Scheduling schools

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Why do I have such a lousy schedule?

- What is this problem?
- Dutch secondary school system
- A working algorithm

What is this problem?

- Not Maximum Entropy
- Not Bayesian
- So, what are we doing here?

Large combinatorial problem

- 9000 lessons per week
- 95 teachers, 50% work part time
- 55 rooms, 2 locations
- 1200 students
- many other constraints and wishes

Dutch school system

- Lower level classes
- years 1 - 3, students 12 - 15 years old
- all classes have the same curriculum
- scheduling classes - teachers - rooms relatively easy

Upper levels

- Years 4 - 6, students 16 - 18 years old
- Common subjects (Dutch, English, Science)
- Branch subjects (math & physics, biology & life science, economy, social science)
- Optional subjects (Spanish, drama, math++)
- Need for scheduling per individual student
- Hard problem

Students - subjects

Name	M/F	Dutch	English	Math	Bio	Drama	Math++
Frank	M	1	1	0	1	0	0
Kevin	m	1	1	1	0	1	1
Frieda	f	1	1	1	0	1	0
Mark	m	1	1	0	1	0	0

Hierarchy in students

- Year: all studs of given year in given level, typically 200 students
- Cluster: associated with subject - students
- Class: 30 students, or less
- Group: all students with exactly the same subjects, typically 1 - 30 students

The partitioning of students in classes is the most computing intensive part

Subjects - hours - teachers

	Hours	Classes					
Dutch	4	5	Abe	Abe	Abe	Eve	Eve
English	3	5	Jan	Kay	Ivy	Jan	Kay
Math	3	3	John	John	John		
Bio	2	2	Mary	Mary			
Drama	2	1	Lucia				
Math++	2	1	John				

Constraints

- Lessons table (9000 lessons per week)
- Block hours, Combi hours
- Part time teaching
- Dutch labour laws
- Classes must fit in assigned rooms

Preferences

- minimize number of idle hours
- minimize movements between locations
- avoid late hours in the day
- balance gender in classes
- balance class sizes
- same lessons always in same rooms
- et-cetera ...

A working algorithm

- Binary problem: student/teacher/room is occupied or not
- Space is huge: $1.6\ 10^{10}$
- Search by MCMC algorithm of John Skilling
- There are many valid schedules
- Choose one - based on preferences

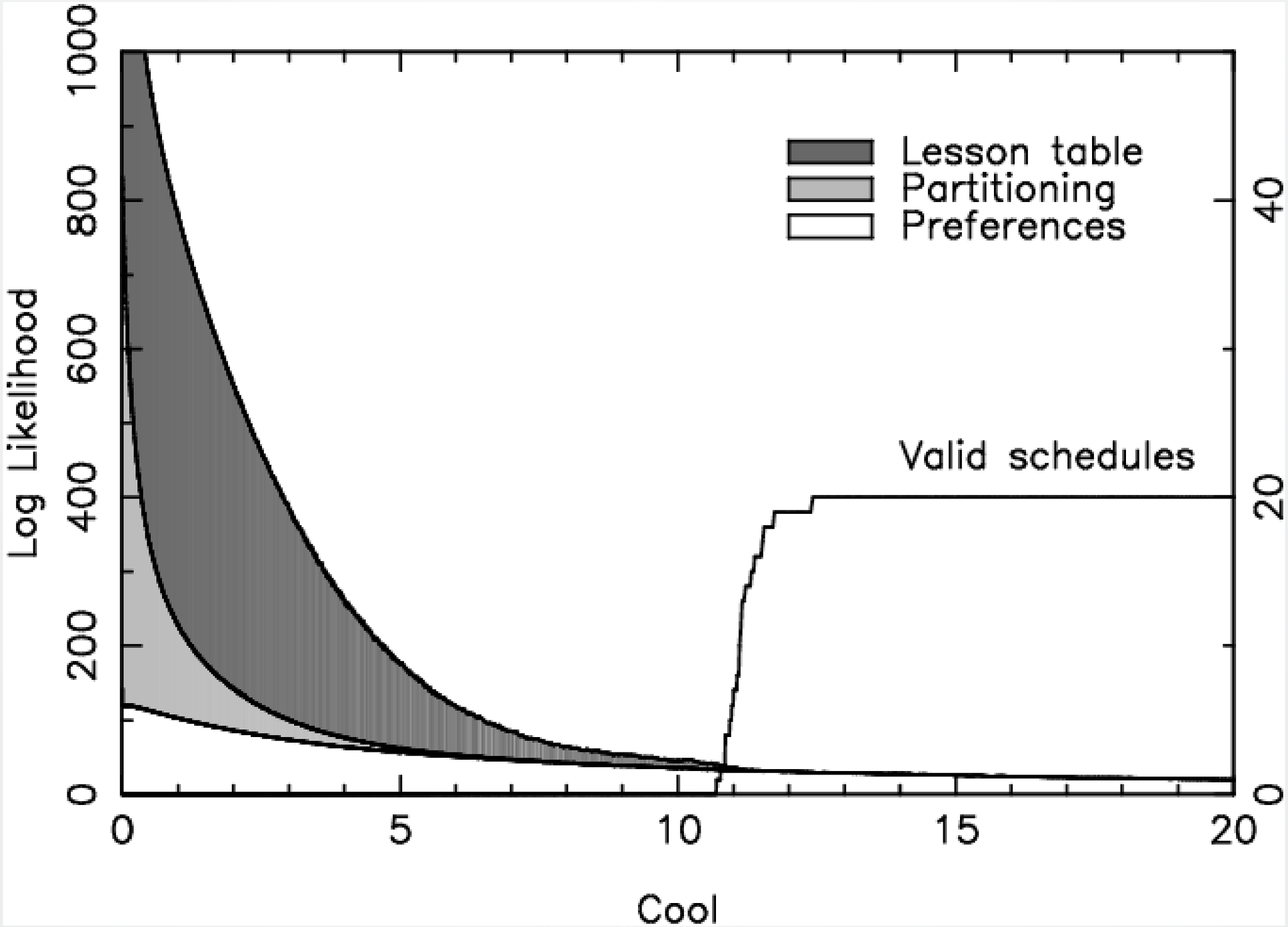
MCMC

- 20 members
- each member a trial schedule
- weighted sum of misfits: likelihood L
- evolve by stepping: compute ΔL
- accept step or reject step
- Decrease temperature of the system

MCMC

- EXAMPLE STEPS:
  - INSERT/DELETE 1 LESSON FOR 1 CLASS
  - SWAP 2 LESSONS/HOURS/TEACHERS IN 1 CLASS
  - MOVE STUDENT GROUP TO ANOTHER CLASS
  - DIFFUSE BETWEEN MEMBERS

MCMC at work


Log Likelihood
0
200
400
600
800
1000
0
5
10
15
20
Cool
0
20
40
Lesson table
Partitioning
Preferences
Valid schedules

Two likelihoods

- Constraints likelihood: L_C
  - any valid schedule MUST have $L_C = 0$
- Preference likelihood: L_P
  - minimization of L_P
  - ideally $L_P = 0$, but this is unattainable in practice

Two phases in the computation

- Convergence phase, when $L_c > 0$
  - Prior is availability of teachers*rooms
- Optimization phase, when $L_c = 0$
  - Prior is set of valid schedules
  - begins here at Cool = 10.7
  - continue optimizing until tired

How many valid schedules are there?

- Careful Combinatorics: about 10^{3770} possibilities at Cool = 0
- Ockham factor: $H = \int_{Cool=0}^{10.7} < L_C + L_W > \ dCool$
- exp(H) $\approx 10^{1370}$
- therefore there are 10^{2400} valid schedules!

Thank you for your

ATTENTION