

RIEMANNIAN OPTIMIZATION METHOD ON THE GENERALIZED FLAG MANIFOLD FOR COMPLEX AND SUBSPACE ICA

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Abstract

Independent component analysis (ICA) can be solved in two steps: whitening followed by orthogonal rotation. Thus ICA can be tackled by an optimization on the manifold of orthogonal matrices. Recently researchers have investigated the use of manifolds and Lie group methods for ICA and other signal processing tasks, including the Stiefel and the Grassmann manifolds. The aim of this paper is to introduce a new class of manifold: the *generalized flag manifold*. The generalized flag manifold is a set of orthogonal subspaces and includes the Stiefel and the Grassmann manifolds as special cases. This new manifold naturally arises when we relax the condition of ICA and consider subspace ICA. Subspace ICA assumes the source signal s is decomposed into d -tuples where signals within a particular tuple are allowed to be dependent on each other, while signals belonging to different tuples are statistically independent. Then the manifold of candidate matrices is no longer just the Stiefel manifold. The statistical dependence of signals within tuples gives the manifold an additional symmetry, which makes it into the generalized flag manifold. Moreover, the demixing matrix for complex ICA is a unitary matrix as ordinary ICA, and the pair of the real and imaginary parts of each column vector of the unitary matrix forms a 2-dimensional real subspace which is orthogonal to each other. Therefore complex ICA can also be tackled by an optimization on this flag manifold. We extend the Riemannian optimization method to the flag manifold utilizing our previous geodesic formula for the Stiefel manifold [1], and based on it propose a new learning algorithm for complex and subspace ICA. Simulations validate the effectiveness of our method. A part of this work was first presented in [2].

References:

- [1] Y. Nishimori and S. Akaho, Learning Algorithms Utilizing Quasi-Geodesic Flows on the Stiefel Manifold, *Neurocomputing*, **67**, pp.106-135, 2005.
- [2] Y. Nishimori, S. Akaho, and M D. Plumbley Riemannian Optimization Method on the Flag Manifold for Independent Subspace Analysis, *Proceedings of 6th International Conference ICA2006*, pp.295-302, 2006.