{0, 1\}$ and E^m be the m-fold Cartesian product $E \times \cdots \times E$ with $E^0 \stackrel{\text{def}}{=} \emptyset$. Let $E^\star = \cup_{m=0}^{\infty} E^m$. Let $\pi_0 = [0; T]$ and for all integer m, $\pi_m = \{B_\varepsilon : \varepsilon \in E^m\}$ be a partition of $[0; T]$ so that sets of π_{m+1} are obtained by a binary split of the sets of π_m. Note that $\Pi = \cup_{m=0}^{\infty} \pi_m$ generates the measurable sets. A probability distribution G on $[0; T]$ has a Pólya tree distribution with parameter $(\Pi, \mathcal{A})$, denoted by $G \sim \mathrm{PT}(\Pi, \mathcal{A})$, if there exists a sequence of nonnegative numbers $\mathcal{A} = \{\alpha_\varepsilon : \varepsilon \in E^\star\}$ and a sequence of random variables $\mathcal{V} = \{V_\varepsilon : \varepsilon \in E^\star\}$ such that

  (i) $\mathcal{V}$ is a sequence of independent random variables,

 (ii) for all ε in $E^\star$, $V_\varepsilon \sim \mathrm{Beta}(\alpha_{\varepsilon 0}, \alpha_{\varepsilon 1})$, and

(iii) for all integer m and $\varepsilon = \varepsilon_1 \cdots \varepsilon_m$ in E^m,

$$G(B_{\varepsilon_1 \cdots \varepsilon_m}) = \prod_{\substack{j=1 \\ \varepsilon_j=0}}^{m} V_{\varepsilon_1 \cdots \varepsilon_{j-1}} \times \prod_{\substack{j=1 \\ \varepsilon_j=1}}^{m} (1 - V_{\varepsilon_1 \cdots \varepsilon_{j-1}})$$

with factors equal to $V_\emptyset$ or $1 - V_\emptyset$ if $j = 1$.

Pólya trees allow to generate continuous distributions, under some conditions on its parameters. Another interesting property of Pólya tree is its conjugacy to the likelihood of observed data lying in a given subset of any partition. Consequently, the posterior distribution associated to a Pólya tree prior is still a Pólya tree. More precisely, given a Pólya tree prior $G \sim \mathrm{PT}(\Pi, \mathcal{A})$ and an i.i.d. sample $\mathbf{X} = (X_1, X_2, \ldots, X_n)$ with common distribution G, then

  (i) $G|\mathbf{X} \sim \mathrm{PT}(\Pi, \mathcal{A}^{X_1, \ldots, X_n})$, where the updated parameters are given for all ε in E^m by

$$\alpha_\varepsilon^{X_1, \ldots, X_n} = \alpha_\varepsilon + n_\varepsilon, \text{with } n_\varepsilon \stackrel{\text{def}}{=} \#\{i \in \{1, \ldots, n\} : X_i \in B_\varepsilon\} \tag{10}$$

(ii) for all integer m and every ε in E^m, the conditional expectation distribution is given by

$$\mathbb{E}(G|\mathbf{X}) = \prod_{i=1}^{m} \frac{\alpha_{\varepsilon_1\cdots\varepsilon_i} + n_{\varepsilon_1\cdots\varepsilon_i}}{\alpha_{\varepsilon_1\cdots\varepsilon_{i-1}0} + \alpha_{\varepsilon_1\cdots\varepsilon_{i-1}1} + n_{\varepsilon_1\cdots\varepsilon_{i-1}}} \tag{11}$$

A more exhaustive description of Pólya trees processes and their properties are given in [13, 12]. We now describe an explicit construction of the Pólya tree prior suitable for the censored counter problem. Recall that in our application, no data is observed during busy periods, hence no information is available to update the distribution over the dead-times. Define $L = \int_0^T Y(u)\,du$ and the distribution G_0 for all Borel set B in $[0,T]$ such that

$$G_0(B) = \frac{1}{L} \int_B Y(u)\,du \,, \tag{12}$$

We assume that $L > 0$ ensuring that there is at least an idle period on $[0;T]$. Since G_0 is a monotonous function, define its associated Levy inverse function for all x in $(0,1)$ as $\widetilde{G_0}(x) \overset{\text{def}}{=} \inf\{t \in [0,T] : G_0(t) \geq x\}$. Given G_0 defined in (12), we construct the binary quantile partitions on the uncensored periods of time as follows: let $B_\emptyset = \widetilde{G_0}(G_0([0,T]))\Omega^\star$ and for all integer m and all $\varepsilon = (\varepsilon_1\cdots\varepsilon_m)$ in E^m:

$$B_{\varepsilon_1\cdots\varepsilon_m} = \left(\widetilde{G_0}\left(2^{-m}\sum_{k=1}^{m}\varepsilon_k 2^{k-1} \right), \widetilde{G_0}\left(2^{-m}\sum_{k=1}^{m}\varepsilon_k 2^{k-1} + 2^{-m} \right) \right) \overset{\text{def}}{=} (a_\varepsilon, b_\varepsilon) \,. \tag{13}$$

In addition to the binary partitions tree, we define the tree parameters $\mathcal{A} = \{\alpha_\varepsilon, \varepsilon \in E^\star\}$ such that for all ε in $E^\star$, $\alpha_{\varepsilon 0} = \alpha_{\varepsilon 1}$. It might be noticed that this particular quantile partitions construction ensures that for all ε in $E^\star$ and $G \sim \mathrm{PT}(\Pi, \mathcal{A})$, $G_0(B_\varepsilon) > 0$ and $\mathbb{E}(G) = G_0$ which would correspond to a uniform prior for λ.

Even if the binary splitting procedure may be pursued infinitely, computational implementation implies the definition of a predefined maximum level M. In the sequel, we will consider Pólya trees partially specified until level M.

We now consider a sample $\mathbf{S} = \{S_1, \ldots, S_{N(T)}\}$ of points of N. The posterior distribution $G|\mathbf{S}$ is computed using (10), which leads to

$$G|\mathbf{S} \sim \mathrm{PT}(\Pi, \mathcal{A}^\mathbf{S}), \text{ with } \alpha_\varepsilon^\mathbf{S} = \alpha_\varepsilon + N(b_\varepsilon) - N(a_\varepsilon) \text{ for all } \varepsilon \text{ in } E^\star \,. \tag{14}$$

The conditional mean is also obtained for all $\varepsilon = \varepsilon_1\cdots\varepsilon_M$ in E^M using (11) and (13):

$$\mathbb{E}(G(B_\varepsilon)|\mathbf{S}) = \prod_{j=1}^{M} \frac{\alpha_{\varepsilon_1\cdots\varepsilon_j} + N(b_{\varepsilon_1\cdots\varepsilon_j}) - N(a_{\varepsilon_1\cdots\varepsilon_j})}{\alpha_{\varepsilon_1\cdots\varepsilon_{j-1}0} + \alpha_{\varepsilon_1\cdots\varepsilon_{j-1}1} + N(b_{\varepsilon_1\cdots\varepsilon_{j-1}}) - N(a_{\varepsilon_1\cdots\varepsilon_{j-1}})} \,. \tag{15}$$

We now provide an estimator $\hat{\lambda} \overset{\text{def}}{=} (\lambda|\mathbf{S}, K)$ of λ which is piecewise constant over the sets of π_M, that is for all ε in E^M and t in B_ε, $\hat{\lambda}(t) = \lambda_\varepsilon$. Under this restriction, $(\lambda|\mathbf{S}, K)$ can be explicitly computed. First, we remark that for the binary quantile partitions set Π and for $G \sim \mathrm{PT}(\Pi, \mathcal{A})$,

$$G(B_\varepsilon) = \int_{B_\varepsilon} \rho(u)\,du = \frac{1}{K}\int_{B_\varepsilon} Y(u)\lambda(u)\,du = \frac{L}{K\,2^M}\lambda_\varepsilon \,.$$

Consequently, draws from $(\lambda|\mathbf{S},K)$ can be generated for all ε in E^M from the three following steps:

(i) generate K from (9) : $(K|N(T)) \sim \Gamma(\mu + N(T), \nu + 1)$

(ii) for all ε in E^M, compute $G(B_\varepsilon)|\mathbf{S}$ using (14),

(iii) for all ε in E^M, compute λ_ε

$$\lambda_\varepsilon = \frac{2^M K}{L} G(B_\varepsilon) . \tag{16}$$

In addition, conditional mean for λ_ε can be expressed from (9) and (15) due to independence of $(K|N(T))$ and $G(B_\varepsilon)$; we get for all $\varepsilon = \varepsilon_1 \cdots \varepsilon_M$ in E^M,

$$\mathbb{E}(\lambda_\varepsilon|\mathbf{S},K) = \frac{2^M}{L} \frac{\mu + N(T)}{\nu + 1} \prod_{j=1}^{M} \frac{\alpha_{\varepsilon_1 \cdots \varepsilon_j} + N(b_{\varepsilon_1 \cdots \varepsilon_j}) - N(a_{\varepsilon_1 \cdots \varepsilon_j})}{\alpha_{\varepsilon_1 \cdots \varepsilon_{j-1}0} + \alpha_{\varepsilon_1 \cdots \varepsilon_{j-1}1} + N(b_{\varepsilon_1 \cdots \varepsilon_{j-1}}) - N(a_{\varepsilon_1 \cdots \varepsilon_{j-1}})} . \tag{17}$$

APPLICATIONS

As mentioned in the previous section, our estimator depends on an underlying fixed partition. Even if parameters are chosen to ensure continuity of generated distributions, a lack of smoothness can appear at partition endpoints for finite Pólya trees. Moreover, in many practical applications, the counting process is an open-ended stream (that is, T is large with respect to the busy period durations) and it could be needed to estimate its intensity sequentially on overlapping subsets of $[0;T]$. For this purpose, we also propose another estimator of λ obtained by averaging conditional expectations of shifted finite Pólya trees estimates. As a side result, this mixture could reduce the observations of discontinuities by averaging distributions across shifted partitions.

Denote by τ_M a shift parameter which must be tuned in order to tackle the dynamics of λ. Denote by $\widetilde{G_U}$ the quantile function of the unnormalized measure $G_U(t) = \int_0^t Y(u)\,du$, and define for all integer i: $t_i = \widetilde{G_U}(i\tau_M)$. Based on (13), different partitions trees are defined on each subset $[t_i;t_{i+2^M}]$ for all integer i and $\varepsilon = (\varepsilon_1, \ldots, \varepsilon_m)$ in E^m as follows:

$$B^i_{\varepsilon_1 \cdots \varepsilon_m} = \left(\widetilde{G_U}\left(t_i + 2^{M-m}\tau_M \sum_{k=1}^{m} \varepsilon_k 2^{k-1}\right), \widetilde{G_U}\left(t_i + 2^{M-m}\tau_M + 2^{M-m}\tau_M \sum_{k=1}^{m} \varepsilon_k 2^{k-1}\right) \right)$$

We can therefore propose another piecewise constant estimator $\hat{\lambda}$ such that for all t in $[t_i, t_{i+1}]$, $\hat{\lambda}(t) = \lambda_i$. Following (17) the conditional expectation estimate of λ_i is simply obtained by averaging the $\kappa_i = \min(2^M, i+1)$ conditional means distributions whose partitions contain t_i. Specifically, we get that

$$\widehat{\lambda_i} = \frac{1}{\kappa_i \tau_M} \sum_{j=0}^{\kappa_i - 1} \left(\frac{\mu + N(B^{i-j}_\emptyset)}{\nu + 1} \prod_{k=1}^{M} \frac{\alpha_{\varepsilon_1 \cdots \varepsilon_k} + N(B^{i-j}_{\varepsilon_1 \cdots \varepsilon_k})}{\alpha_{\varepsilon_1 \cdots \varepsilon_{k-1}0} + \alpha_{\varepsilon_1 \cdots \varepsilon_{k-1}1} + N(B^{i-j}_{\varepsilon_1 \cdots \varepsilon_{k-1}})} \right) , \tag{18}$$

where $\varepsilon_1 \cdots \varepsilon_M$ is the M-digits binary representation of j and $N(B) \stackrel{\text{def}}{=} \int_B dN$.

We now discuss on the choice on the prior parameters. Since our degree of belief on K is very vague, beta prior parameters μ and ν should be set very small. Tree parameters $\mathcal{A}$ control how generated random distributions deviate from G_0 and require more attention. A default method usually suggested is to choose α_ε depending only of the tree level m : for all $\varepsilon \in E^m$, $\alpha_\varepsilon = a_m$. According to the Kraft theorem [13], a condition to generate absolutely continuous distributions with probability one with respect to Lebesgue measure is $\sum_{m=0}^{\infty} a_m^{-1} < \infty$. This condition is respected for example when $a_m \propto \eta^m$ with $\eta > 1$. In addition the growth rate η controls Pólya tree smoothness.

We present results on two simulated sequences corresponding to Type I and Type II counters. The "true" λ is taken as a mixture of Gamma distribution in both cases. The realization of the input Poisson process is the same for both sequences. The parameters are set to: $\tau_M = 1$, $M = 13$, $a_m = 0.1 \cdot 3^m$. We observe $N(T) = 577$ (resp. $N(T) = 434$) for Type I (resp. Type II) recorded events, with a 30% (resp. 36%) dead-time percentage. We compare our shifted Pólya tree based estimate $\hat{\lambda}$ given in (18) to a nonparametric estimation based on assumption that intensity is piecewise constant on regular intervals, given for all h by $\lambda^\star(h) = (N((h+1)\tau^\star) - N(h\tau^\star))/\int_{h\tau^\star}^{(h+1)\tau^\star} Y(u)du$, with $\tau^\star = 256$ chosen such that the denominator of $\lambda^\star$ is positive. We also present an estimate without dead time correction given for all h by $\lambda_b^\star(h) = (N((h+1)\tau^\star) - N(h\tau^\star))/\tau^\star$. Results are displayed in Figure 1 for the Type I counter, and in Figure 2 for the Type II counter.
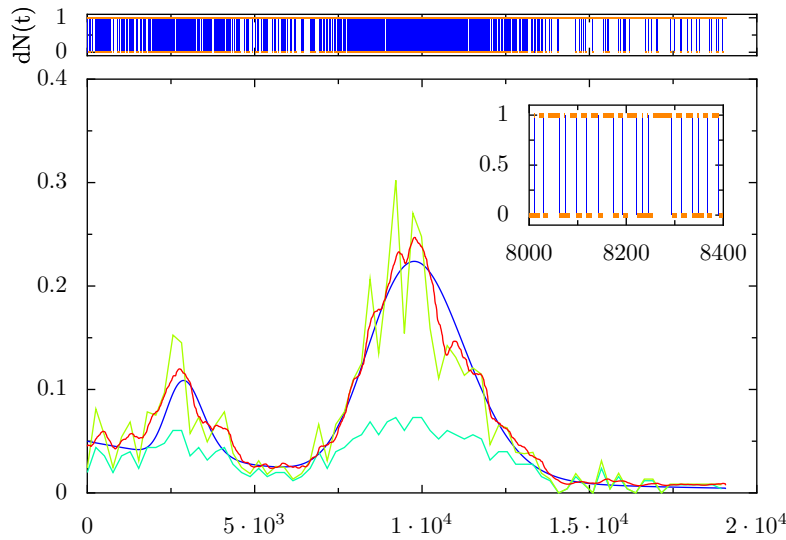

FIGURE 1. Type-I counters data and estimators: "true" $\lambda(t)$ (blue), estimator $\lambda_b^\star$ (cyan), estimator $\lambda^\star$ (green), estimator $\hat{\lambda}$ (red). Upper and detailed plots : $\Delta N(t)$ (blue), $Y(t)$ (orange).

Since the posterior distribution are explicit, our algorithm is very efficient and behaves as a nonlinear moving average on the counts data. As expected by the use of a random measure prior, the algorithm smoothes more the regions with few recorded jumps. We encounter those regions when λ_0 is small (few Poisson arrivals) and, for Type-II counters only, when λ_0 increases such we observe few uncensored jumps. A Monte-Carlo MSE estimate leads for Type-I (resp. Type-II) to: $\mathbb{E}((\lambda - \hat{\lambda})^2) \approx 1.5\,10^{-4}$ (resp. $2.1\,10^{-4}$), and $\mathbb{E}((\lambda - \lambda^\star)^2) \approx 5.3\,10^{-4}$ (resp. $1.1\,10^{-3}$) for the chosen λ and parameters.
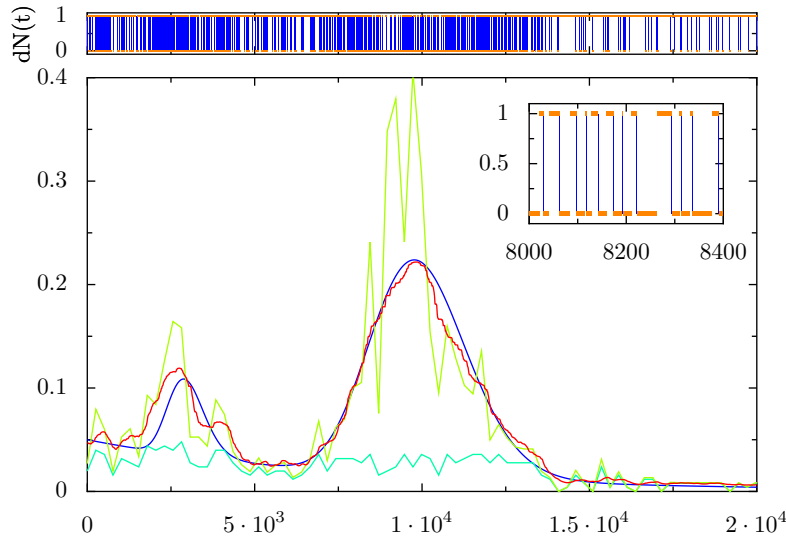
FIGURE 2. Type-II counters data and estimators: "true" $\lambda(t)$ (blue), estimator $\lambda_b^\star$ (cyan), estimator $\lambda^\star$ (green), estimator $\widehat{\lambda}$ (red). Upper and detailed plots : $\Delta N(t)$ (blue), $Y(t)$ (orange).

CONCLUSION

We have built an estimator of the intensity of a censored Poisson process, which gives good results on simulations. Experiments show that a nonparametric Bayesian approach outperforms empirical procedures and seems flexible enough to capture a wide range of intensity profiles. Theoretical aspects of our estimator will be developed in future contributions.

REFERENCES

1. T. Trigano, *Traitement statistique du signal spectrométrique : étude du désempilement de spectre en énergie pour la spectrométrie Gamma*, Ph.D. thesis, ENST (2005).
2. J. D. Scargle, and G. J. Babu, "Point processes in astronomy: Exciting events in the universe," in *Handbook of Statistics*, 2003, vol. 21, pp. 795–825.
3. T. J. Loredo, and D. Q. Lamb, *Physical Review D* **65**, 063002 (2002).
4. N. H. Bingham, and S. Pitts, *Annals of the Institute of Statistical Mathematics* **51**, 71–97 (1999).
5. G. F. Knoll, *Radiation detection and measurement*, Wiley, 1989, 2 edn.
6. R. Pyke, *Annals of Mathematical Statistics* **2**, 71–80 (1958).
7. E. Moulines, F. Roueff, A. Souloumiac, and T. Trigano, *accepted for publication in Bernoulli* (2006).
8. O. O. Aalen, *Annals of Statistics* **6**, 701–726 (1978).
9. P. K. Andersen, Ø. Borgan, R. D. Gill, and N. Keiding, *Statistical models based on counting processes*, Springer-Verlag, 1993.
10. Y. Kim, *Annals of Statistics* **27**, 562–588 (1999).
11. J. K. Ghosh, and R. V. Ramamoorthi, *Bayesian Nonparametrics*, Springer, 2003.
12. R. D. Mauldin, W. D. Sudderth, and S. C. Williams, *Annals of Statistics* **20**, 1202–1221 (1992).
13. M. Lavine, *Annals of Statistics* **20**, 1222–1235 (1992).