NONPARAMETRIC BAYESIAN ESTIMATION OF CENSORED COUNTERS INTENSITY FROM THE INDICATOR DATA

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Abstract

Physical counting devices are usually imperfect in the sense that they are unable to record all particles present at their input. After a particle is registered, the counter is inhibited for a positive duration. This censoring period is referred to as dead time (or busy period). Following [1], we denote by $N(t)$ the number of recorded particles at t and $M(t)$ the input poisson process with intensity function λ_t, then $N(t) = \int_0^t Y(s)\, dM(s)$ where $Y(t) = I(t \geq S_{N(t-)} + C_{N(t-)})$, S_i is the time of the i^{th} recorded particle and C_i the corresponding dead time. We thus consider the problem of estimating λ_t given a sample path of the indicator data Y. Since it might be cumbersome to infer from the busy distribution, we propose a bayesian nonparametric method leaning only on idle periods. For all t where $Y(t) = 0$, we define the *lifetime* $t^\star = t - \sum_{j=1}^{N(t-)} C_j$. A Polya tree [2] prior defined over the lifetime space is used for the normalized intensity. Due to the data-dependent partition, the problem leads to a nonhomogeneous poisson intensity $\lambda^\star_{t^\star}$ estimation. With a gamma prior for the integrated intensity, the posterior remains the product of a Polya tree and a gamma distribution. The intensity for the idle periods is then achieved by setting $\lambda_t = \lambda^\star_{t^\star}$. For busy periods, an interpolation scheme can be used. For application purposes involving an open-ended stream, we propose an estimator of the intensity based on the posterior expectation of a shifted polya trees finite mixture which leads to a finite response nonlinear filter. Assuming small λ_t variations during the busy periods, this method is suitable for various kinds of censoring mechanism because no assumption is made about the dead time distribution.

For the usual case of type-II counter [3], formally an $M_t/G/\infty$ queue, $Y(t) = I(Q(t) > 0)$ with $Q(t)$ the number of clients in the queue, we improve the method by using additional information from the distribution of the number of particles participating to a busy period conditionally to its length.

References:

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