MaxEnt Velocity Profiles in Laminar to Turbulent Flow

Robert K. Niven
School of Aerospace, Civil and Mechanical Engineering
The University of New South Wales at ADFA
Northcott Drive, Canberra, ACT, 2600, Australia.
Email: r.niven@adfa.edu.au
28 February 2006

Abstract

This work applies the differential equation method developed by Chiu and co-workers[1] - based on Jaynes' maximum entropy method[2] - to determine the "most probable" steady-state velocity profile $u(y)$ in three systems of "classical" fluid mechanics: (i) axial flow in a cylindrical pipe (Poiseuille flow) (previously examined by Chiu[1]); (ii) flow between stationary parallel plates; and (iii) flow between moving parallel plates (Couette flow). In each case, the analysis yields an analytical solution for the velocity profile over the complete spectrum of laminar to turbulent flow. The profiles are obtained as functions of the maximum velocity u_m and parameter $M = -u_m\lambda_1$, where λ_1 is the Lagrangian multiplier for the conservation of mass constraint. M can be interpreted as a "temperature of turbulence", with M=0 indicating laminar flow and $M \to \infty$ complete turbulence. The main elements of this analysis, which have been presented briefly[3], are reproduced here.

For the axial flow system, the predicted profiles and their moments reduce to the well-known laminar solution at M=0. For M>0, the resulting solution can be used in place of existing semi-empirical correlations for the velocity profile in axial flow[1,4]. For the plane parallel flows, in order to match both the laminar profiles and higher order moments at M=0, it is necessary to make use of the relative entropy (Kullback-Liebler cross-entropy) function, incorporating a different Bayesian prior (Jaynes' invariant) distribution. A method to determine this prior distribution is described.

The analysis is then used to derive a new maximum-entropy laminar-turbulent boundary layer theory, for the velocity profile in steady flow along a flat plate. For M=0, this reduces to the laminar boundary layer theory given in some texts[4], which approximates the Prandtl-Blasius solution to the Navier-Stokes equation[5]. For turbulent flow, it yields a previously unreported solution.

Keywords: MaxEnt; fluid mechanics; velocity profile; turbulent flow; boundary layers.

References:

[1] C.-L. Chiu (1987) *J. Hydraul. Eng.-ASCE* 113(5) 583; C.-L. Chiu (1988) *J. Hydraul. Eng.-ASCE* 114(7): 738; C.-L. Chiu, G.-F. Lin, J.-M. Lu, (1993) *J. Hydraul. Eng.-ASCE* 119(6): 742.
[2] E.T. Jaynes (1957) *Phys. Rev.* 106: 620.
[3] R.K. Niven (2005) 3rd Int. Conf.: NEXT-Sigma-Phi, 3-18 August 2005, Kolymbari, Crete, Greece.
[4] R.L. Street, G.Z. Watters, J.K. Vennard (1996) Elementary Fluid Mechanics, 7th ed., John Wiley, NY.
[5] H. Schlichting, K. Gersten (2001), Boundary Layer Theory, 8th ed., Springer, NY.