A FAST METHOD FOR SPARSE COMPONENT ANALYSIS BASED ON ITERATIVE DETECTION-PROJECTION

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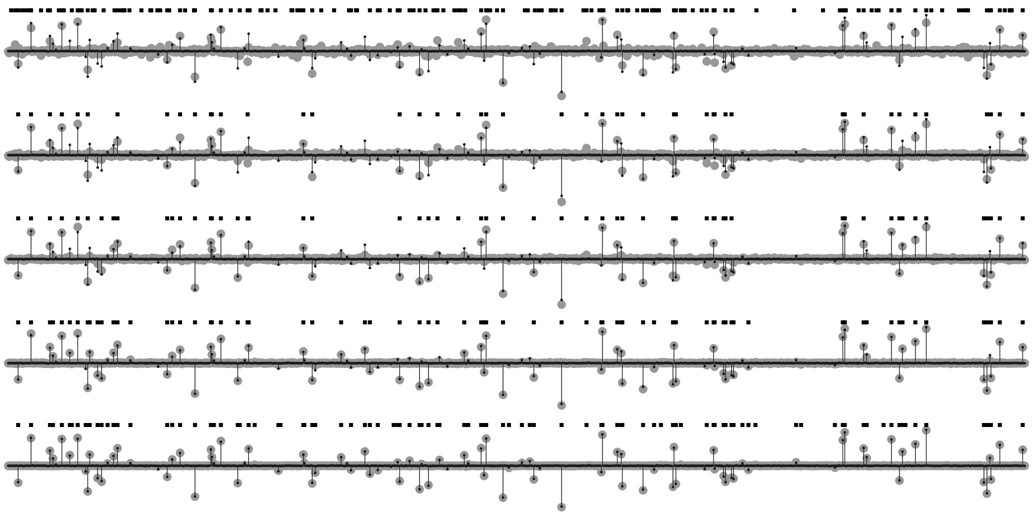
Abstract

We introduce a new iterative algorithm for Sparse Component Analysis (SCA). The algorithm is essentially a method to find sparse solutions of underdetermined linear systems of equations. In the SCA context, the method solves the source separation part of the problem, provided that the mixing matrix is known (*i.e.* estimated). The method is not restricted to SCA and may be used in any context in which such a problem arises. For example, it may be used to find sparse decomposition of a signal in an overcomplete dictionary. For the purpose of discussion, however, we will use the SCA notation and terminology. More specifically, we are given the system $\mathbf{x} = \mathbf{A}\mathbf{s}$ where $\mathbf{x}$ is the known $n \times 1$ mixture vector, $\mathbf{s}$ is the unkown $m \times 1$ source vector and $\mathbf{A}$ is the known $n \times m$ mixing matrix. The system is underdetermined, *i.e.* $n < m$. We wish to find the sparsest source vector satisfying the system.

The idea is to first detect which components of the source vector are active, *i.e.* having a considerable value. The test for activity is carried for each component (*i.e.* each source) separately. We will use a Gaussian mixture to model a (sparse) source. It is found that the optimal test for activity of one source requires the knowledge of all the other sources. We will replace those other sources with their estimates obtained from a previous iteration. The suboptimal test for activity of the i-th source then reduces to comparison of an activity function $g_i(\mathbf{x}, \hat{\mathbf{s}})$ against a threshold. After determination of the activity status of all the sources, the new estimate for source vector will be obtained by finding a solution of $\mathbf{x} = \mathbf{A}\mathbf{s}$ which is closest (in 2-norm) to the subspace specified by the detection step. We will call this step "projection into the activity subspace". Explicit solution of the projection step will be given in terms of pseudo-inverses, for the cases of interest. It is found experimentally that repeated use of the two-step iteration, with proper choices of thresholds, quickly yields the sparsest solution for most well-posed problems.

We will compare the performance of the proposed algorithm against the minimum l^1 norm solution obtained by Linear Programming (LP). It is found by experiment that, with the proper choices of thresholds, the algorithm performs nearly two orders of magnitude faster than interior-point LP solvers while providing the same (or better) accuracy. The figure below shows the typical evolution toward

the solution for a five-iteration instance of the algorithm. We took $m = 1024$ and $n = \lfloor 0.4\, m \rfloor = 409$. Each plot shows the original source vector (black) and its estimate (gray) at the end of an iteration. Sources detected to be active are also marked with a black square above them. With five iterations, we were able to achieve a MSE of the order of 10^{-5} which is the same order of MSE obtained by LP. However, the total CPU time required for the proposed algorithm was measured to be nearly 4 seconds versus approximately 123 seconds required by a LP interior-point solver.



$s_1(\hat{s}_1)$ $s_{1024}(\hat{s}_{1024})$

Key Words: sparse component analysis, source separation