Fusion and Inversion of SAR Data to Obtain a Superresolution Image

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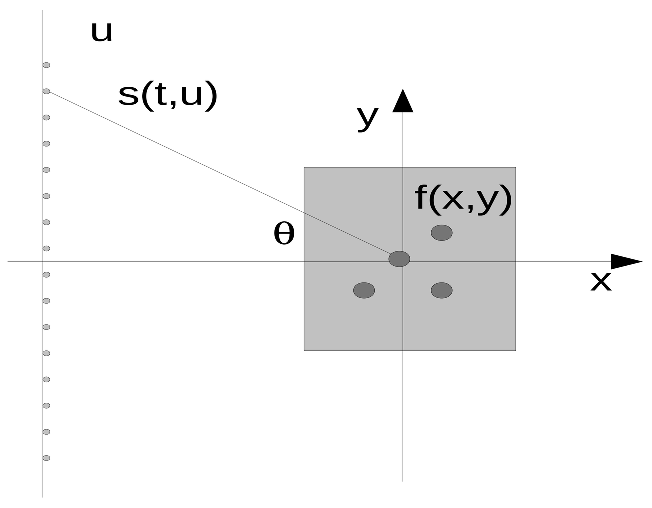
Absract

The Synthetic Aperture Radar (SAR) data obtained from a single emitter and a single receiver gives information in the Fourier domain of the scene over a line segment whose width is related to the bandwidth of the emitted signal. The mathematical problem of image reconstruction in SAR then becomes a Fourier Synthesis (FS) inverse problem. When there are more than one emitter and/or receiver looking the same scene, the problem becomes fusion and inversion. In this paper we report on a Bayesian inversion framework to obtain a Super Resolution (SR) image doing jointly data fusion and inversion. We applied the proposed method on some synthetic data to compare its performances to other classical methods and on experimental data obtained at ONERA.

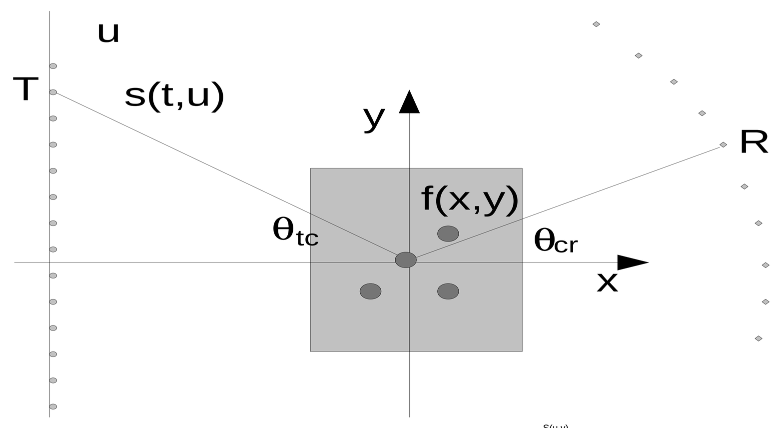
Synthetic Aperture Radar (SAR) imaging

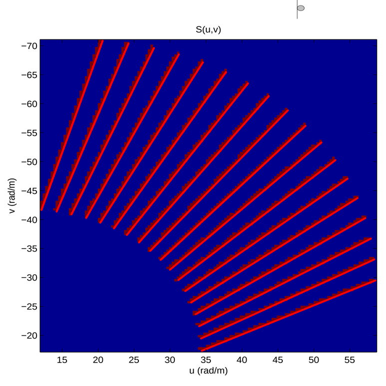


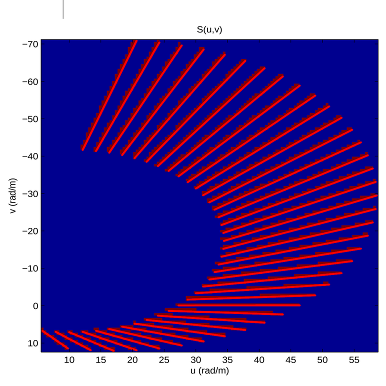
$$s(t,u)=\int f(x,y)\,p(t-\tau(x,y,u))\,\mathrm{d}x\,\mathrm{d}y$$

$$k=\begin{bmatrix}k_x\\k_y\end{bmatrix}=\begin{bmatrix}k\cos(\theta)\\k\sin(\theta)\end{bmatrix}\qquad |k|=k=\omega/c$$

$$s(\omega,u)=\int f(x,y)\,\exp\left[-j\omega\tau(x,y,\theta(u))\right]\,\mathrm{d}x\,\mathrm{d}y$$
$$=\int f(x,y)\,\exp\left[-j(k_x x+k_y y)\right]\,\mathrm{d}x\,\mathrm{d}y$$







Monostatic — Bistatic & Multistatic

$$\begin{cases}k_x=k\,\cos(\theta)\\k_y=k\,\sin(\theta)\end{cases}\quad\begin{cases}k_x=k\,(\cos(\theta_{tc})+\cos(\theta_{cr}))\\k_y=k\,(\sin(\theta_{tc})+\sin(\theta_{cr}))\end{cases}$$

Forward model: Fourier Synthesis

$$f(x,y)\rightarrow G(k_x,k_y),M(k_x,k_y)$$

Inverse problem:

$$G(k_x,k_y),M(k_x,k_y)\rightarrow f(x,y)$$

$$g(u_i,v_i)=\int f(x,y)\exp\left[-j(u_i x+v_i y)\right]\mathrm{d}x\,\mathrm{d}y$$

$$g=\mathcal{H}f+\epsilon$$

Bayesian Estimation Approach

- Forward model $\mathcal{M}$: $g=Hf+\epsilon$
- Likelihood: $p(g|f;\mathcal{M})=p_\epsilon(g-Hf)$
- A priori information: $p(f|\mathcal{M})$
- Bayes :

$$p(f|g;\mathcal{M})=\frac{p(g|f;\mathcal{M})\,p(f|\mathcal{M})}{p(g|\mathcal{M})}$$

- Estimators:
  - Mode (Maximum A Posteriori)
  - Mean (Posterior Mean)
  - Marginal modes

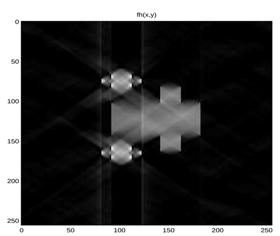
Proposed Bayesian Approach

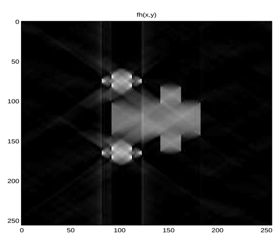
$$\widehat{f}=\arg\min_f\left\{J(f)=-\ln p(g|f)-\ln p(f)\right\}$$

$$p(g|f)\propto\exp\left[\frac{1}{2\sigma_\epsilon{}^2}\|g-Hf\|^2\right]$$

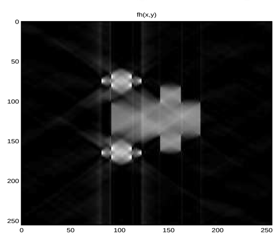
- Generalized Gaussian:
$$p(f)\propto\exp\left[\gamma\sum_j|f_j|^2\right]$$
- Cauchy:
$$p(f)\propto\exp\left[\gamma\sum_j\ln(1-|f_j|^2\right]$$
- Generalized Gauss-Markov
$$p(f)\propto\exp\left[\gamma\sum_j|f_j-f_{j-1}|^\beta\right]$$
- Gauss-Markov-Potts

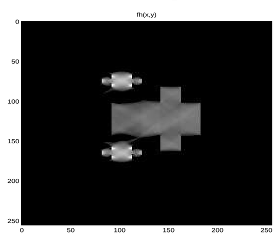
Comparison with classical methods





Gerchberg-Papoulis — Least Squares





Quad. Reg. — Proposed MAP

Multistatic data fusion methods

Method 1:
Data Fusion followed by inversion

$$\left.\begin{array}{l}G_1(u,v)\\M_1(u,v)\\G_2(u,v)\\M_2(u,v)\end{array}\right\}\mapsto\begin{array}{c}G(k_x,k_y)\\M(u,v)\end{array}\rightarrow\boxed{\text{Inversion}}\rightarrow\widehat{f}(x,y)$$

with
$$G(u,v)=\begin{cases}\frac{G_1(u,v)+G_2(u,v)}{2} & (u,v)\in M_1(u,v)\cap G_2(u,v)\\ G_1(u,v) & (u,v)\in M_1(u,v)\\ G_2(u,v) & (u,v)\in M_2(u,v)\end{cases}$$

and $M(k_x,k_y)=M_1(u,v)\cup M_2(u,v)$

Method 2:
Separte inversion followed by image fusion

$$\begin{array}{l}G_1(u,v),M_1(u,v)-\boxed{\text{Inversion}}-\widehat{f}_1(x,y)\\ G_2(u,v),M_2(u,v)-\boxed{\text{Inversion}}-\widehat{f}_2(x,y)\end{array}\mapsto\boxed{\text{Fusion}}\rightarrow\widehat{f}(x,y)$$

- Image fusion
  - Coherent addition
  $\widehat{f}(x,y)=(\widehat{f}_1(x,y)+\widehat{f}_2(x,y))/2$
  - Incoherent addition
  $\widehat{f}(x,y)=(|\widehat{f}_1(x,y)|+|\widehat{f}_2(x,y)|)/2$

Bayesian Simultaneous Data Fusion and Inversion

$$\left.\begin{array}{l}G_1(u,v)\\M_1(u,v)\\G_2(u,v)\\M_2(u,v)\end{array}\right\}\rightarrow\boxed{\begin{array}{c}\text{Joint Fusion}\\\text{and}\\\text{Inversion}\end{array}}\rightarrow\widehat{f}(x,y)$$

$$\begin{cases}g_1=H_1f+\epsilon_1\\g_2=H_2f+\epsilon_2\end{cases}$$

$$p(f|g_1,g_2)\propto p(g_1|f)\,p(g_2|f)\,p(f)$$

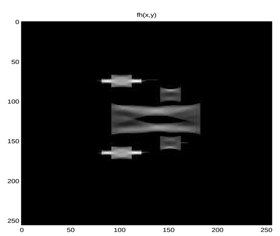
MAP :

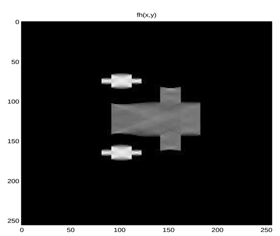
$$\widehat{f}=\arg\max_f\left\{p(f|g_1,g_2)\right\}=\arg\min_f\left\{J(f)\right\}$$

with

$$J(f)=-\ln p(g_1|f)-\ln p(g_2|f)-\ln p(f)$$
$$=\frac{\|g_1-H_1f\|^2}{2\sigma_{\epsilon_1}^2}+\frac{\|g_2-H_2f\|^2}{2\sigma_{\epsilon_2}^2}+\gamma\sum_j[Df]_j|^\beta$$

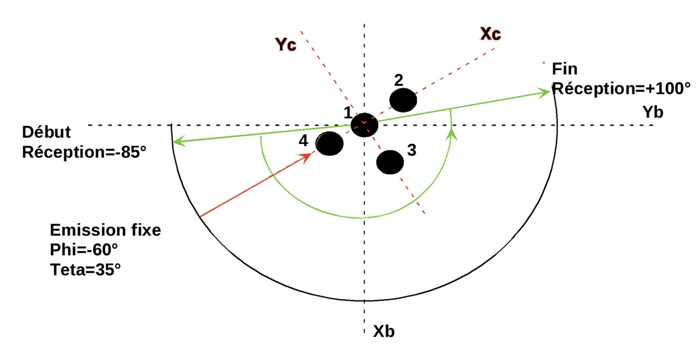
Simulated data



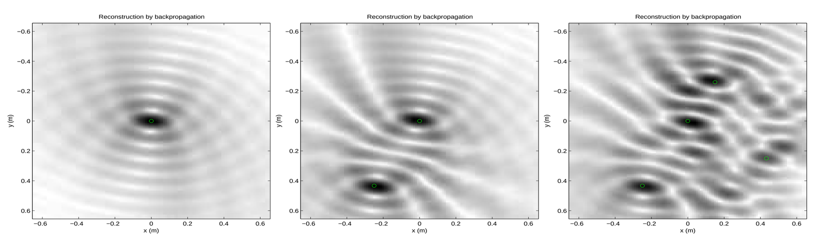


Data Fusion followed by Inversion — Joint Fusion and Inversion

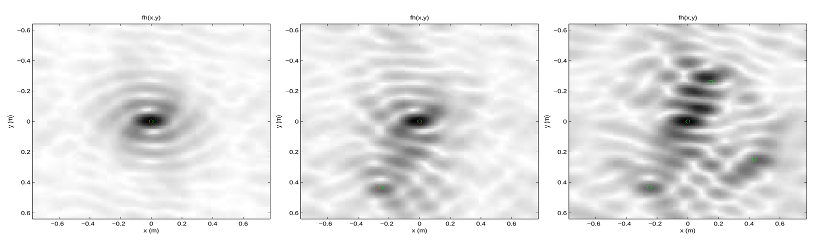
Experimental data (Vv polarisation)



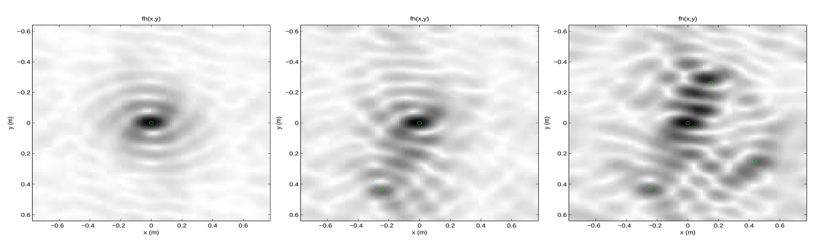
BF1 band



BF2 band



BF1 & BF2



References

A. Mohammad-Djafari, F. Daout & Ph. Fargette, Fusion et inversion des signaux SAR pour obtenir une image super résolue, *GRETSI 2009*, 7-10 Sept., Dijon, France.

A. Mohammad-Djafari, Sh. Zhu, F. Daout & Ph. Fargette, Fusion of Multistatic Synthetic Aperture Radar Data to obtain a Superresolution Image, *WIO 2009*, 20-24 July, Paris, France.