Bayesian Variational Approximation Implementation for linear inverse problem with infinite Gaussian mixture model

Ali Mohammad Djafari, Leila Gharsalli, Mircea Dumitru
Laboratoire des signaux et systems (L2S)
UMR 8506 CNRS-SUPELEC-Univ. Paris Sud 11
Supelec, Plateau de Moulon 01192 Gif-sur-Yvette, FRANCE
e-mail: djafari, leila.gharsalli, mircea.dumitru @lss.supelec.fr,

Abstract

We consider a Bayesian approach to linear inverse problems where an Infinite Gaussian Mixture Model (IGMM) is used as the a priori to enforce the sparsity of the solution. This IGMM is used in a hierarchical way via the hidden variables representing the inverse variances which we want to estimate jointly with with the unknowns. For this the joint posterior probabilistic low of the unknowns and these hidden variables is approximated by a fully separable probabilistic law via the Variational Bayesian Approximation (VBA) approach where the criterion used is the Kullback-Leibler divergence. To obtain the solution this criterion can be optimized either by a classical alternate optimization or by a gradient based algorithm [1] [2]. In this paper we focus on the implementation issues of this algorithms for great dimensional linear inverse problems where we do not have access directly to the huge dimensional matrix representing the forward operator. In many real applications of inverse problems we have access to the results of the forward and the adjoint operators without needing to have explicitly the matrix corresponding to the operators. We consider then particularly these algorithms in this context. At the end we show the results of two applications: estimating the components of a periodic signal and the image reconstruction in X-ray computed tomography.

Key Words: Inverse Problems, Sparse priors, VBA, IGMM, Period Estimation, Biological time series

References

[1] A. Mohammad-Djafari, "Bayesian approach with prior models which enforce sparsity in signal and image processing," *EURASIP Journal on Advances in Signal Processing*, vol. Special issue on Sparse Signal Processing, p. 12:52, 2012.

[2] A. Fraysse and T. Rodet, "A measure-theoretic variational bayesian algorithm for large dimensional problems," 2012.