NONPARAMETRIC BAYESIAN ESTIMATION OF X/γ-RAY SPECTRA USING A HIERARCHICAL POLYA TREE – DIRICHLET MIXTURE MODEL

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Abstract

We address the problem of X/γ-ray spectra estimation in the fields of nuclear physics and astrophysics. Bayesian estimation of experimental backgrounds has been studied in [1] involving splines. Since Dirichlet Processes (DP) sit on discrete measures, they provide an appealing prior for photopeaks. On the other hand, in order to tackle the complexity of experimental backgrounds, we consider a Polya Tree Mixture (PTM) – with suitable parameters yielding distribution continuity – for which predictive densities exhibit better smoothness properties than a single Polya Tree. Furthermore, it is easy to introduce some physical Compton line approximation formula (e.g. Klein-Nishina) in the base measure of the Polya Tree, or some physically driven local modifications of the PTM prior parameters. As backgrounds depend on photopeaks locations, we propose a hierarchical model where the PTM is conditioned on the DP. We use a beta prior for the mixing proportion between the DP and the PTM. Energies are not directly observed due to detection devices noises which introduce a convolution of both discrete and continuous measures by an assumed gaussian kernel whose variance is an unknown linear function of energy. Thus, the proposed semiparametric model for experimental data becomes a hierarchical Polya Tree–Dirichlet mixture of normal kernels. The quantities of interest for physicists are usually posterior functionals of the DP mixing distribution. This implies an inverse problem which is carried out in the framework of finite stick-breaking representation. To allow finite representation of PTM, we assume infinite prior parameters after a certain stage. The *blocked Gibbs sampler* of [2] is extended to update simultaneously the hidden allocation variables either from a DP component or from a set of the PTM latest informative partition. Thanks to conjugacy and conditioning on the hidden allocation variables, draws from the posterior DP and PTM are easily obtained. With minor modifications the algorithm can deal with binned data which turns out to be computationally attractive for huge datasets.

References:

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