GraphMaxEnt

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Abstract. Assume a undirected graph G on a finite domain X and a probability distribution P on X. Graph entropy, defined in terms of the vertex packing polytope, is recast as

$$H(G,P) = \min_R - \sum p_i \log \mathrm{Pl}^{(R)}(x_i),$$

where plausibility $Pl^{(R)}$ is defined wrt probability distribution R on the stable sets Y (independent subsets of G)

$$\mathrm{Pl}^{(R)}(x) = \sum_{Y: x \in Y} R(Y).$$

The plausibility which obtains from the minimising R is called the *plausibility wrt* P on X

$$\mathrm{Pl}_P(x) = \mathrm{Pl}^{(R)}(x), \quad R = \arg\min H(G,P).$$

It serves to define the *graph information distance*

$$D(G,Q\|P) = \sum q_i \log \frac{\mathrm{Pl}_Q(x_i)}{\mathrm{Pl}_P(x_i)}$$

for two distributions Q and P on X, given a (fixed) graph structure G. It is straightforward to offer a similar definition wrt the change of G, though useful results require some restrictions on that change.

One verifies the usual properties of additivity and subadditivity wrt weak products of the supporting graphs. The method of GraphMaxEnt can be formulated accordingly. It is postulated it admits an axiomatisation akin to that for MaxEnt.

Applied to probability kinematics, it permits resolving several problems arising from the AGM belief revision. One obtains

- imaging ('nonproportional conditioning') as minimisation of graph information distance
- Jeffrey-like imaging
- inverse imaging

Applied to trust updating, based on reported experiences, one obtains

- recognition of repeated reports
- recognition of dependent reports

Several further directions are being pursued. On the computational side

- defining DigraphEnt - entropy over directed graphs, or, at least, DAG's
- constructing continuous domain analogs to $H(G,P)$ and $D(G,Q\|P)$
- use of entropy generating functions

On the foundational side, the main question is physical interpretation of graph entropy. It is hoped that a 'virtual reality' can be produced, one that assigns a statistical mechanical meaning to probability kinematics on graph structures.

Key Words: Graph entropy, probability kinematics, AGM model, belief revision, inverse conditioning.