THE MINIMUM CROSS-ENTROPY METHOD: A GENERAL ALGORITHM FOR ONE-DIMENSIONAL PROBLEMS

J.C. Cuchí[1], A. Zarzo[2]
(1) Departament d'Enginyeria Agroforestal, ETSEA,
Universitat de Lleida, 25006-Lleida, Spain.
(e-mail: cuchi@eagrof.udl.es)
(2) Depto. Matemática Aplicada, E.T.S. Ingenieros Industriales,
Universidad Politécnica de Madrid, 28006-Madrid, Spain.
and
Instituto Carlos I, Facultad de Ciencias,
Universidad de Granada, 18071-Granada, Spain.
(e-mail: azarzo@etsii.upm.es, http://dmaii.etsii.upm.es/∼azarzo/)

Abstract

We consider in this paper the well known minimum cross-entropy method (MinxEnt) as applied to the problem of constructing approximations to a non-negative function, $f(x)$, when partial information about it is given as a set of constraints of the form:

$$\mu_0 = \int_a^b f(x)dx \quad \mu_j = \int_a^b k_j(x)f(x)dx \quad j = 1, \ldots, n\,,$$

i.e. its normalization, μ_0, and a set of expectation values of certain functions, $k_j(x)$ ($j = 1, \ldots, n$). On applying the MinxEnt method to this problem, a minimum of the cross-entropy functional

$$\mathcal{E}[f : f_0] = \int_D f(x) \log\left(\frac{f(x)}{f_0(x)}\right) dx$$

has to be computed, where $f_0(x)$ is a prior approximation to $f(x)$, usually obtained from the knowledge of the specific problem in which $f(x)$ and the constraints appears.

One can find in the literature a number of algorithms to deal with this problem which works for some particular situations (see e.g. [1]–[4] among others). Our intention here is to discuss the behavior of the standard optimization methods (Newton, quasi-Newton, ... with line–search of several types) with the aim of developing a general algorithm to solve the minimization problem in the sense that it could be applied to a wide set of densities and constraints, ranging from the discrete to the continuous cases. As illustration, the density of zeros of several families of orthogonal polynomials (discrete case) and also some problems related with the charge density in atomic systems (continuous case) are discussed.

References:

[1] J. Darroch, D. Ratcliff, Generalized iterative scaling for Log-linear Models, Annals of Mathematical Statistics 43 (5) (1972) 1470–1480.

[2] L. R. Mead and N. Papanicolaou, Maximum entropy in the problem of moments, J. Math. Phys. 25 (1984) 2404–2417.

[3] J.M. Borwein and W.Z. Huang, A fast heuristic method for polynomial moment problems with Boltzmann–Shannon entropy, SIAM J. Opt. 5 (1995) 68–99.

[4] K. Bandyopadhyay, A. K. Bhattacharya, Parthapratim Biswas and D. A. Drabold, Maximum entropy and the problem of moments: A stable algorithm, Phys. Rev E 71 (2005).

Key Words: MinxEnt method, optimization methods, zeros of polynomials, charge densities in atomic systems