Phase space methods in continuous tensor products of Hilbert spaces

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A continuum of quantum mechanical oscillators in a straight line, is considered. The Hilbert space of the system is a continuous tensor product of the Hilbert spaces describing the individual oscillators.

A mode-position operator u_x is introduced whose expectation value describes the location of a quantum state within the chain of oscillators. A mode-momentum operator u_p is also introduced whose expectation value describes the change of the mode position with time. Both of these operators involve all oscillators and are very different from the position and momentum operators of individual oscillators. Their exponentials are displacement operators which propagate a quantum state in the continuum of oscillators; and also change its momentum. They are collective transformations and are very different from displacements in the phase space of individual oscillators.

Entropic quantities which describe correlations and entanglement between the various oscillators are also studied.

The work is presented in a quantum mechanical language; but it is also applicable to related areas like time-frequency analysis in signal processing, applied harmonic analysis, etc. In this more general context it studies phase-space methods for problems described with a continuous tensor product of the Hilbert spaces.