AN APPLICATION OF ENTROPIC DYNAMICS ON CURVED STATISTICAL MANIFOLDS

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Abstract

Any attempt to unify the classical theory of gravity with quantum theories of electromagnetic, weak and strong forces in a single unified theory has been unsuccessful so far. Entropic Dynamics (ED), namely the combination of principles of inductive inference (Maximum Entropy Methods) and methods of Information Geometry (IG), is a theoretical framework constructed to explore the possibility that laws of physics, either classical or quantum, might be laws of inference rather than laws of nature. The ultimate goal of such an ED concerns the derivation of Einstein's theory of gravity from an underlying "statistical geometrodynamics" [1].

Our objective here is to show explicitly all the steps needed to derive an ED model and to underline the most delicate aspects of it. The first step is to identify the appropriate variables describing the system, and thus the corresponding space of macrostates. This is by far the most difficult step because there does not exist any systematic way to search for the right macro variables; it is a matter of taste and intuition, trial and error. In the ED model here presented we do not specify the nature of our system, it might be a thermal system or something else. We will make connections to conventional physical systems only later in the formulation of the ED model. We only assume that the space of microstates is 2D and that all the relevant information to study the dynamical evolution of such a system is contained in a 3D space of macrostates. The second step is to define a quantitative measure of change from one macrostate to another. Maximum Entropy Methods lead to the assignment of a probability distribution to each macrostate, while methods of IG lead to the assignment of the Fisher-Rao information metric quantifying the extent to which one distribution can be distinguished from another. The ED is defined on the space of probability distributions $\mathcal{M}_s$. The geometric structure of $\mathcal{M}_s$ is studied in detail. We show that $\mathcal{M}_s$ is a 3D pseudosphere with constant negative Ricci scalar curvature, $R = -1$. The final step concerns the study of irreversible and reversible aspects of such ED on $\mathcal{M}_s$. In the former case, we study the evolution of the system from a given macrostate to an unknown final macrostate. This study is used to show that the microstates of the model undergo an irreversible diffusion process. In the latter, we study the evolution of the system from a given initial macrostate to a given final state. The trajectories of the system are shown to be hyperbolic curves on $\mathcal{M}_s$, and the surface of evolution of the statistical parameters describing $\mathcal{M}_s$ is plotted. Finally, similarities and possible connections between ED methods and established physics are highlighted.

References:

[1] A. Caticha: "The Information Geometry of Space and Time", Presented at MaxEnt2005, the 25th International Workshop on Bayesian Inference and Maximum Entropy Methods (August 7-12, 2005, San Jose, California, USA).