ENTROPIC INFERENCE FOR ASSIGNING PROBABILITIES: SOME DIFFICULTIES IN AXIOMATICS AND APPLICATIONS

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Abstract

The question how to assign probabilities is inescapable in order to develop a theory of plausible inference, yet it is an extraordinarily difficult one. Among the most attractive candidate procedures for this task we find entropic methods, characterized by the extremization of an entropy functional subject to probability constraints representing available information. Notwithstanding several precedents in concrete disciplines, the first proposal of an entropy method as a general scientific inference procedure was due to Jaynes, in the form of the Principle of Maximum Entropy, with extensions in the method of minimum relative entropy of Kullback, and other generalized formulations.

Here we briefly review the different interpretations and uses of entropy methods. Likewise, we examine the various justifications that have been put forward to support them, in particular the appealing attempts to axiomatically derive a unique mathematical expression for entropy procedures in compliance with consistency requirements. In addition to summarizing the state of the art of the rational foundations for entropic inference, which remain controversial and open, we underline the major difficulties analysts encounter in practice when trying to apply these methods. They are: the selection of an entropy functional, the choice of constraints and the selection of reference measures. The latter issues, about which entropy formalisms remain silent, constitute more than just practical obstacles, and ultimately manifest the incompleteness of inference theory.

In our opinion progress can only be attained with a change of perspective. We propose a logical viewpoint of plausible inference, understood not merely as an optimization problem starting from given probability inputs, but as the representation or encoding of the knowledge that basic evidences and other higher-order information or assumptions provide about conjectures. More specifically, a novel generic scheme for inference is presented, which considers two stages and three inference levels.