Modèles markoviens en image et leurs utilisations pour les problèmes inverses en imagerie

Ali MOHAMMAD-DJAFARI
Laboratoire des signaux et systèmes
Supélec, Plateau de Moulon
91192 Gif-sur-Yvette Cedex, FRANCE.

djafari@lss.supelec.fr
http://djafari.free.fr
http://www.lss.supelec.fr/perso/djafari

Plan de l'exposé

- Approche bayésienne pour les problèmes inverses en imagerie
- **Modélisation a priori des signaux et des images**
- Modèles séparables
- **Modèles markoviens** :
  - simples
  - avec variables cachées lignes, contours et régions
- **Modèles de Gauss-Markov-Potts**
- **Aspects mis en oeuvre et calcul bayésien**
- **Exemples d'applications** :
  Tomographie X, Imagerie microondes, Fusion d'images, Super-résolution, Séparation de sources, Imagerie hyperspectrale
- **Conclusions**

Estimation bayésienne pour les problèmes inverses

- Modèle directe : $\boldsymbol{g} = \boldsymbol{H}(\boldsymbol{f}) + \boldsymbol{\epsilon}$
- Vraisemblance : $p(\boldsymbol{g}|\boldsymbol{f}) = p_{\epsilon}(\boldsymbol{g} - \boldsymbol{H}(\boldsymbol{f}))$
- *A priori* $p(\boldsymbol{f})$
- Bayes :

$$p(\boldsymbol{f}|\boldsymbol{g}) = \frac{p(\boldsymbol{g}|\boldsymbol{f})\, p(\boldsymbol{f})}{p(\boldsymbol{g})} \propto p(\boldsymbol{g}|\boldsymbol{f})\, p(\boldsymbol{f})$$

- Choix d'un estimateur:
  - Maximum A posteriori (MAP) :

$$\widehat{\boldsymbol{f}} = \arg\max_{\boldsymbol{f}} \{\ln p(\boldsymbol{f}|\boldsymbol{g}) = \ln p(\boldsymbol{g}|\boldsymbol{f}) + \ln p(\boldsymbol{f})\}$$

  - Espérance *a posteriori* (EAP) :

$$\widehat{\boldsymbol{f}} = \int \boldsymbol{f}\, p(\boldsymbol{f}|\boldsymbol{g})\, \mathrm{d}\boldsymbol{f}$$

Cas linéaire et modèles a priori Gaussiens

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon}$$

- Hypothèse sur les erreurs: $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma_\epsilon^2 \boldsymbol{I})$
  $\boldsymbol{g}|\boldsymbol{f} \sim \mathcal{N}(\boldsymbol{H}\boldsymbol{f}, \beta\boldsymbol{I}) \longrightarrow p(\boldsymbol{g}|\boldsymbol{f}) \propto \exp\left[-\beta\|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2\right], \beta = \frac{1}{2\sigma_\epsilon^2}$

- Modèle a priori $\boldsymbol{f}$:
  $\boldsymbol{f} \sim \mathcal{N}(\mathbf{0}, \sigma_f^2(\boldsymbol{D}^t\boldsymbol{D})^{-1}) \longrightarrow p(\boldsymbol{f}) \propto \exp\left[-\alpha\|\boldsymbol{D}\boldsymbol{f}\|^2\right], \alpha = \frac{1}{2\sigma_f^2}$

- A posteriori:

$$\begin{aligned} p(\boldsymbol{f}|\boldsymbol{g}) &\propto \exp\left[-\|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2 + \lambda\|\boldsymbol{D}\boldsymbol{f}\|^2\right], \lambda = \frac{\sigma_\epsilon^2}{\sigma_f^2} = \frac{\alpha}{\beta} \\ &= \mathcal{N}(\widehat{\boldsymbol{f}}, \widehat{\boldsymbol{P}}) \text{ avec } \widehat{\boldsymbol{f}} = \widehat{\boldsymbol{P}}\boldsymbol{H}^t\boldsymbol{g},\ \widehat{\boldsymbol{P}} = \left(\boldsymbol{H}^t\boldsymbol{H} + \lambda\boldsymbol{D}^t\boldsymbol{D}\right)^{-1} \end{aligned}$$

- EAP = MAP : $\widehat{\boldsymbol{f}} = \arg\max_{\boldsymbol{f}} \{p(\boldsymbol{f}|\boldsymbol{g})\} = \arg\min_{\boldsymbol{f}} \{J(\boldsymbol{f})\}$
  avec $J(\boldsymbol{f}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2 + \lambda\|\boldsymbol{D}\boldsymbol{f}\|^2$

Modélisation a priori des signaux et des images

- Modèles séparables $p(\boldsymbol{f}) = \prod_j p_j(f_j) \propto \exp\left[-\beta \sum_j \phi(f_j)\right]$

$$p(\boldsymbol{f}) \propto \exp\left[-\beta \sum_{\boldsymbol{r}\in\mathcal{R}} \phi(f(\boldsymbol{r}))\right]$$

- Modèles markoviens simples
$p(f_j|f_{j-1}) \propto \exp\left[-\beta\phi(f_j - f_{j-1})\right]$

$$p(\boldsymbol{f}) \propto \exp\left[-\beta \sum_{\boldsymbol{r}\in\mathcal{R}} \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} \phi(f(\boldsymbol{r}), f(\boldsymbol{r}'))\right]$$

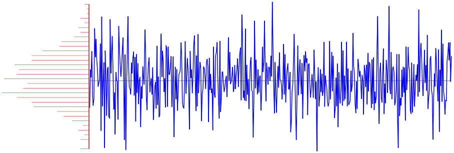
- Modèles markoviens avec variables cachées
$z(\boldsymbol{r})$ (lignes, contours, frontières et régions)

$$p(\boldsymbol{f}|\boldsymbol{z}) \propto \exp\left[-\beta \sum_{\boldsymbol{r}\in\mathcal{R}} \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} \phi(f(\boldsymbol{r}), f(\boldsymbol{r}'), z(\boldsymbol{r}), z(\boldsymbol{r}'))\right]$$

Modèles séparables iid

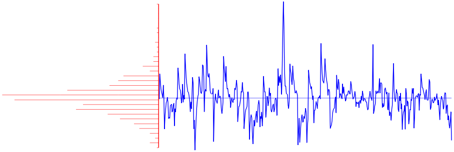
- Gaussienne:
$$p(f_j) \propto \exp\left[-\alpha|f_j|^2\right] \longrightarrow \quad \Omega(\boldsymbol{f}) = \alpha \sum_j |f_j|^2$$
- Gaussienne généralisée:
$$p(f_j) \propto \exp\left[-\alpha|f_j|^p\right], \quad 1 \leq p \leq 2 \longrightarrow \quad \Phi(\boldsymbol{f}) = \alpha \sum_j |f_j|^p,$$
- Gamma: $f_j > 0$
$$p(f_j) \propto f_j^{\alpha} \exp\left[-\beta f_j\right] \longrightarrow \quad \Omega(\boldsymbol{f}) = \alpha \sum_j \ln f_j + \beta \sum_j f_j,$$
- Béta: $1 > f_j > 0$
$$p(f_j) \propto f_j^{\alpha}(1-f_j)^{\beta} \longrightarrow \quad \Omega(\boldsymbol{f}) = \alpha \sum_j \ln f_j + \beta \sum_j \ln(1-f_j),$$

Modèles séparables iid



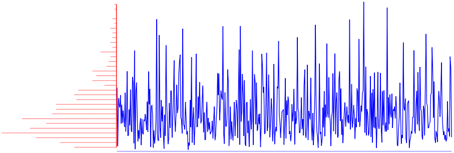
Gaussienne

$p(f_j) \propto \exp\left[-\alpha|f_j|^2\right]$



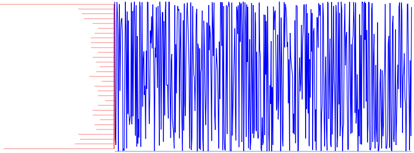
Gaussienne généralisée

$p(f_j) \propto \exp\left[-\alpha|f_j|^p\right], \quad 1 \leq p \leq 2$



Gamma

$p(f_j) \propto f_j^{\alpha} \exp\left[-\beta f_j\right]$



Béta

$p(f_j) \propto f_j^{\alpha}(1 - f_j)^{\beta}$

Modèles markoviens simples

$$p(f_j|\boldsymbol{f}) \propto \exp\left[-\alpha \sum_{i\in v_j} \phi(f_j, f_i)\right] \longrightarrow \quad \Phi(\boldsymbol{f}) = \alpha \sum_j \sum_{i\in V_j} \phi(f_j, f_i)$$

- Cas 1D et un seul voisin $V_j = j-1$:

$$\Phi(\boldsymbol{f}) = \alpha \sum_j \phi(f_j - f_{j-1})$$

- Cas 1D et deux voisins $V_j = \{j-1, j+1\}$:

$$\Phi(\boldsymbol{f}) = \alpha \sum_j \phi\,(f_j - \beta(f_{j-1} + f_{j-1}))$$

- Cas 2D et les quatres voisins:

$$\Phi(\boldsymbol{f}) = \alpha \sum_{\boldsymbol{r}\in\mathcal{R}} \phi\left(f(\boldsymbol{r}) - \beta \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} f(\boldsymbol{r}')\right)$$

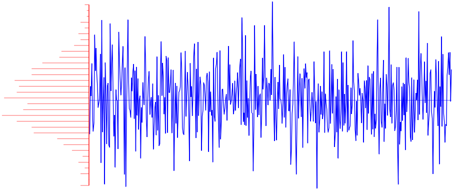
- $\phi(t) =$

$|t|^\gamma$: Gaussienne généralisée

$\begin{cases} |t|^2 & \text{if } |t| < \alpha, \\ \alpha^2 & \text{else,} \end{cases}$, $\begin{cases} t^2 & \text{if } |t| < \alpha, \\ 2\alpha t - \alpha^2 & \text{else,} \end{cases}$, $\frac{\alpha^2 t^2}{1+t^2}$, $\log\cosh(t/\alpha)$

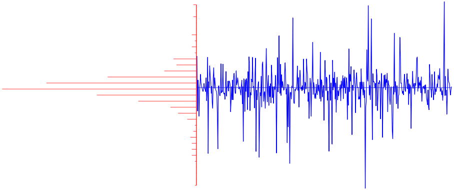
Modèles markoviens simples

Séparable



Gaussienne iid

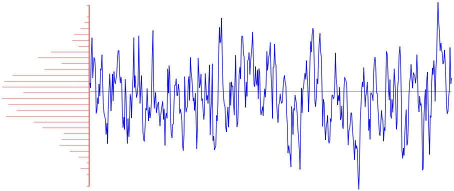
$p(f_j) \propto \exp\left[-\alpha |f_j|^2\right]$



Gaussienne généralisée iid

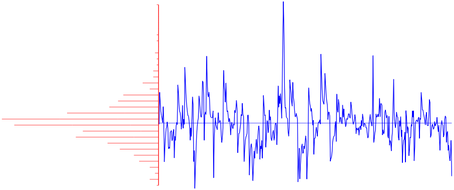
$p(f_j) \propto \exp\left[-\alpha |f_j|^p\right]$

Markovien



Gauss-Markov

$p(f_j | f_{j-1}) \propto \exp\left[-\alpha |f_j - f_{j-1}|^2\right]$



Gaussienne généralisée markovienne

$p(f_j | f_{j-1}) \propto \exp\left[-\alpha |f_j - f_{j-1}|^p\right]$

Modèles gaussiens en terme de variables cachées

- Mode Analyse

$$\begin{cases} \boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon} \\ \boldsymbol{f} \sim \mathcal{N}\left(\mathbf{0}, \sigma_f^2(\boldsymbol{D}^t\boldsymbol{D})^{-1}\right)\end{cases} = \begin{cases} \boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon} \\ \boldsymbol{f} = \boldsymbol{C}\boldsymbol{f} + \boldsymbol{z} \ \text{ avec} \\ \boldsymbol{z} \sim \mathcal{N}(\mathbf{0}, \sigma_f^2\boldsymbol{I}) \text{ et } \boldsymbol{D} = (\boldsymbol{I} - \boldsymbol{C})\end{cases}$$

$$\boldsymbol{f}|\boldsymbol{g} \sim \mathcal{N}(\widehat{\boldsymbol{f}}, \widehat{\boldsymbol{P}}) \ \text{ avec } \ \widehat{\boldsymbol{f}} = \widehat{\boldsymbol{P}}\boldsymbol{H}^t\boldsymbol{g}, \quad \widehat{\boldsymbol{P}} = \left(\boldsymbol{H}^t\boldsymbol{H} + \lambda\boldsymbol{D}^t\boldsymbol{D}\right)^{-1}$$

$$\ln p(\boldsymbol{f}|\boldsymbol{g}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2 + \lambda\|\boldsymbol{D}\boldsymbol{f}\|^2$$

- Mode Synthèse

$$\begin{cases} \boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon} \\ \boldsymbol{f} \sim \mathcal{N}\left(\mathbf{0}, \sigma_f^2(\boldsymbol{D}\boldsymbol{D}^t)\right)\end{cases} = \begin{cases} \boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon} \\ \boldsymbol{f} = \boldsymbol{D}\boldsymbol{z} \ \text{ avec } \ \boldsymbol{z} \sim \mathcal{N}(\mathbf{0}, \sigma_f^2\boldsymbol{I})\end{cases}$$

$$\boldsymbol{z}|\boldsymbol{g} \sim \mathcal{N}(\widehat{\boldsymbol{z}}, \widehat{\boldsymbol{P}}), \widehat{\boldsymbol{z}} = \widehat{\boldsymbol{P}}\boldsymbol{D}^t\boldsymbol{H}^t\boldsymbol{g}, \ \widehat{\boldsymbol{P}} = \left(\boldsymbol{D}^t\boldsymbol{H}^t\boldsymbol{H}\boldsymbol{D} + \lambda\boldsymbol{I}\right)^{-1}, \widehat{\boldsymbol{f}} = \boldsymbol{D}\widehat{\boldsymbol{z}}$$

$$\ln p(\boldsymbol{z}|\boldsymbol{g}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{D}\boldsymbol{z}\|^2 + \lambda\|\boldsymbol{z}\|^2$$

$\boldsymbol{z}$ coefficients de décomposition sur une base
(colonnes de $\boldsymbol{D}$ forment une base)

Parcimonie et "Compressed Sensing"

$$\left\{ \begin{array}{l} \boldsymbol{g} = \boldsymbol{H} \boldsymbol{f} + \boldsymbol{\epsilon} \\ \boldsymbol{f} = \boldsymbol{D} \boldsymbol{z} \end{array} \right.$$

- $\boldsymbol{D}$ "code book", $\boldsymbol{z}$ coefficients de décomposition
- Gaussienne : $p(\boldsymbol{z}) = \mathcal{N}(\mathbf{0}, \sigma_f^2 \boldsymbol{I}) \propto \exp\left[-\alpha \sum_j |z_j|^2\right]$

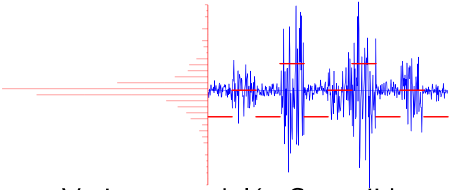
$$J(\boldsymbol{z}) = -\ln p(\boldsymbol{z}|\boldsymbol{g}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{D}\boldsymbol{z}\|^2 + \lambda \sum_j |z_j|^2$$

- Gaussienne généralisée : $p(\boldsymbol{z}) \propto \exp\left[-\alpha \sum_j |z_j|^\beta\right]$

$$J(\boldsymbol{z}) = -\ln p(\boldsymbol{z}|\boldsymbol{g}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{D}\boldsymbol{z}\|^2 + \lambda \sum_j |z_j|^\beta$$

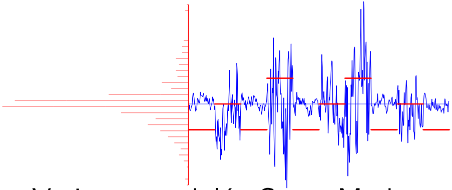
- $\widehat{\boldsymbol{z}} = \arg\min_{\boldsymbol{z}} \{J(\boldsymbol{z})\} \longrightarrow \widehat{\boldsymbol{f}} = \boldsymbol{D}\widehat{\boldsymbol{z}}$

Signaux non stationnaires



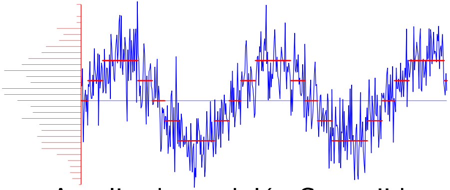
Variance modulée Gauss iid

$$p(f_j|z_j) = \mathcal{N}(0, v(z_j))$$



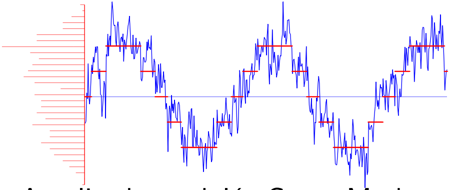
Variance modulée Gauss-Markov

$$p(f_j|f_{j-1}, z_j) = \mathcal{N}(f_{j-1}, v(z_j))$$



Amplitude modulée Gauss iid

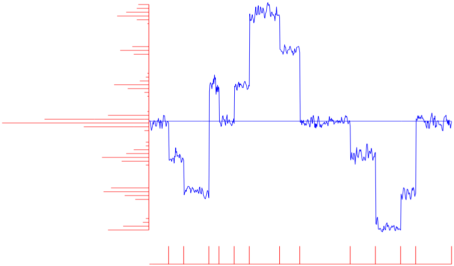
$$p(f_j|z_j) = \mathcal{N}(a(z_j), 1)$$



Amplitude modulée Gauss-Markov

$$p(f_j|f_{j-1}, z_j) = \mathcal{N}(a(f_{j-1}, z_j), 1)$$

Modèles markoviens avec des variables cachées

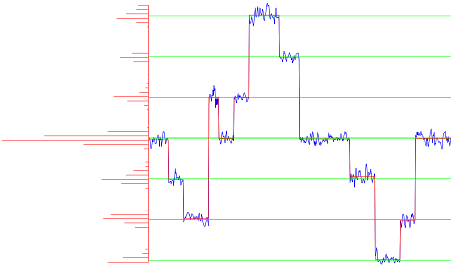


Gaussiennes par morceaux

var. cachées lignes ou contours q_j

$$p(f_j|q_j, f_{j-1}) = \mathcal{N}\left((1-q_j)f_{j-1}, \sigma_f^2\right)$$

$$p(\boldsymbol{f}|\boldsymbol{q}) \propto \exp\left[-\alpha \sum_j |f_j - (1-q_j)f_{j-1}|^2\right]$$



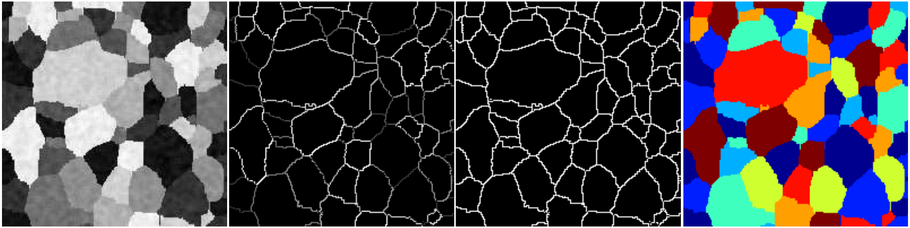
Mélange de gaussiennes

var. cachées étiquettes des régions z_j

$$p(f_j|z_j = k) = \mathcal{N}\left(m_k, \sigma_k^2\right), \; z_j \text{ markovien}$$

$$p(\boldsymbol{f}|\boldsymbol{z}) \propto \exp\left[-\alpha \sum_k \sum_{j\in\mathcal{R}_k} \left(\frac{f_j - m_k}{\sigma_k}\right)^2\right]$$

Modèles markoviens avec des variables cachées

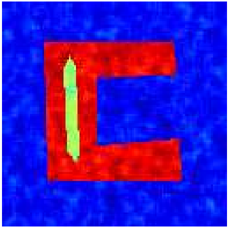


$f(\boldsymbol{r})$	$l(\boldsymbol{r}) \in [0,1]$	$q(\boldsymbol{r}) \in \{0,1\}$	$z(\boldsymbol{r}) \in \{1,\cdots,K\}$
intensités	lignes	contours	étiquettes régions

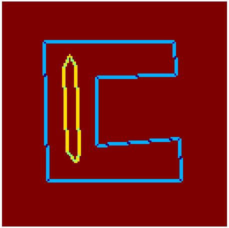
$$\mathcal{R} = \{(i,j) : 1 \leq i \leq m, 1 \leq j \leq n\} \quad \text{sites pixels}$$
$$\mathcal{R}_k = \{\boldsymbol{r} : \boldsymbol{r} \in \mathcal{R}_k\}\,, \quad \cup_k \mathcal{R}_k = \mathcal{R} \quad \text{régions}$$

$$\boldsymbol{f} = \{f(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{R}\} \quad \text{image (intensités)}$$
$$\boldsymbol{l} = \{l(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{R}\} \quad \text{lignes} \quad l(\boldsymbol{r}) \in [0,1]$$
$$\boldsymbol{q} = \{q(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{R}\} \quad \text{contours} \quad q(\boldsymbol{r}) \in \{0,1\}$$
$$\boldsymbol{z} = \{z(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{R}\} \quad \text{étiquettes régions} \quad z(\boldsymbol{r}) \in \{1,\cdots,K\}$$

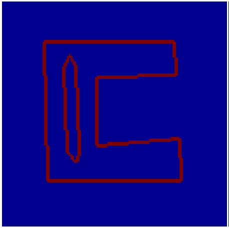
Modèles markoviens avec des variables cachées



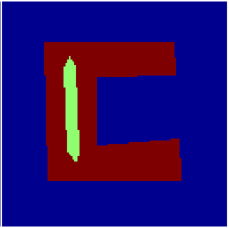
$f(\boldsymbol{r})$



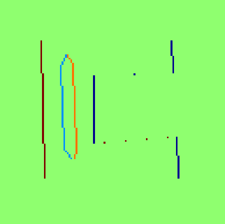
$l(\boldsymbol{r})$



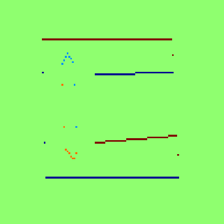
$q(\boldsymbol{r})$



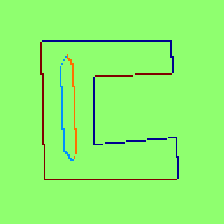
$z(\boldsymbol{r})$



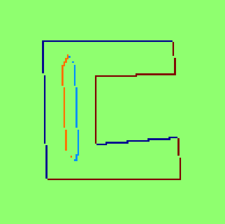
$l_x(\boldsymbol{r})$



$l_y(\boldsymbol{r})$



$l_{yx}(\boldsymbol{r})$



$l_{xy}(\boldsymbol{r})$

Modèles markoviens avec des variables cachées

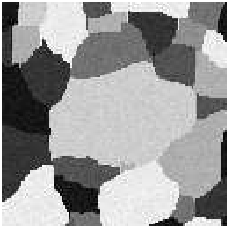
A chaque pixel de l'image est associées 2 variables $f(\boldsymbol{r})$ et $z(\boldsymbol{r})$

- $\boldsymbol{f}|\boldsymbol{z}$ Gaussienne iid, $\boldsymbol{z}$ iid :
  Mélange de gaussiennes (MoG)

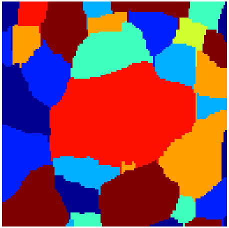
- $\boldsymbol{f}|\boldsymbol{z}$ Gauss-Markov, $\boldsymbol{z}$ iid :
  Mélange de Gauss-Markov (MoGM)

- $\boldsymbol{f}|\boldsymbol{z}$ Gaussienne iid, $\boldsymbol{z}$ Potts-Markov :
  Mélange de gaussiennes iid
  (avec var. cachée Potts)

- $\boldsymbol{f}|\boldsymbol{z}$ Markov, $\boldsymbol{z}$ Potts-Markov :
  Mélange de Gauss-Markov
  (avec var. cachée Potts)

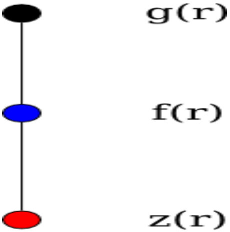


$f(\boldsymbol{r})$

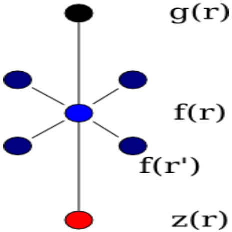


$z(\boldsymbol{r})$

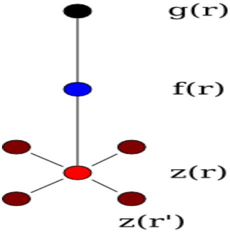
Quatre modèles



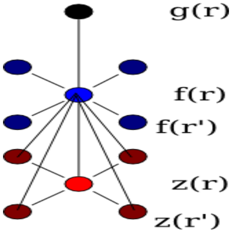
Modèle 1: MoG



Modèle 2: MoGM

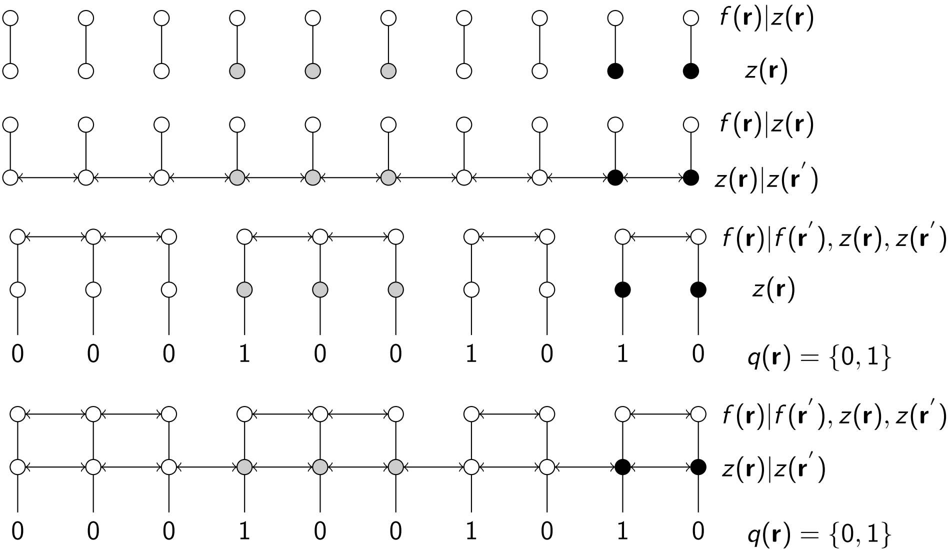


Modèle 3: Gauss-Markov-Potts 1



Modèle 4: Gauss-Markov-Potts 2

Quatre modèles



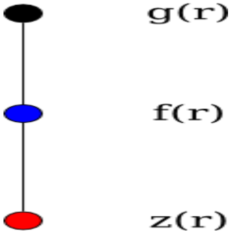
Modèle 1: $\quad f|z$ Gaussienne iid, $\quad z$ iid

Independent Mixture of Independent Gaussians (IMIG):

$$p(f(\boldsymbol{r})|z(\boldsymbol{r})=k)=\mathcal{N}(m_k,v_k), \quad \forall \boldsymbol{r}\in\mathcal{R}$$

$$p(f(\boldsymbol{r}))=\sum_{k=1}^K \alpha_k \mathcal{N}(m_k,v_k), \text{with } \sum_k \alpha_k=1.$$

$$p(\boldsymbol{z})=\prod_{\boldsymbol{r}} p(z(\boldsymbol{r})=k)=\prod_{\boldsymbol{r}} \alpha_k=\prod_k \alpha_k^{n_k}$$



Notant $\quad \mathcal{R}_k=\{\boldsymbol{r}: z(\boldsymbol{r})=k\}, \quad \mathcal{R}=\cup_k \mathcal{R}_k,$

$$m_z(\boldsymbol{r})=m_k, v_z(\boldsymbol{r})=v_k, \alpha_z(\boldsymbol{r})=\alpha_k, \forall \boldsymbol{r}\in\mathcal{R}_k$$

on a:

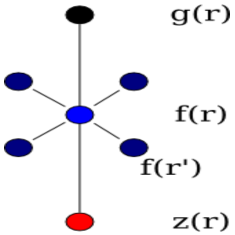
$$\begin{aligned}
p(\boldsymbol{f}|\boldsymbol{z}) &= \prod_{\boldsymbol{r}\in\mathcal{R}} \mathcal{N}(m_z(\boldsymbol{r}), v_z(\boldsymbol{r})) \\
p(\boldsymbol{z}) &= \prod_{\boldsymbol{r}} \alpha_z(\boldsymbol{r}) = \prod_k \alpha_k^{\sum_{\boldsymbol{r}\in\mathcal{R}} \delta(z(\boldsymbol{r})-k)} = \prod_k \alpha_k^{n_k}
\end{aligned}$$

Modèle 2: $\boldsymbol{f}|\boldsymbol{z}$ Gauss-Markov, $\boldsymbol{z}$ iid

Independent Mixture of Gauss-Markov (IMGM) :

$$p(f(\boldsymbol{r})|z(\boldsymbol{r}), z(\boldsymbol{r}'), f(\boldsymbol{r}'), \boldsymbol{r}' \in \mathcal{V}(\boldsymbol{r}))$$

$$= \mathcal{N}(\mu_z(\boldsymbol{r}), v_z(\boldsymbol{r})), \forall \boldsymbol{r} \in \mathcal{R}$$



$$\begin{aligned}
\mu_z(\boldsymbol{r}) &= \frac{1}{|\mathcal{V}(\boldsymbol{r})|} \sum_{\boldsymbol{r}' \in \mathcal{V}(\boldsymbol{r})} \mu_z^*(\boldsymbol{r}') \\
\mu_z^*(\boldsymbol{r}') &= \delta(z(\boldsymbol{r}') - z(\boldsymbol{r}))\, f(\boldsymbol{r}') + (1 - \delta(z(\boldsymbol{r}') - z(\boldsymbol{r}))\, m_z(\boldsymbol{r}') \\
&= \quad (1 - c(\boldsymbol{r}')) \quad f(\boldsymbol{r}') \;+ \qquad c(\boldsymbol{r}') \qquad m_z(\boldsymbol{r}')
\end{aligned}$$

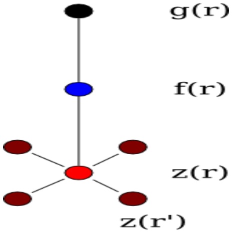
$$\begin{aligned}
p(\boldsymbol{f}|\boldsymbol{z}) &\propto \prod_{\boldsymbol{r}} \mathcal{N}(\mu_z(\boldsymbol{r}), v_z(\boldsymbol{r})) & &\propto \prod_k \alpha_k \mathcal{N}(m_k \mathbf{1}, \boldsymbol{\Sigma}_k) \\
p(\boldsymbol{z}) &= \prod_{\boldsymbol{r}} v_z(\boldsymbol{r}) & &= \prod_k \alpha_k^{n_k}
\end{aligned}$$

with $\mathbf{1}_k = 1, \forall \boldsymbol{r} \in \mathcal{R}_k$ and $\boldsymbol{\Sigma}_k$ a covariance matrix $(n_k \times n_k)$.

Modèle 3: $\boldsymbol{f}|\boldsymbol{z}$ Gauss iid, $\boldsymbol{z}$ Potts

Gauss iid comme dans le cas du Modèle 1 :

$$\begin{aligned} p(\boldsymbol{f}|\boldsymbol{z}) &= \prod_{\boldsymbol{r}\in\mathcal{R}} \mathcal{N}(m_z(\boldsymbol{r}), v_z(\boldsymbol{r})) \\ &= \prod_k \prod_{\boldsymbol{r}\in\mathcal{R}_k} \mathcal{N}(m_k, v_k) \end{aligned}$$



Potts-Markov:

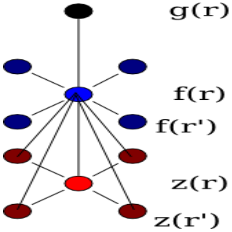
$$p(\boldsymbol{z}) \propto \exp\left[\gamma \sum_{\boldsymbol{r}\in\mathcal{R}} \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} \delta(z(\boldsymbol{r}) - z(\boldsymbol{r}'))\right]$$

Modèle 4: $\boldsymbol{f}|\boldsymbol{z}$ Gauss-Markov, $\boldsymbol{z}$ Potts

Gauss-Markov comme dans le cas du modèle 2 :

$p(f(\boldsymbol{r})|z(\boldsymbol{r}), z(\boldsymbol{r}'), f(\boldsymbol{r}'), \boldsymbol{r}' \in \mathcal{V}(\boldsymbol{r})) =$

$\mathcal{N}(\mu_z(\boldsymbol{r}), v_z(\boldsymbol{r})), \forall \boldsymbol{r} \in \mathcal{R}$



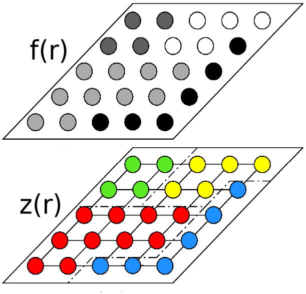
$$p(\boldsymbol{f}|\boldsymbol{z}) \quad \propto \prod_{\boldsymbol{r}} \mathcal{N}(\mu_z(\boldsymbol{r}), v_z(\boldsymbol{r})) \propto \prod_k \alpha_k \mathcal{N}(m_k \mathbf{1}, \boldsymbol{\Sigma}_k)$$

Potts-Markov comme dans le cas du Modèle 3:

$$p(\boldsymbol{z}) \propto \exp\left[\gamma \sum_{\boldsymbol{r}\in\mathcal{R}} \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} \delta(z(\boldsymbol{r}) - z(\boldsymbol{r}'))\right]$$

Les deux modèles Gauss-Markov-Potts retenus en imagerie

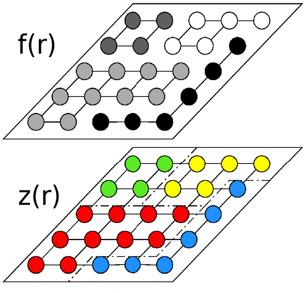
Gauss-Markov-Potts 1



$\boldsymbol{f}|\boldsymbol{z}$ Gaussienne iid
$\boldsymbol{z}$ Potts-Markov

(MIG with Hidden Potts)

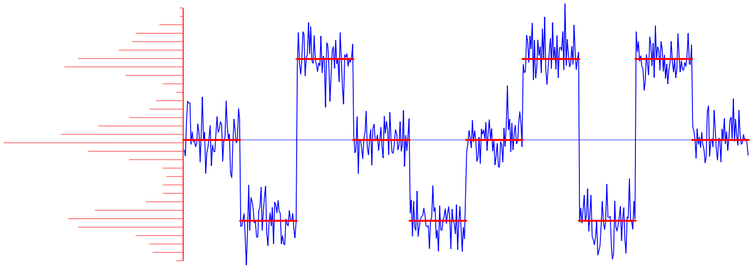
Gauss-Markov-Potts 2

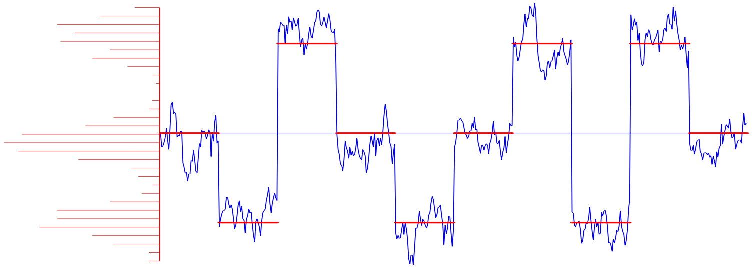


$\boldsymbol{f}|\boldsymbol{z}$ Markov
$\boldsymbol{z}$ Potts-Markov

(MGM with hidden Potts)

Les deux modèles retenus (Cas 1D)





Modèles a priori choisis et inférence bayésienne

- Modèle d'observation:

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon} \longrightarrow p(\boldsymbol{g}|\boldsymbol{f}) = \mathcal{N}(\boldsymbol{H}\boldsymbol{f}, {\sigma_\epsilon}^2\boldsymbol{I})$$

- Modèle a priori

$$\begin{array}{ll} p(\boldsymbol{f},\boldsymbol{z}) = p(\boldsymbol{f}|\boldsymbol{z})\, p(\boldsymbol{z}) & \\ p(\boldsymbol{f}|\boldsymbol{z}) = \prod_k p(\boldsymbol{f}_k) \;\text{ avec }\; p(\boldsymbol{f}_k) = \mathcal{N}(\boldsymbol{f}_k | m_k \mathbf{1}_k, \boldsymbol{\Sigma}_k) & \\ p(\boldsymbol{z}) \quad \propto \exp\left[\gamma \sum_{\boldsymbol{r}\in\mathcal{R}} \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} \delta(z(\boldsymbol{r}) - z(\boldsymbol{r}'))\right] & \end{array}$$

- Hyperparamètres $\boldsymbol{\theta} = \{{\sigma_\epsilon}^2, (m_k, \sigma_k^2), k = 1, \cdots, K\}$, $\{\gamma, K\}$

$$\begin{array}{ll} p(m_k) = \mathcal{N}(m_k | m_{k0}, {\sigma_k}_0^2), & p(\sigma_k^2) = \mathcal{IG}(\sigma_k^2 | \alpha_{k0}, \beta_{k0}), \\ p(\boldsymbol{\Sigma}_k) = \mathcal{IW}(\boldsymbol{\Sigma}_k | \alpha_{k0}, \boldsymbol{\Lambda}_{k0}), & p({\sigma_\epsilon}_i^2) = \mathcal{IG}({\sigma_\epsilon}_i^2 | \alpha_0^{\epsilon_i}, \beta_0^{\epsilon_i}). \end{array}$$

- Loi *a posteriori* conjointe de $\boldsymbol{f}$, $\boldsymbol{z}$ et $\boldsymbol{\theta}$

$$p(\boldsymbol{f},\boldsymbol{z},\boldsymbol{\theta}|\boldsymbol{g}) \propto p(\boldsymbol{g}|\boldsymbol{f},\boldsymbol{\theta}_1)\; p(\boldsymbol{f}|\boldsymbol{z},\boldsymbol{\theta}_2)\; p(\boldsymbol{z}|\boldsymbol{\theta}_3)\; p(\boldsymbol{\theta})$$

Estimation conjointe de $(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta})$ utilisant $p(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta}|\boldsymbol{g})$

$$
\left\{
\begin{array}{lcl}
\widehat{\boldsymbol{\theta}} & \sim & p(\boldsymbol{\theta}|\widehat{\boldsymbol{f}}, \widehat{\boldsymbol{z}}, \boldsymbol{g}) \quad \text{ou} \quad p(\boldsymbol{\theta}|\widehat{\boldsymbol{z}}, \boldsymbol{g}) \\
\widehat{\boldsymbol{z}} & \sim & p(\boldsymbol{z}|\widehat{\boldsymbol{f}}, \widehat{\boldsymbol{\theta}}, \boldsymbol{g}) \quad \text{ou} \quad p(\boldsymbol{z}|\widehat{\boldsymbol{\theta}}, \boldsymbol{g}) \\
\widehat{\boldsymbol{f}} & \sim & p(\boldsymbol{f}|\widehat{\boldsymbol{z}}, \widehat{\boldsymbol{\theta}}, \boldsymbol{g})
\end{array}
\right.
$$

$\sim$ signifie :

soit arg max (MAP conjointe)

soit échantillonner suivant (Gibbs)

$$
\left\{
\begin{array}{rcll}
p(\boldsymbol{f}|\boldsymbol{z}, \boldsymbol{\theta}, \boldsymbol{g}) & = & p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{\theta}_1)\, p(\boldsymbol{f}|\boldsymbol{z}, \boldsymbol{\theta}_2) / p(\boldsymbol{g}|\boldsymbol{z}, \boldsymbol{\theta}) & \text{Gaussienne} \\
p(\boldsymbol{z}|\boldsymbol{f}, \boldsymbol{\theta}, \boldsymbol{g}) & \propto & p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{\theta}_1)\, p(\boldsymbol{f}|\boldsymbol{z}, \boldsymbol{\theta}_2)\, p(\boldsymbol{z}) & \text{Potts} \\
p(\boldsymbol{z}|\boldsymbol{\theta}, \boldsymbol{g}) & = & p(\boldsymbol{g}|\boldsymbol{z}, \boldsymbol{\theta})\, p(\boldsymbol{z}) / p(\boldsymbol{g}|\boldsymbol{\theta}) & \text{Potts} \\
p(\boldsymbol{\theta}|\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{g}) & = & p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{\theta}_1)\, p(\boldsymbol{f}|\boldsymbol{z}, \boldsymbol{\theta}_2)\, p(\boldsymbol{\theta}) / p(\boldsymbol{g}) & \text{Gaussienne/Inverse Gam} \\
p(\boldsymbol{\theta}|\boldsymbol{z}, \boldsymbol{g}) & = & p(\boldsymbol{g}|\boldsymbol{z}, \boldsymbol{\theta})\, p(\boldsymbol{\theta}) / p(\boldsymbol{g}) & \text{Gaussienne/Inverse Gam}
\end{array}
\right.
$$

$$
\boldsymbol{\theta}_1 = {\sigma_\epsilon}^2,\ \boldsymbol{\theta}_2 = \{(m_k, \sigma_k^2), k = 1, \cdots, K\},\ \boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)
$$

Aspects calculs

$$p(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta}|\boldsymbol{g}) \propto p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{z}, v_\epsilon)\, p(\boldsymbol{f}|\boldsymbol{z}, \boldsymbol{m}, \boldsymbol{v})\, p(\boldsymbol{z}|\boldsymbol{\alpha}, \gamma, K)\, p(\boldsymbol{\theta})$$

$$\boldsymbol{\theta} = \{v_\epsilon, (\alpha_k, m_k, v_k), k = 1, \cdot, K\}$$

- L'expression de $p(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta}|\boldsymbol{g})$ est trop complexe
- Approximations possibles :
  - Gauss-Laplace
  - Exploration utilisant les méthodes MCMC
  - Approximations Variationnelles
- Principale idée dans les approches approximations variationnelles :
  Approcher $p(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta}|\boldsymbol{g})$ par une loi plus simple $q(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta})$
  - Choix du critère d'approximation : $KL(q : p)$
  - Choix de familles et de la structure des lois approchantes
    $q(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta}) = q_1(\boldsymbol{f}|\boldsymbol{z})\ q_2(\boldsymbol{z})\ q_3(\boldsymbol{\theta})$
    $q(\boldsymbol{f}, \boldsymbol{z}, \boldsymbol{\theta}) = q_1(\boldsymbol{f})\ q_2(\boldsymbol{z})\ q_3(\boldsymbol{\theta})$
    Familles exponentielles, Lois conjuguées

Applications

- Tomographie X (X ray Computed Tomography)
- Tomographie microondes (Microwave Diffraction Tomography)
- Tomographie Optique de Diffraction (Optical Diffraction Tomography)
- Fusion d'images
- Imagerie hyperspectrale
- Séparation de sources en imagerie satelitaires
- Segmentation d'une séquence vidéo
- Super-résolution
- Restauration des documents ancients par sparation d'images
- Imagerie gamma 3D en CND (coll. avec EDF)
- Tomographie X 3D pour des micro structures (coll. avec CEA)
- TEP Spatio-Temporelle (CEA LIST, SHFJ)
- Imagerie SAR Passive (ONERA)

Modèles de Gauss-Markov (signaux et images continus)

$$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon}$$

$f(\boldsymbol{r})$ image continue : Gauss-Markov

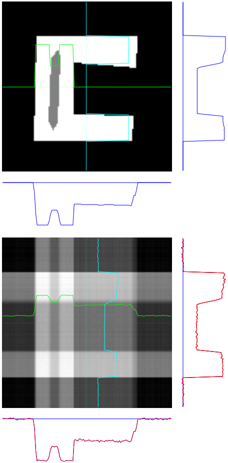
$$p(f(\boldsymbol{r})|f(\boldsymbol{r}')) = \mathcal{N}\left(\beta \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} f(\boldsymbol{r}'), \sigma_f^2\right)$$

MAP :

$$\widehat{\boldsymbol{f}} = \arg\max_{\boldsymbol{f}} \{p(\boldsymbol{f}|\boldsymbol{g})\} = \arg\min_{\boldsymbol{f}} \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2 + \sum_{\boldsymbol{r}} \left(f(\boldsymbol{r}) - \beta \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} f(\boldsymbol{r}')\right)^2$$

MAP =Rgularisation quadratique



Modèles markoviens avec variables cachées lignes

$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon},$

$f(\boldsymbol{r})$ images continues par morceaux :

Variables cachées lignes ou contours: $q(\boldsymbol{r})$

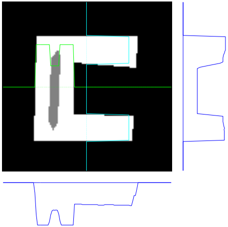
$$p(f(\boldsymbol{r})|q(\boldsymbol{r}), f(\boldsymbol{r}')) = \mathcal{N}\left(\beta(1-q(\boldsymbol{r}))\sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} f(\boldsymbol{r}'), \sigma_f^2\right)$$

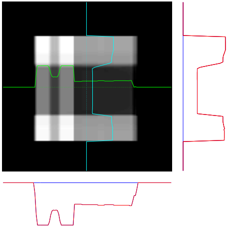
$$\text{MAP} : (\widehat{\boldsymbol{f}}, \widehat{\boldsymbol{q}}) = \arg\max_{\boldsymbol{f},\boldsymbol{q}} \{p(\boldsymbol{f}, \boldsymbol{q}|\boldsymbol{g})\}$$

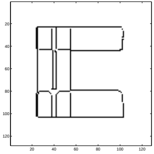
$$\widehat{\boldsymbol{f}} = \arg\max_{\boldsymbol{f}} \{p(\boldsymbol{f}|\boldsymbol{g}, \boldsymbol{q})\} = \arg\min_{\boldsymbol{f}} \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2 + \sum_{\boldsymbol{r}}(1-q(\boldsymbol{r}))\left(f(\boldsymbol{r}) - \beta\sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} f(\boldsymbol{r}')\right)^2$$

$$\widehat{\boldsymbol{q}} = \arg\max_{\boldsymbol{q}} \{p(\boldsymbol{q}|\boldsymbol{g})\}$$







Lien entre modèles à variables cachées et modèles avec potentiels non convexes

$$(\widehat{\boldsymbol{f}}, \widehat{\boldsymbol{q}}) = \arg\max_{\boldsymbol{f},\boldsymbol{q}} \{p(\boldsymbol{f}, \boldsymbol{q}|\boldsymbol{g})\}$$

$$\widehat{\boldsymbol{f}} = \arg\max_{\boldsymbol{f}} \{p(\boldsymbol{f}|\boldsymbol{g}, \boldsymbol{q})\} = \arg\min_{\boldsymbol{f}} \{J(\boldsymbol{f})\}$$

$$J(\boldsymbol{f}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2 + \sum_{\boldsymbol{r}} (1 - q(\boldsymbol{r})) \left(f(\boldsymbol{r}) - \beta \sum_{\boldsymbol{r}' \in \mathcal{V}(\boldsymbol{r})} f(\boldsymbol{r}') \right)^2$$

$$\widehat{\boldsymbol{q}} = \arg\max_{\boldsymbol{q}} \{p(\boldsymbol{q}|\boldsymbol{g})\}$$

$=$

$$\widehat{\boldsymbol{f}} = \arg\min_{\boldsymbol{f}} \{J_1(\boldsymbol{f}|\boldsymbol{g})\}$$

$$J_1(\boldsymbol{f}) = \|\boldsymbol{g} - \boldsymbol{H}\boldsymbol{f}\|^2 + \sum_{\boldsymbol{r}} \phi \left(f(\boldsymbol{r}) - \beta \sum_{\boldsymbol{r}' \in \mathcal{V}(\boldsymbol{r})} f(\boldsymbol{r}') \right)$$

$$\phi(t) = \begin{cases} |t|^2 & \text{if } |t| < \alpha, \\ \alpha^2 & \text{else,} \end{cases}$$

Modèles markoviens avec variables cachées étiquettes régions

$\boldsymbol{g} = \boldsymbol{H}\boldsymbol{f} + \boldsymbol{\epsilon}$

$\boldsymbol{f}$ représente l'image d'un objet composés d'un nombre finie de matériaux homogènes

Variables cachées étiquettes

$z(\boldsymbol{r}) = k, \quad k = 1, \cdots, K$

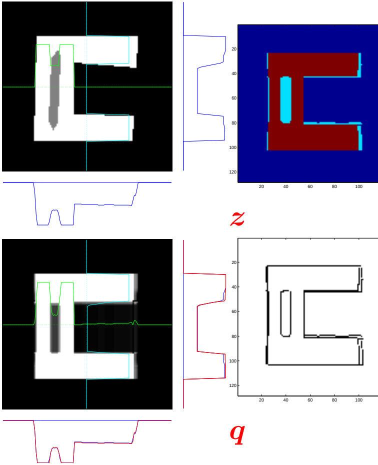
$\mathcal{R}_k = \{\boldsymbol{r} \;:\; z(\boldsymbol{r}) = k\}, \quad \mathcal{R} = \cup_k \mathcal{R}_k$

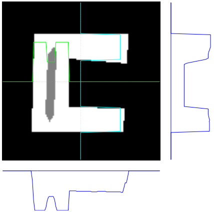
$p(f(\boldsymbol{r})|z(\boldsymbol{r}) = k) = \mathcal{N}(f(\boldsymbol{r})|m_k, \sigma_k^2)$

$\boldsymbol{z} = \{z(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{R}\}$ image segmentée

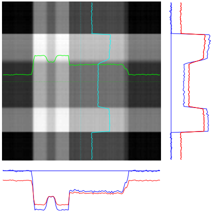
Champ de Potts :

$$p(\boldsymbol{z}) \propto \exp\left[\alpha \sum_{\boldsymbol{r}\in\mathcal{R}} \sum_{\boldsymbol{r}'\in\mathcal{V}(\boldsymbol{r})} \delta(z(\boldsymbol{r}) - z(\boldsymbol{r}'))\right]$$

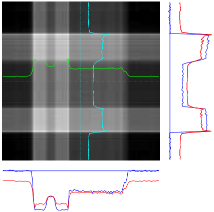




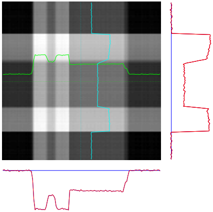
Original



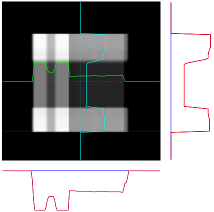
Backprojection



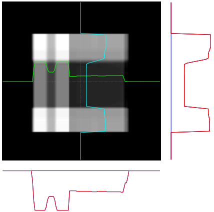
Filtered BP



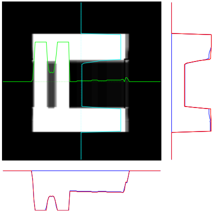
LS



Gauss-Markov+pos+supp $\boldsymbol{f}$

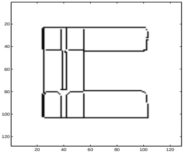


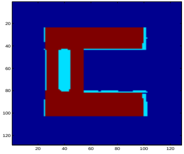
GM+Line process $\boldsymbol{f}, \boldsymbol{q}$

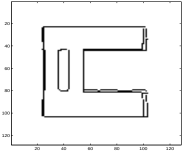


GM+Label process $\boldsymbol{f}, \boldsymbol{z}$

$\boldsymbol{q}$







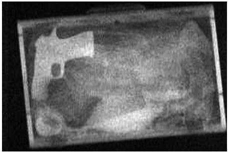
Fusion d'image et segmentation jointe

[Olivier FÉRON]

$$\begin{cases} g_i(\boldsymbol{r}) = f_i(\boldsymbol{r}) + \epsilon_i(\boldsymbol{r}) \\ p(f_i(\boldsymbol{r})|z(\boldsymbol{r}) = k) = \mathcal{N}(m_{i\,k}, \sigma_{i\,k}^2) \\ p(\underline{\boldsymbol{f}}|\boldsymbol{z}) = \prod_i p(\boldsymbol{f}_i|\boldsymbol{z}) \end{cases}$$

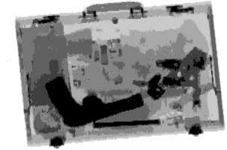
$\boldsymbol{g}_1$

$\boldsymbol{g}_2$

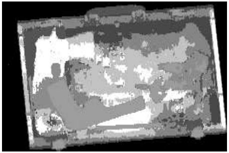


$\longrightarrow$

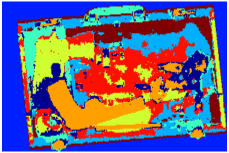
$\widehat{\boldsymbol{f}}_1$



$\widehat{\boldsymbol{f}}_2$

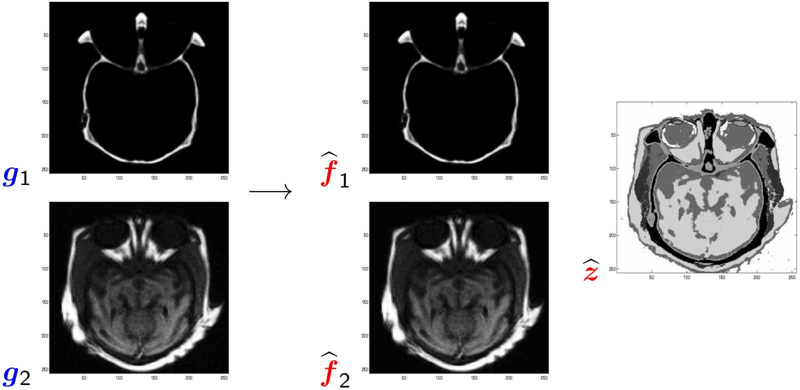


$\widehat{\boldsymbol{z}}$



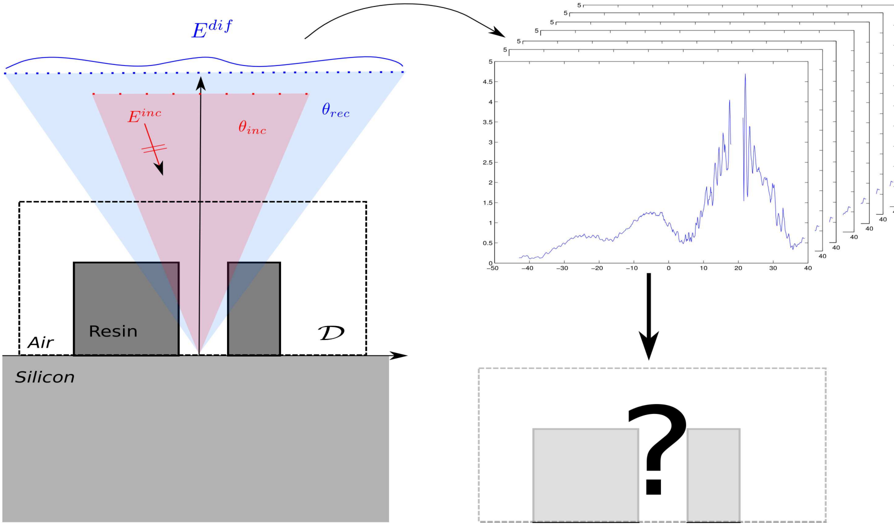
Fusion d'image en imagerie médicale

[Olivier FÉRON]



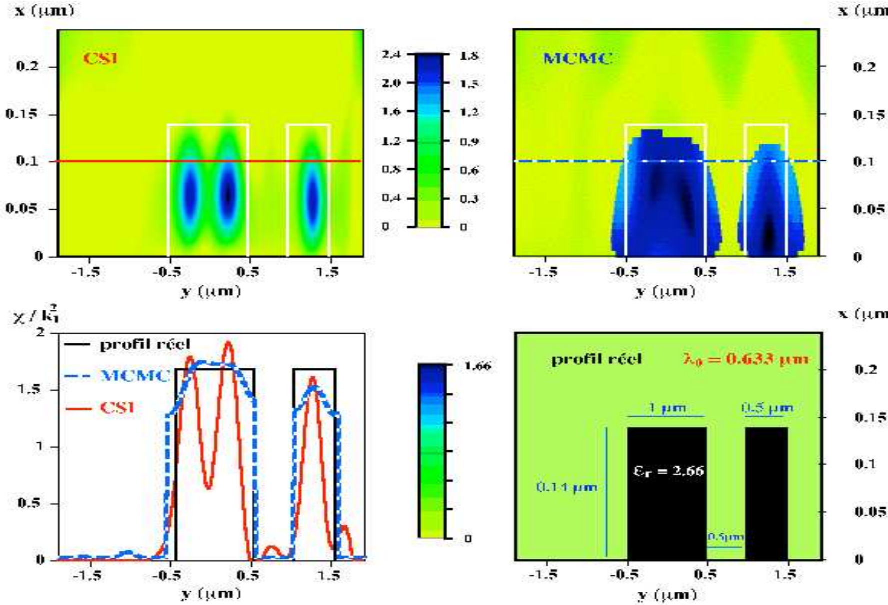
Imagerie optique de diffraction (ODT)

[H. Ayasso, B. Duchêne]



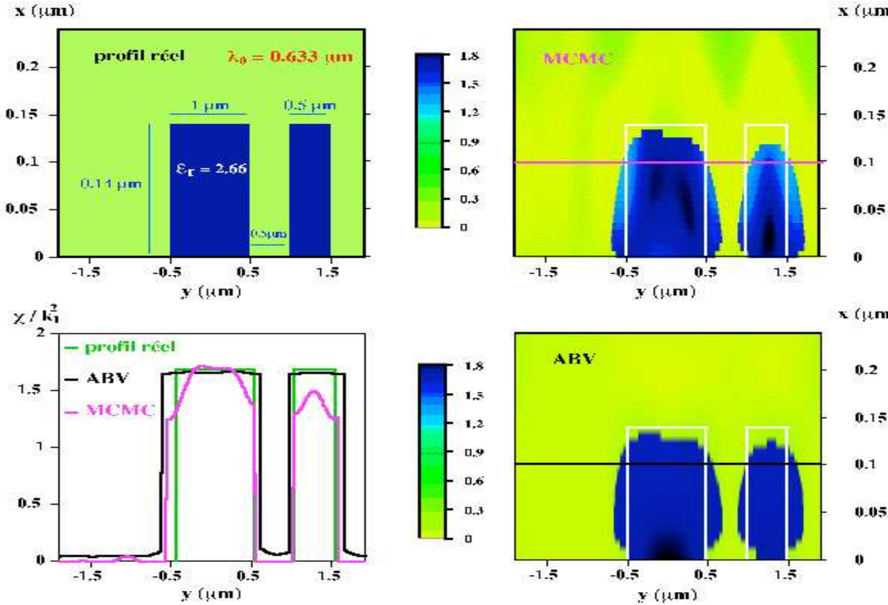
Imagerie optique de diffraction (ODT)

[H. Ayasso, B. Duchêne]



Imagerie optique de diffraction (ODT)

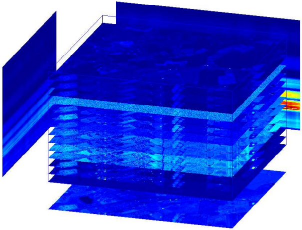
[H. Ayasso, B. Duchêne]

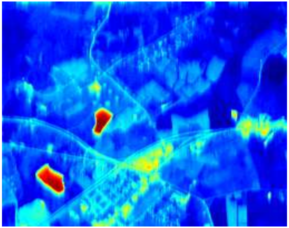


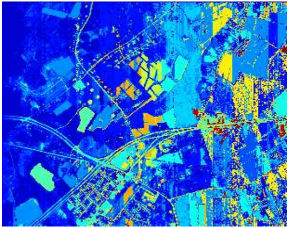
Sgmentation conjoint des images hyper-spectrales

[Adel MOHAMMADPOUR]

$$\left\{\begin{array}{l} g_i(\boldsymbol{r}) = f_i(\boldsymbol{r}) + \epsilon_i(\boldsymbol{r}) \\ p(f_i(\boldsymbol{r})|z(\boldsymbol{r}) = k) = \mathcal{N}(m_{i\,k}, \sigma^2_{i\,k}), \quad k = 1, \cdots, K \\ p(\underline{\boldsymbol{f}}|\boldsymbol{z}) = \prod_i p(\boldsymbol{f}_i|\boldsymbol{z}) \\ m_{i\,k} \;\; \text{markovienne le long de l'index} \;\; i \end{array}\right.$$







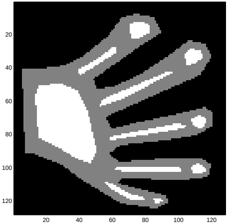
Synthèse de Fourier en imagerie microondes

[O. Féron en collaboration avec A. Joisel (DRÉ)]

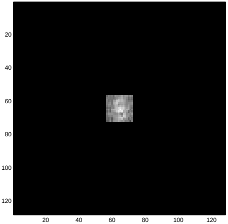
$$\color{blue}{g(\boldsymbol{\omega})} = \int \color{red}{f(\boldsymbol{r})} \exp\left[-j(\boldsymbol{\omega}.\boldsymbol{r})\right] \, \mathrm{d}\boldsymbol{r} + \epsilon(\boldsymbol{\omega})$$

$$\color{blue}{g(u,v)} = \int \color{red}{f(x,y)} \exp\left[-j(ux+vy)\right] \, \mathrm{d}x \, \mathrm{d}y + \epsilon(u,v)$$

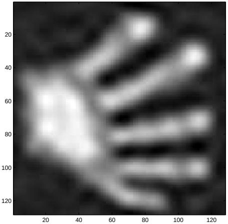
$$\color{blue}{\boldsymbol{g}} = \boldsymbol{H}\color{red}{\boldsymbol{f}} + \boldsymbol{\epsilon}$$



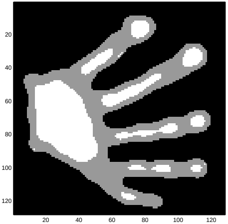
$\color{red}{f(x,y)}$



$\color{blue}{g(u,v)}$



$\color{red}{\widehat{f}}$ par FFTI



$\color{red}{\widehat{f}}$ méthode proposé

Conclusion

Approche bayésienne
et
Modèles markoviens simples ou avec des variables cachées

sont des outils d'inférence
bien appropriés
pour grand nombre de problèmes inverses
en
traitement du signal et d'image

Merci à tous mes collègues, ancients thésards et thésards

- H. Snoussi : Séparation de sources 1D et 2D
- M. Ichir : Séparation de sources dans le domaine des ondelettes
- S. Moussaoui : Séparation de sources positives et application en spéctrométrie
- O. Féron : Fusion d'images et problèmes inverses en imagerie microonde
- P. Brault : Segmentation de séquence d'images
- A. Mohammadpour : Classification et segmentation d'images hyper-spectrales,
- F. Su : Restauration de documents ancients
- F. Humblot : Super-résolution
- N. Bali : Séparation de sources pour la classification et réduction de données en imagerie hyper-spectrale
- S. Fékih-Salem, A. Vabre (CEA-LIST) : Tomographie 3D des micro structures (collaboration avec CEA)
- L. Robillard : Tomographie 3D en contrôle non destructif (CND) (collaboration avec EDF)

Merci à tous mes collègues, ancients thésards et thésards

- H. Ayasso, B. Duchêne (DRÉ) :
  Imagerie microondes et optique de diffusion
- D. Pougaza (thèse 3 ème année) : Copules et Tomographie
- D. Fall (thèse 4 ème année), E. Barat (CEA), C. Comptat (SHFJ) :
  Bayésienne non paramétrique en TEP
- Sh. Zhu (thèse 3 ème année), Ph. Fargette (ONERA), F. Daout (IUT Ville d'Averay) : Imagerie passive en SAR
- C. CAI (thèse 2ème année), Th. Rodet, S. Le Goupil (CEA-LIST) :
  Tomographie X multi-énérgies
- Gh. Khodabandelou (thèse 1ère année), J. Lapuyade (post-doc), F. Lévi (INSERM) : Analyse multi variée des données complexes en biologie
- R. Prénon (thèse 1ère année), P. Grangeat (CEA-LETI) :
  Traitement de l'information au niveau moléculaire
- Th. Boulay (thèse 1ère année), J. Lagoutte (Thompson Air Systèmes) : Méthodes de reconnaissances non coopératives en Radar
- N. Chu (thèse 1ère année), N. Gac, J. Picheral (Supélec) :
  Méthodes de déconvolution en localisation de sources en acoustiques

Quelques références

- A. Mohammad-Djafari (Ed.) Problèmes inverses en imagerie et en vision (Vol. 1 et 2), Hermes-Lavoisier, Traité Signal et Image, IC2, 2009
- A. Mohammad-Djafari (Ed.) Inverse Problems in Vision and 3D Tomography, ISTE, Wiley and sons, ISBN: 9781848211728, December 2009, Hardback, 480 pp.
- H. Ayasso and Ali Mohammad-Djafari Joint NDT Image Restoration and Segmentation using Gauss-Markov-Potts Prior Models and Variational Bayesian Computation, IEEE Trans. on Image Processing, Avril 2010.
- H. Ayasso, B. Duchene and A. Mohammad-Djafari, Bayesian Inversion for Optical Diffraction Tomography Journal of Modern Optics, 2008.
- A. Mohammad-Djafari, Gauss-Markov-Potts Priors for Images in Computer Tomography Resulting to Joint Optimal Reconstruction and segmentation, International Journal of Tomography & Statistics 11: W09. 76-92, 2008.

Quelques autres références

- A Mohammad-Djafari, Super-Resolution : A short review, a new method based on hidden Markov modeling of HR image and future challenges, The Computer Journal doi:10,1093/comjnl/bxn005 :, 2008.
- O. Féron, B. Duchène and A. Mohammad-Djafari, Microwave imaging of inhomogeneous objects made of a finite number of dielectric and conductive materials from experimental data, Inverse Problems, 21(6):95-115, Dec 2005.
- M. Ichir and A. Mohammad-Djafari,
  Hidden markov models for blind source separation, IEEE Trans. on Signal Processing, 15(7):1887-1899, Jul 2006.
- F. Humblot and A. Mohammad-Djafari,
  Super-Resolution using Hidden Markov Model and Bayesian Detection Estimation Framework, EURASIP Journal on Applied Signal Processing, Special number on Super-Resolution Imaging: Analysis, Algorithms, and Applications:ID 36971, 16 pages, 2006.
- O. Féron and A. Mohammad-Djafari,
  Image fusion and joint segmentation using an MCMC algorithm, Journal of Electronic Imaging, 14(2):paper no. 023014, Apr 2005.
- H. Snoussi and A. Mohammad-Djafari,
  Fast joint separation and segmentation of mixed images, Journal of Electronic Imaging, 13(2):349-361, April 2004.
- A. Mohammad-Djafari, J.F. Giovannelli, G. Demoment and J. Idier,
  Regularization, maximum entropy and probabilistic methods in mass spectrometry data processing problems, Int. Journal of Mass Spectrometry, 215(1-3):175-193, April 2002.