COMPARING CLASSIC, BAYES AND PARAMETRIC EMPIRICAL BAYES RISKS

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The assumptions on the structure of data or modelling data is a basic problem in the statistical decision making. In this paper we compare the estimator's risks in the different frameworks, with respect to false assumption(s) on the structure of data. More precisely, we compare the robustness of classic, Bayes and parametric empirical Bayes estimators with respect to the prior choice.

Consider independent sample $(X_1, \theta_1), \ldots, (X_n, \theta_n)$ of (X, θ), where $X_i|\theta_i$ has probability density function (pdf) $f(x_i|\theta_i)$ and consider the following cases:

1. In the classical statistics, the prior $\pi(\theta_i)$ is a dirac function, centered on the unknown value θ. That is, $X_1, \ldots, X_n$ are independent and identically distributed random variables with pdf $f(x_i|\theta)$.

2. In the Bayesian case, the prior pdf, $\pi(\theta)$ is no more a degenerate dirac function but is assumed to be perfectly known. In this case, we have only a random parameter θ, i.e. $\theta = \theta_1 = \ldots = \theta_n$, and so the sample $(X_1, \theta), \ldots, (X_n, \theta)$ is not independent and as a result the marginal distributions of X_is are not independent. However, conditional on θ, X_is are independent and have a common pdf $f(x_i|\theta)$.

3. In the parametric empirical Bayes framework, θ_is have pdf $\pi(\theta_i|\tau)$ with the hyper-parameter τ. Conditional on θ_i, X_i has a pdf $f(x_i|\theta_i)$. The common marginal pdf of each X_i is
$$m(x_i|\tau) = \int_{-\infty}^{\infty} f(x_i|\theta_i)\ \pi(\theta_i|\tau)\ \mathrm{d}\theta_i,$$
and $X_1, \ldots, X_n$ are independent.

In this paper, we compare the robustness of estimators in the above frameworks with respect to the false model on data by a Monte Carlo study.

Key Words: Calssic, Bayes and empirical Bayes estimators.

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