COMPETITIVE BIDDING IN A CERTAIN CLASS OF AUCTIONS

Mathias Johansson[1,2]
(1) Signals and Systems Group, Uppsala University, Sweden,
e-mail: mj@signal.uu.se
(2) Dirac Research AB, Sweden, e-mail: mj@dirac.se

Abstract

We consider a problem of determining the amount to bid in a certain type of auctions in which customers submit one sealed bid. The bid reflects the price a customer is willing to pay for one unit of the offered goods. The auction is repeated and at each auction each customer requests a certain amount of goods, an amount that we call the capacity of the customer and that varies among customers and over time. At each auction, only the customer with the largest bid-capacity product obtains any goods. The price paid by the winner equals his/her bid-capacity product, and the amount of goods obtained in return equals the winner's capacity. The auction is repeated many times, with only limited information concerning winning bid-capacity products being announced to the customers. This situation is motivated in for example wireless communication networks in which a possible way of obtaining a desired service level is to use dynamic pricing and competitive bidding. In this application, the capacity is typically uncertain when the bid is made. We derive bidding rules and loss functions for a few typical service requirements.

We assume that the auctioneer announces only some limited aggregate statistics from previous auctions. Consequently, we use the maximum entropy principle in assigning probabilities for other customers' bids and capacities.

Our approach is to minimize the expected loss, conditional on the limited information I available to the customer. Let a particular customer u's probability that he or she will have the largest bid-capacity product of all customers be denoted by $P(u \mid I)$. Then $P(u \mid I)$ is equal to the probability that the customer v with the largest bid-capacity product of all other customers has a lower bid-capacity product than customer u. Let q_v denote the bid of v, c_v the corresponding capacity, and $y = q_v c_v$ the largest bid-capacity product among all customers except u. We can then find the probability that u wins as follows: first determine the probability that $y < c_u q_u$ assuming knowledge of c_u, i.e. $\int_0^{c_u q_u} P(y \mid c_u I) dy$. Then multiply this with the probability distribution for c_u given I to obtain the joint probability for c_u and $y < c_u q_u$. Integrating the result over all possible capacities c_u, we have

$$P(u \mid I) = \int P(c_u \mid I) \int_0^{c_u q_u} P(y \mid c_u I) dy dc_u \ . \tag{1}$$

In the full paper, we compute this probability explicitly for some particular states of knowledge I and illustrate how customers behave using the suggested strategy.