
Joint Super-Resolution and segmentation from a set of Low Resolution images using a Bayesian approach with a Gauss-Markov-Potts Prior

M. Mansouri*

ICD/LM2S,

University of Technology of Troyes,
France

E-mail: majdi.mansouri@utt.fr

*Corresponding author

A. Mohammad-Djafari

LSS, CNRS-Supélec-UNIV PARIS SUD 11,

Gif-sur-Yvette,

France

E-mail: djafari@lss.supelec.fr

Abstract: This paper addresses the problem of creating a Super-Resolution (SR) image from a set of Low Resolution (LR) images. SR image reconstruction can be viewed as a three-task process: registration or motion estimation, Point Spread Function (PSF) estimation and High Resolution (HR) image reconstruction. In the current work, we propose a new method based on the Bayesian estimation with a Gauss-Markov-Potts Prior Model (GMPPM) where the main objective is to get a new HR image from a set of severely blurred, noisy, rotated and shifted LR images. As a by-product of our prior model, we obtain jointly an SR image and an optimal segmentation of it. The proposed algorithm is unsupervised. A comparison of the performances of the proposed method with some classical and recent SR methods is provided in simulation.

Keywords: Bayesian estimation; motion estimation; PSF estimation; prior modelling; segmentation; super-resolution.

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Biographical notes: Mansouri Majdi received the Engineer Diploma in Telecommunications from High School of Communications of Tunis (SUP'COM) in 2006 and the Master Diploma in Signal and Image processing from High School of Electronic, Informatique and Radiocommunications in Bordeaux (ENSEIRB) in 2008. He is PhD student at Troyes University of Technology, France, since October 2008. His current research interests include statistical signal processing and wireless sensors networks. He is the author of over 20 papers.

Ali Mohammad-Djafari received the BSc in Electrical Engineering from Polytechnique of Teheran in 1975, the Diploma Degree (MSc) from École Supérieure d'Électricité (Supélec), Gif sur Yvette, France, in 1977 and the "Docteur-Ingénieur" (PhD) Degree and "Doctorat d'Etat" in Physics from the Université Paris-Sud (UPS), Orsay, France, respectively, in 1981 and 1987. He was Associate Professor at UPS for two years (1981–1983). Since 1984, he has a permanent position at "Centre National de la Recherche Scientifique (CNRS)" and works at "Laboratoire des Signaux et Systèmes" (L2S) at Supélec. From 1998 to 2002, he has been at the head of Signal and Image Processing division at this laboratory. Presently, he is "Directeur de recherche" and his main scientific interests are in developing new probabilistic methods. The main application domains of his interests are Computed Tomography (X-rays, PET, SPECT, MRI, microwave, ultrasound and eddy current imaging) either for medical imaging or for non-destructive testing (NDT) in industry.

1 Introduction

The SR problem addresses of generating an HR image from a set of LR images. These LR images can have many origins: camera motion (Sun et al., 2008; Foroosh et al., 2002; Vandewalle et al., 2006; Capel and Zisserman, 2003; Irani and Peleg, 1991), change of focus (Yang et al., 2008; Foroosh et al., 2002; Vandewalle et al., 2006), or a combination of these two (Zomet et al., 2001; Farsiu et al., 2004a; Segall et al., 2004).

The first task of SR is registration or motion estimation we are interested in the refinement of coarsely registered images to sub-pixel accuracy, where by coarse registration we imply pixel-level registration. We will assume that the error in registration at this refinement level is a translation and a rotation between the LR images and the HR image. Therefore, we confine our attention to the task of sub-pixel translation and rotation estimation.

The second task of SR is a restoration. The objective of restoration task methods is to obtain the estimate of the unknown image. There are two main approaches to the restoration problem (Kundur and Hatzinakos, 1996; Pickup et al., 2009). With the first one, the blur PSF is identified separately from the original image and later used in combination with one of the known image restoration algorithms, while with the second one the blur identification step is incorporated into the restoration procedure. Most of the work done to date mainly involves motion compensated temporal filtering techniques with appropriate 2D or 3D Wiener filter for noise suppression, 2D/3D median filtering or more appropriate morphological operators for removing impulsive noise. Usually, segmentation and image restoration are tackled separately in image sequence restoration. In this paper, the segmentation and the image restoration parts are done in a coupled way, allowing the segmentation part to positively influence the restoration part and vice versa.

Many methods and algorithms have been proposed in the literature. The different works proposed by Foroosh et al. (2002), Vandewalle et al. (2007), Capel and Zisserman (2003), Irani and Peleg (1991) and Sun et al. (2008) give best results for the motion estimation. Also the works in Nguyen et al. (2001), Elad and Hel-Or (2001) and Farsiu et al. (2004b) have used regularisation methods, to estimate the HR image. And, the works in Gerchberg (1974), Berthod et al. (1996), Foroosh et al. (2002), Rochefort et al. (2006) and He and Kondi (2006), have used a Least Squares (LS) method, as a criterion to estimate the HR image, but they have neglected the prior information of the HR image. The work in Elad and Feuer (1997) has used the Bayesian estimation to estimate the PSF by giving a Gaussian model as the prior information. The Gaussian model is not enough rich to model images with regions and contours. While the work in Park et al. (2003) has used the Constant Modulus (CM) technique to restore the LR images, and it assumed

that the displacement is limited to an integer-shift. And, the work proposed in Yang et al. (2008) has assumed that the LR images were not shifted and the blurred is known, but in fact the noise is unknown and LR and HR images are related by a sub-pixellic shift and a planar motion.

The method that we propose is a generalised framework using the Bayesian estimation with a GMPPM, which is much richer than the classical simple Markovian models thanks to the fact that they account for contours and regions in the images. The proposed algorithms here make other extensions to the previous works by estimating not only the sub-pixel shift but also the rotation parameters of the registration step.

This paper is organised as follows. Section 2 presents a forward model linking HR image to the LR images. The method for computing the subpixel shift and rotation between the LR images is presented in Section 3. Section 4 describes the proposed method, which is based on a Bayesian estimation with a Gauss-Markov-Potts Prior (MGMP) modelling of the HR image that accounts for the fact that, in general, all images are composed of statistically homogeneous regions. Even if this method has been recently presented elsewhere (Humblot and Mohammad-Djafari, 2005, 2006), we give here new extensions and new implementation algorithms when the noisy and blurred images are related by a sub-pixellic shift and a planar motion. We will describe in Section 5 the classical SR reconstruction. The results are discussed in Section 6, and finally concluding remarks are given in Section 7.

2 Image observation model

The image observation model between the m th LR image g_m and the HR image f is given by:

$$g_m = \mathbf{M}_m \mathbf{D} \mathbf{B} f + \epsilon_m = \mathbf{H}_m f + \epsilon_m, \quad m = 1, \dots, M \quad (1)$$

where $\mathbf{B}$ is the space invariant PSF of the camera for the LR images, $\mathbf{D}$ is the sub-sampling matrix, $\mathbf{M}_m$ represents the movement of the camera (or the scene), ϵ_m is the additive noise and $\mathbf{H}_m$ represents the global operator linking the HR image f and the LR image g_m .

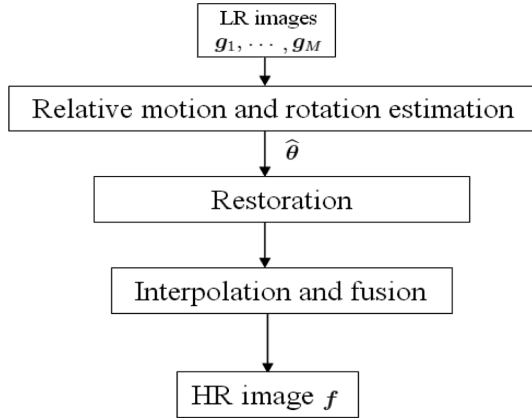
A very simple method to obtain an estimated HR image $\hat{f}$ is just using the adjoint operators:

$$\hat{f} = \sum_{m=1}^M \mathbf{B}^t \mathbf{D}^t \mathbf{M}_m^t g_m, \quad m = 1, \dots, M \quad (2)$$

where $\mathbf{M}_m^t$, $\mathbf{D}^t$ and $\mathbf{B}^t$ are the adjoint operators of $\mathbf{M}_m$, $\mathbf{D}$ and $\mathbf{B}$, respectively. This expression is interesting and is the basis of many classical methods of SR which shows the different steps to obtain an SR image, which are: motion estimation by computing $\mathbf{M}_m^t$, fusion by computing $\mathbf{D}^t$ and PSF estimation and restoration by computing $\mathbf{B}^t$.

Figure 1 describes the different steps of SR, which are presented in detail in the next sections.

Figure 1 Super-resolution model



3 Motion estimation

The context of this work is limited to a global movement of the scene. We then have to estimate the motion parameters between a reference frame and each of other frames. We propose to use a Phase Correlation (CP) method. This method was first proposed by Kuglin and Hines (1975) in the context of SR. It is based on the Fourier shift theorem between the reference image g_1 and its translated and rotated version g_2 . Let denote $\mathbf{r} = (x, y)$, $\mathbf{s} = (dx, dy)$, $\boldsymbol{\omega} = (u, v)$, $\boldsymbol{\theta} = (s, \rho)$ and

$$\mathbf{R} = \begin{bmatrix} \cos(\rho) & -\sin(\rho) \\ \sin(\rho) & \cos(\rho) \end{bmatrix},$$

where $\mathbf{r}$ is the spixel cartesian coordinate, $\mathbf{s}$ is the shift translation vector, $\boldsymbol{\omega}$ is the planar coordinate vector and $\boldsymbol{\theta}$ is the motion vector. Then, the relation between the two images g_1 and g_2 and their Fourier transforms G_1 and G_2 is given by:

$$\begin{aligned} G_2(\boldsymbol{\omega}) &= \int_{\mathbf{r}} g_2(\mathbf{r}) e^{-2\pi j(\boldsymbol{\omega} \cdot \mathbf{r})} d\mathbf{r} \\ &= \int_{\mathbf{r}} g_1(\mathbf{R} \cdot (\mathbf{r} + \mathbf{s})) e^{-2\pi j(\boldsymbol{\omega} \cdot \mathbf{r})} d\mathbf{r} \\ &= e^{2\pi j(\boldsymbol{\omega} \cdot \mathbf{s})} \int_{\mathbf{r}'} g_1(\mathbf{R} \cdot \mathbf{r}') e^{-2\pi j(\boldsymbol{\omega} \cdot \mathbf{r}') } d\mathbf{r}'. \end{aligned} \quad (3)$$

Hence, the relation between the two is:

$$|G_2(\boldsymbol{\omega})| = |G_1(\mathbf{R}^{-1}\boldsymbol{\omega})| \quad (4)$$

where the angle ρ is the spatial domain rotation between the $|G_2(\boldsymbol{\omega})|$ and $|G_1(\boldsymbol{\omega})|$ (see Figure 2(a)) and (b)). Hence, using equation (4), we can first estimate the rotation angle ρ . After that, the shift $\mathbf{s}$ can be computed from the phase difference between $G_1(\boldsymbol{\omega})$ and $G_2(\boldsymbol{\omega})$. In Section 3.1, we describe a precise rotation estimation method. A subpixel shift estimation method is presented in Section 3.2.

3.1 Rotation estimation

To linearise the relationship between $|G_2(\boldsymbol{\omega})|$ and $|G_1(\boldsymbol{\omega})|$, these amplitudes are transformed in polar coordinates, hence the rotation over the angle ρ is reduced to a shift over ρ . Then we compute the Fourier of spectra $|G_2(\boldsymbol{\omega})|$ and $|G_1(\boldsymbol{\omega})|$, and compute ρ as the shift between the two (as it was also done by Tsai (2000)). This requires a transformation of the spectrum to polar coordinates and we can obtain the regular r , α -grid by interpolating the data from the regular x , y . Vandewalle et al. (2006) propose also an extension of this method, their solution is based on computing the frequency content p_ϕ as a function of the angle ϕ by integrating over the coordinate polar:

$$p_\phi = \int_{\phi-\Delta\phi}^{\phi+\Delta\phi} \int_0^\infty |G(r, \alpha)| dr d\alpha. \quad (5)$$

After computing the function p_ϕ for both $|G_1(\boldsymbol{\omega})|$ and $|G_2(\boldsymbol{\omega})|$, the exact rotation angle can be obtained by computing the value for which their correlation is a maximum (see Figure 6(c)).

3.2 Shift estimation

The displacement in the spatial domain $\mathbf{s}$ can thus be computed as the phase difference between $G_1(\boldsymbol{\omega})$ and $G_2(\boldsymbol{\omega})$. To get rid of the luminance variation influence, we normalise the Cross-Power Spectrum SPC by its magnitude

$$S(\boldsymbol{\omega}) = \frac{G_2(\boldsymbol{\omega}) G_1^*(\boldsymbol{\omega})}{|G_2(\boldsymbol{\omega}) G_1^*(\boldsymbol{\omega})|}. \quad (6)$$

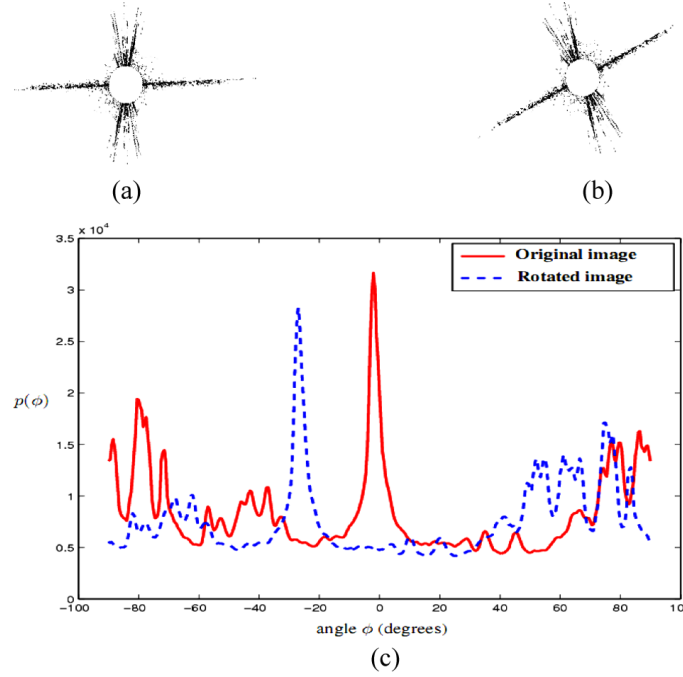
By combining equations (4) and (6) we get

$$S(\boldsymbol{\omega}) = e^{-2\pi j(\boldsymbol{\omega} \cdot \mathbf{s})}. \quad (7)$$

The inverse Fourier transform of (7) is $\delta(\mathbf{r} - \mathbf{s})$, which is the dirac function centred at $\mathbf{s}$ corresponding to the spatial shift between the images g_1 and g_2 . The shift can be computed easily by detecting a highest peak of $S(\boldsymbol{\omega})$ computed using equation (6). This method has assumed that the shift is pixellic between the two images, and when the translation is a subpixel shift, it returns an integer value.

Foroosh et al. (2002) have proposed an extension of this method; their model is based on the assumption that the shift between images is integer value and they were reduced to a subpixel shift by a downsampling bias. Argyriou and Vlachos (2003) have proposed an extension of the work in Foroosh et al. (2002); their model is based on the using of the information obtained from gradients images $g_1(\mathbf{r})$ and $g_2(\mathbf{r})$ before computing their Fourier transform. The aim of using gradients is that the gradient isolates some important information of images such as borders or dominant transitions, which provide the reference points in the case of motion estimation.

Figure 2 (a) Frequency values of the reference image for $0.1 < r < 1$; (b) frequency values of the rotated image ($\phi = 25$ degrees) for $0.1 < r < 1$ and (c) average value as a function of the angle p_ϕ for both $|G_1(\omega)|$ and $|G_2(\omega)|$ (see online version for colours)



The discrete values of vertical k^v and horizontal k^h gradients are estimated using the central difference for each pixel in the frame $g(r)$ (except for pixels on edge):

$$\begin{aligned} k^h(x, y) &= g^h(x+1, y) - g^h(x-1, y) \quad \text{and} \\ k^v(x, y) &= g^h(x, y+1) - g^h(x, y-1) \end{aligned} \quad (8)$$

Both terms are then combined to produce a single image in the complex form:

$$k(x, y) = k^h(x, y) + jk^v(x, y). \quad (9)$$

Each pixel in the complex image contains information on phase and amplitude. The method of CP described in Foroosh et al. (2002) was applied to a set of two images gradients k_t and k_{t+1} .

In the current work, we have used the two extensions: Argyriou and Vlachos (2003) to estimate the sub-pixel shift s , and Vandewalle et al. (2006) to estimate the rotation parameter ρ .

We present in the next section our proposed method in detail, which jointly estimates the SR image and its segmentation version.

4 Super-resolution with Bayesian estimation methods

4.1 First inversion and reconstruction

On the basis of the forward model (1) and using a simple Gauss-Markov prior and the MAP estimation approach, and assuming that the motion parameters and PSF of the blurring affects are known, we can estimate the HR

image f deduced from the LR images g . Some prior modelling of the HR image f , using some tools, are casted in the following main points:

- Least squares (LS) methods (Gerchberg, 1974; Berthod et al., 1996; Foroosh et al., 2002)
- Regularisation methods (Nguyen et al., 2001; Elad and Hel-Or, 2001; Farsiu et al., 2004b)
- Bayesian estimation methods (Molina et al., 2003a, 2003b; Humblot et al., 2005; Humblot and Mohammad-Djafari, 2005; Vega et al., 2006; Mansouri et al., 2010).

On the basis of the points of these three tools, we can estimate the HR image f by minimising a criterion $J(f)$:

$$\hat{f} = \arg \min_f J(f) \quad (10)$$

where $J(f)$ can be expressed:

- LS methods:

$$J(f) = \sum_m \|g_m - H_m f\|^\beta \quad (11)$$

with $1 \leq \beta \leq 2$ for the general case and $\beta = 2$ for the LS case.

- Regularisation methods:

$$J(f) = \sum_m \|g_m - H_m f\|^{\beta_1} + \lambda \|Df\|^{\beta_2} \quad (12)$$

with $1 \leq \beta_1, \beta_2 \leq 2$, D represents a high-pass filter operator and λ is a regularisation parameter.

- Bayesian MAP estimation methods.

$$J(\mathbf{f}) = -\ln p(\mathbf{f}|\mathbf{g}) = -\ln p(\mathbf{g}|\mathbf{f}) - \ln p(\mathbf{f}) + c. \quad (13)$$

If we suppose that the noises ϵ_m are assumed centred, iid and Gaussian with given variance σ_ϵ^2 , the likelihood is expressed as

$$p(\mathbf{g}|\mathbf{f}) \propto \exp \left\{ -\frac{1}{2\sigma_\epsilon^2 \sum_m \|\mathbf{g}_m - \mathbf{H}_m \mathbf{f}\|^2} \right\}$$

where: $\mathbf{g} = \{\mathbf{g}_m, k = 1, \dots, M\}$ represents all the LR images, and $p(\mathbf{f})$ is a priori model on the HR image. We consider, now that we know the prior information of the HR image $\mathbf{f}$, its average (is zero), and its covariance matrix $E[\mathbf{f}\mathbf{f}^t] = \sigma_f^2 \mathbf{P}_0$. If we assumed that the reverse of the covariance matrix $\mathbf{P}^{-1}$ exists and can be written as $\mathbf{D}^t \mathbf{D}$, then with this prior information, we can compute the $\hat{\mathbf{f}}_{\text{MAP}}$ by minimising the criterion:

$$J(\mathbf{f}) = \sum_m \|\mathbf{g}_m - \mathbf{H}_m \mathbf{f}\|^2 + \frac{1}{\sigma_f^2} \|\mathbf{D} \mathbf{f}\|^2.$$

We can also modelise the prior information by generalised Gauss-Markov model, so in these two cases the MAP criterion becomes equivalent to the regularisation criterion.

So, the general expression of the criterion is expressed as:

$$J(\mathbf{f}) = \sum_m \|\mathbf{g}_m - \mathbf{H}_m \mathbf{f}\|^2 + \lambda \|\mathbf{D} \mathbf{f}\|^2.$$

If $\mathbf{D}^t$ and $\mathbf{H}^t$ are the adjoint operators of $\mathbf{D}$ and $\mathbf{H}$, the solution is obtained by minimising this criterion, which verifies the following equation:

$$\nabla J(\hat{\mathbf{f}}) = -2 \sum_m \mathbf{H}_m^t (\mathbf{g}_m - \mathbf{H}_m \hat{\mathbf{f}}) + 2\lambda \mathbf{D}^t \mathbf{D} \hat{\mathbf{f}} = 0 \quad (14)$$

which gives this equation:

$$\left[\sum_m \mathbf{H}_m^t \mathbf{H}_m + \lambda \mathbf{D}^t \mathbf{D} \right] \hat{\mathbf{f}} = \sum_m \mathbf{H}_m^t \mathbf{g}_m \quad (15)$$

and finally the solution is:

$$\hat{\mathbf{f}} = \left[\sum_m \mathbf{H}_m^t \mathbf{H}_m + \lambda \mathbf{D}^t \mathbf{D} \right]^{-1} \sum_m \mathbf{H}_m^t \mathbf{g}_m. \quad (16)$$

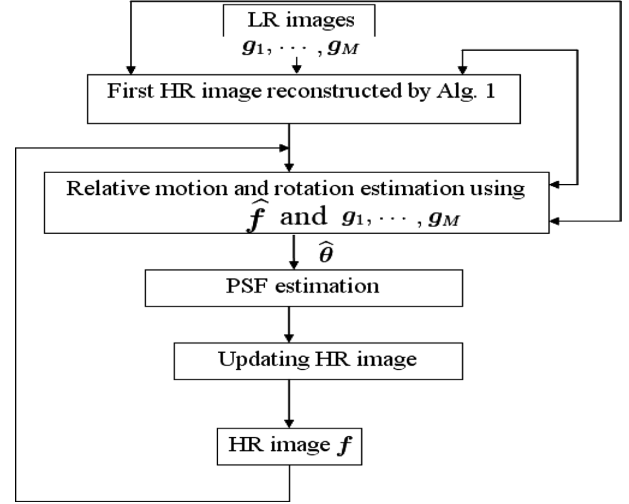
We can notice that we compute this solution via an iterative optimisation algorithm, where at each iteration (i), we use the previous result with an increment, which needs the computation of the gradient:

$$\begin{cases} \mathbf{f}^{(i+1)} = \mathbf{f}^{(i)} - \alpha^{(i)} \nabla J(\mathbf{f}^{(i)}) \text{ with} \\ \nabla J(\mathbf{f}) = \sum_m \mathbf{H}_m^t (\mathbf{g}_m - \mathbf{H}_m \mathbf{f}) + \lambda \mathbf{D}^t \mathbf{D} \mathbf{f}. \end{cases} \quad (17)$$

where $\alpha^{(i)}$ are either const. or can be adapted at each iteration (for example steepest gradient).

The first proposed algorithm (see Algorithm 1) and its diagram (see Figure 3) are described here.

Figure 3 First algorithm diagram



4.2 Gauss-Markov-Potts prior model

A simple Gauss-Markov prior model is used in the previous section which results to a simple SR method needing a quadratic criterion optimisation algorithm. Here, a more sophisticated, and so much richer prior model is proposed which accounts for the existence of homogeneous regions in the images. The main idea is to introduce a hidden field $z(\mathbf{r})$, which takes discrete values $l = 1, \dots, L$ and which represents the labels of the pixels $f(\mathbf{r})$. To model the fact that all the pixel labels of the image are grouped in compact regions, we propose to use a Potts Markov field model:

$$p(\mathbf{z}) \propto \exp \left\{ \varphi \sum_{\mathbf{r} \in \mathcal{R}} \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \delta(z(\mathbf{r}) - z(\mathbf{r}')) \right\} \quad (18)$$

where $\mathbf{z} = \{z(\mathbf{r}), \mathbf{r} \in \mathcal{R}\}$ and $\mathcal{R}$ is the set of all pixel positions. $\mathcal{V}(\mathbf{r})$ is the set of the four nearest neighbours of $\mathbf{r}$ and φ represents here the degree of spatial dependence.

All the pixels $f(\mathbf{r})$ with the same label $z(\mathbf{r})$ share some common statistics. If we note by $\mathcal{R}_k = \{\mathbf{r} : z(\mathbf{r}) = k\}$ and by $\mathbf{f}_k = \{f(\mathbf{r}), \mathbf{r} \in \mathcal{R}_k\}$, then the prior knowledge of homogeneity can be modelled by Gauss-Markov fields:

$$p(\mathbf{f}_k) = \mathcal{N}(m_k \mathbf{1}_k, \Sigma_k) \quad (19)$$

which can also be expressed in conditional form:

$$p(f(\mathbf{r})|z(\mathbf{r}) = k) = \mathcal{N}(m_k, \sigma_k^2) \quad (20)$$

where the means m_k , the variances σ_k^2 and covariances Σ_k parameters depend on the region labels k . On the basis of this model, the pixels of an image are classified in K independent classes, and we can write:

$$\begin{aligned} p(\mathbf{f}|\mathbf{z}, \boldsymbol{\theta}_f) &= \prod_{k=1}^K \mathcal{N}(m_k \mathbf{1}_k, \Sigma_k) \\ &\propto \prod_{k=1}^K \prod_{\mathbf{r} \in \mathcal{R}_k} \exp \left\{ -\frac{1}{2} \left(\frac{f(\mathbf{r}) - m_k}{\sigma_k^2} \right)^2 \right\} \end{aligned} \quad (21)$$

Algorithm 1: Pseudo-code of the first algorithm

— Initialization:

- 1) Estimate the motion parameters: sub-pixel shift s_m and rotation ρ_m for the LR image

$g_m (m = 2, \dots, M)$ with respect to the reference image g_1 ;

- a) Estimate the rotation ρ_m according to Sub-section III-A;
- b) Estimate the sub-pixel shift s_m according to Sub-section III-B;

— Iterations:

- 1) Estimate a HR image $f(r)$ based on LS or quadratic regularization using the equation (17).

where $\theta_f = \{(m_k, \sigma_k^2), k = 1, \dots, K\}$.

Now, using the likelihood

$$p(g|f, \theta_\epsilon) \propto \exp \left\{ -\frac{1}{2\sigma_\epsilon^2} \sum_m \|g_m - H_m f\|^2 \right\}, \quad (22)$$

where $\theta_\epsilon = \{\sigma_\epsilon^2\}$.

Now, we have all the priors to write down the expression of the posterior probability law

$$p(f, z, \theta|g) \propto p(g|f, \theta_\epsilon) p(f|z, \theta_f) p(z) p(\theta) \quad (23)$$

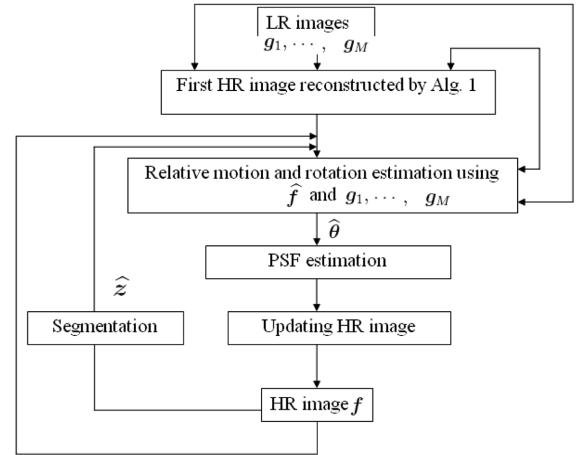
where $\theta = \{\theta_f, \theta_\epsilon\}$. We have used conjugate priors for all the hyperparameters θ , i.e., Gaussians for the means, Inverse Gammas ($\mathcal{IG}$) for the variances and Inverse Wishart ($\mathcal{IW}$) for the covariance matrix. For more details on these, refer to Humblot et al.'s (2005). So, by using the MAP, we can develop an algorithm to estimate jointly all the unknowns: the HR image $\hat{f}$, its segmentation $\hat{z}$ and the hyper-parameters $\hat{\theta}$. As before for the LS, the MAP estimator reduces to a large set of sparse equations that can be solved iteratively. Therefore, to estimate the HR image f given the two other unknowns z and θ , we minimise the criterion, for example the regularisation one where the regularisation term depends on the segmentation z obtained in previous steps. Given previous value of f and θ , we can estimate z by assimilating to a dynamic segmentation step. Finally, to estimate the hyper-parameters θ , we use the information given by the previous values of f and θ . The results are the HR image $\hat{f}$, its segmentation $\hat{z}$, and statistical properties (means, variances and correlation coefficients) of pixels in each of those homogeneous regions, as well as the statistical properties (mainly the common variance) of the noises. We notice that the expression of the likelihood depends on the motion parameters, so we must estimate once at the beginning these unknown parameters, translations and rotation movements from LR images (Humblot et al., 2005; Humblot and Mohammad-Djafari, 2005).

The different steps of the second algorithm (see Algorithm 2) with extensions are described here by their models (see Figure 4).

The main advantage of this second algorithm is that it can be applied more easily for the video sequence SR.

Also the SR reconstruction result can be available from the first arrival of the LR image and its resolution and precision is increased by the arrival of the new LR images. Figures 6–8 show an example of the results of the different steps of this algorithm.

Figure 4 Second algorithm diagram



5 Reconstruction

This section explains the image reconstruction process. Image reconstruction is a process by which information from a set of restored, deblurred and registered LR images are incorporated into an HR image. There are different steps for image reconstruction. The importance of image reconstruction lies in the fact that each observation image contains complementary information. When this complementary information is integrated with that of another observation, an image with the maximum amount of information is obtained. The different steps used to obtain an estimated SR image are:

- i image registration using the approach mentioned in Section 2
- ii image deconvolution and denoising using the Bayesian approach with an MGMPP presented in Section 3.

Algorithm 2: Pseudo-code of the second algorithm

— Initialization:

- 1) Estimate the motion parameters: sub-pixel shift s_m and rotation ρ_m between the LR images $g_m(\mathbf{r})$ (as the algorithm 1);
- 2) Estimate a first HR image $\hat{f}(\mathbf{r})$ based on LS or quadratic regularization;

— Iterations:

- 1) Estimate a segmentation $\hat{z}(\mathbf{r})$ for the HR image $\hat{f}(\mathbf{r})$ based on the Potts Markov modeling;
- 2) Estimate the parameters $\hat{\theta}$ of Gaussian mixtures model for the HR image $\hat{f}(\mathbf{r})$ using the segmentation result $\hat{z}(\mathbf{r})$;
- 3) Update the HR image using the gradient of log posterior which writes here

$$-\ln p(\mathbf{f}|\hat{\mathbf{z}}, \hat{\theta}, \mathbf{g}) = \frac{1}{2\sigma_\epsilon^2} \sum_m \|\mathbf{g}_m - \mathbf{H}_m \mathbf{f}\|^2 + \sum_m \|\mathbf{f}_t - m_m \mathbf{1}_m\|_{\Sigma_m}^2 \quad (24)$$

In the reconstruction algorithm, the samples of the different images are first expressed in the coordinate frame of the reference image. Then, these known samples are blended or combined, by interpolating the image values on an HR image grid. Figure 4 shows the diagram explaining how the HR image is estimated from a set of LR images.

6 Results

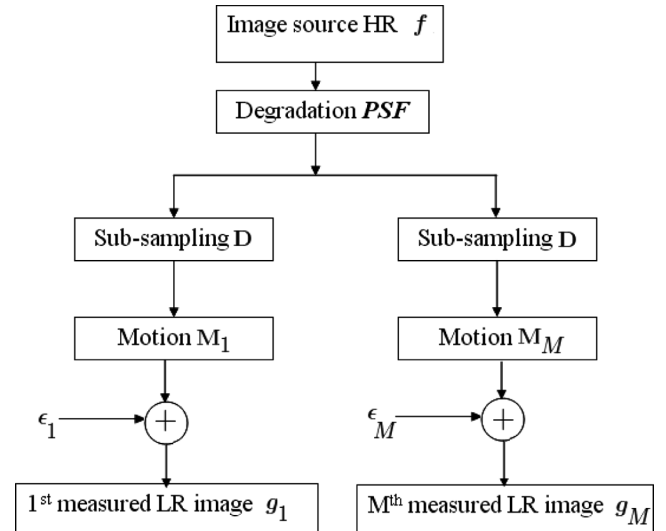
The SR algorithm described earlier is tested in simulations, so some experiments and their results are described (see Algorithm 3). In the first step, we describe a simulation where a set of LR images were created from an original HR image. In the second step an experiment is described with a real data where an HR image is created from a set of LR images, captured by digital camera, with unknown parameters.

6.1 Simulation

In the simulation, we have used an HR image (256×256 pixels) to create a set of an LR images. These images are obtained by the following steps: First, we have undersampled the HR image by a factor 4, the four images are filtered with a gaussian PSF of 4 by 4 pixels and variance $\sigma_{\text{psf}}^2 = 1$. Then, we have generated a severely blurred image with variance $\sigma_\epsilon^2 = 2$. After that, the filtered images are rotated and shifted in both horizontal and vertical directions. Figure 5 describes how to create the LR images from an HR image. Figure 6 illustrates the results for the intermediate steps according to the configuration described before.

By using these four LR images, we can apply our algorithm which provides not only a restored SR image

Figure 5 Diagram explaining how the LR images are obtained from a HR image



but also simultaneously an optimal segmentation of the HR image.

The results using different methods are summarised in Table 1. The SR results with our method are much better than the other methods by Marcel et al. (1998), Lucchese and Cortelazzo (2002), Vandewalle et al. (2006) and Keren et al. (2002). The motion estimation using the method proposed by Lucchese and Cortelazzo still accurate up to subpixel shift and neglected the both steps deblurring and restoration, while the method proposed by Vanderwalle resolved the two problems rotation and motion estimation but to reconstruct the HR image it used a simple interpolation by neglecting the PSF estimation. The algorithm proposed by Marcel et al. performs much worse in estimating the rotation angle. The results obtained by Keren et al. are similar to those

Algorithm 3: Pseudo-code of extension

— Initialization:

- 1) Estimate a first HR image $f(\mathbf{r})$ just by interpolating the first LR images, or by interpolation and superposition of all the LR images;

— Iterations:

- 1) Estimate the motion parameters: sub-pixel shift s_m and rotation ρ_m between the LR images $g_m(\mathbf{r})$ which is interpolated to the HR dimensions (as the algorithm 1) by taking the first HR image as reference;
- 2) Estimate the blurring PSF. We limited here this step on choosing a PSF between a finite number of choices. The selected PSF minimizes the LS criterion.
- 3) Estimate a segmentation $\hat{z}(\mathbf{r})$ for the HR image $f(\mathbf{r})$ based on the Potts Markov modeling;
- 4) Estimate the parameters $\hat{\theta}$ of Gaussian mixtures model for the HR image $f(\mathbf{r})$ using the classification and segmentation result $\hat{z}(\mathbf{r})$;
- 5) Update the HR image using the gradient of log posterior as in previous case, but also using the segmentation results, $z(\mathbf{r})$ and $\hat{\theta}$ and return to 1) until convergency.

with our algorithm in motion estimation, but this work has used the Bayesian estimation to estimate the PSF by giving a Gaussian model as the prior information, while the Gaussian model is not enough rich to model image with regions and contours.

Table 1 Results of simulation

	$L1$	$L2$	$PSNR$
Our method 1	0.0745109	0.041774	29.31032
Our method 2	0.07340	0.040792	29.4567
Our method 3	0.07223	0.039145	29.47589
Marcel et al.	0.12069	0.0470	22.00109
Lucchese et al.	0.16421	0.2109	21.466
Keren et al.	0.0753	0.04240	29.0472
Vandewalle et al.	0.110029	0.247	23.07

Our proposed method is a generalised framework using the Bayesian estimation with a GMPPM which is much richer than the Gaussian and classical simple Markovian models thanks to the fact that they account for contours and regions in the images. The proposed algorithm provides not only the sub-pixel shift and rotation estimation but also the restoration and segmentation steps.

The reconstructed image using our method is very close to the original and our method gives better results and outperforms the other methods. This performance can be confirmed by looking at Figures 6 and 7, which show that the algorithm using the Bayesian estimation with MGMPP is sophisticated best for all test sets. And, our output is not only the HR image but also its segmentation. Figure 8 shows the segmentation

performance comparison. Performance measures used are: the distance L_1 , a quadratic distance L_2 , and the Peak Signal-to-Noise Ratio (PSNR) between the source HR image $\mathbf{f}$ and the reconstructed HR image $\hat{\mathbf{f}}$, where:

$$L_\beta = \frac{\|\mathbf{f} - \hat{\mathbf{f}}\|^\beta}{\|\mathbf{f}\|^\beta} \quad \text{with } 1 \leq \beta \leq 2$$

$$\text{and } \|\mathbf{f}\|^\beta = \sum_j |\mathbf{f}_j|^\beta, \quad (25)$$

and

$$\text{PSNR} = 10 \cdot \log_{10} \frac{d^2}{\text{MSE}}, \quad (26)$$

where

$$\text{MSE} = E(\|\mathbf{f} - \hat{\mathbf{f}}\|^2). \quad (27)$$

and d is the maximum possible pixel value of the image. When the pixels are represented using 8 bits per sample, this is 255.

6.2 Practical experiment using a real images

Our algorithm was applied to a set of four images that were taken with a black and white digital camera. In our case, the camera was fixed, and in each capture of image we have moved the scene that allows an horizontal and vertical shifts and a planar rotation parallel to the scene plane. So four images shifted and rotated of a planar scene are captured (174×174). We apply the estimators of the subpixel shift and rotation to the four images to compute these parameters, after

Figure 6 (a) Original image (256×256); (b) one of the four LR images (40×40) before estimation; (c) LR image (40×40 pixels) after motion estimation; (d) reconstructed HR image using our first method; (e) reconstructed HR image using our second method and (f) reconstructed HR image using our third method

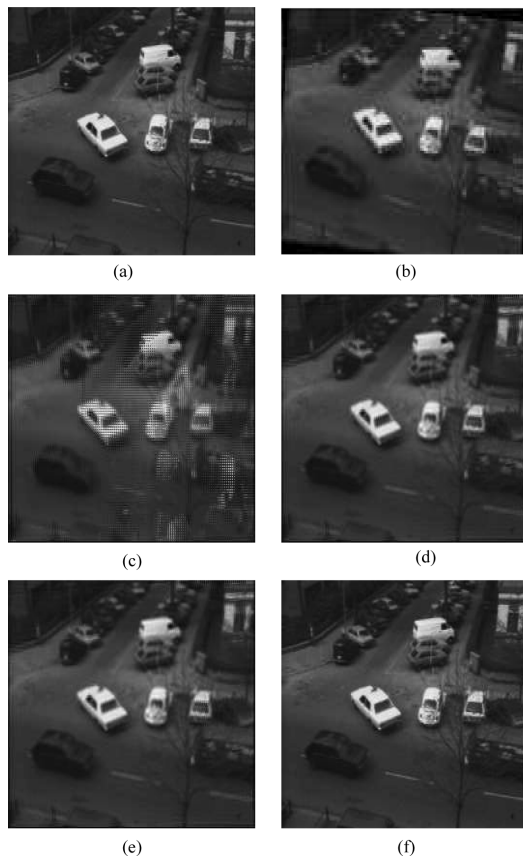


Figure 7 (a) Reconstructed HR image by Marcel et al. method; (b) reconstructed HR image by Lucchese and Cortelazzo method; (c) reconstructed HR image by Vanderwalle et al. method and (d) reconstructed HR image by Keren et al. method



that we apply our Bayesian estimator with a Model of GMPP. Using sub-shift, rotation and PSF parameters, we can reconstruct the HR image (704×704) using cubic interpolation and we can show its segmented version. Figures 9 and 10 illustrate this fact.

Figure 8 (a) Source HR image segmentation and (b) reconstructed HR image segmentation PSNR= 29.39032 (see online version for colours)

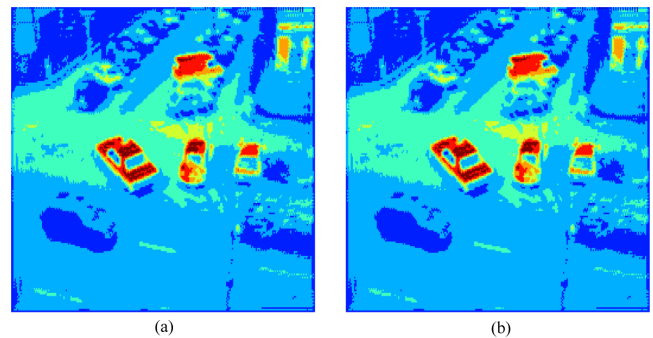


Figure 9 (a) One of the four LR images (174×174); (b) reconstructed HR image using our first method; (c) reconstructed HR image using our second method; (d) reconstructed HR image using our third method and (e) segmentation of HR image (see online version for colours)

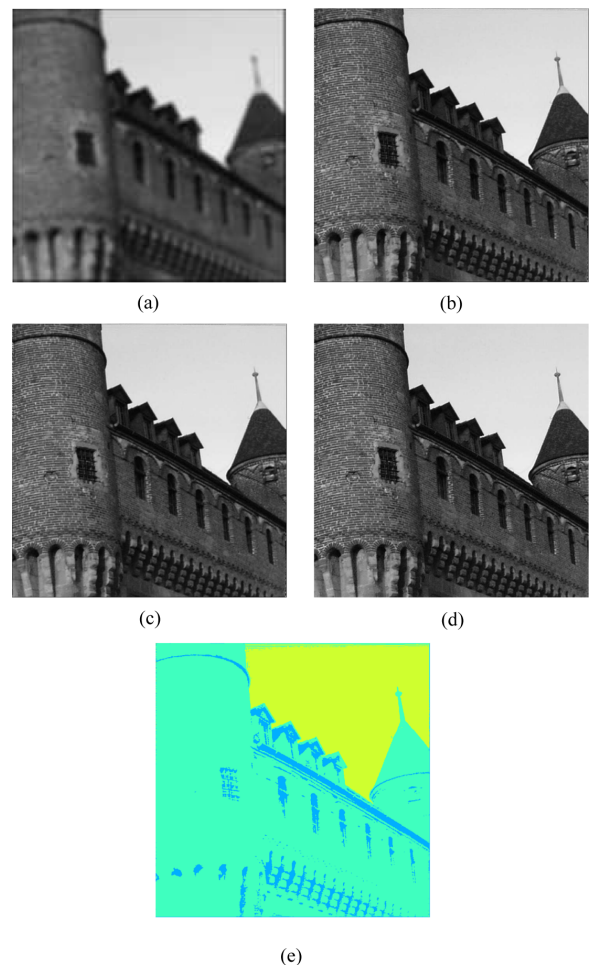
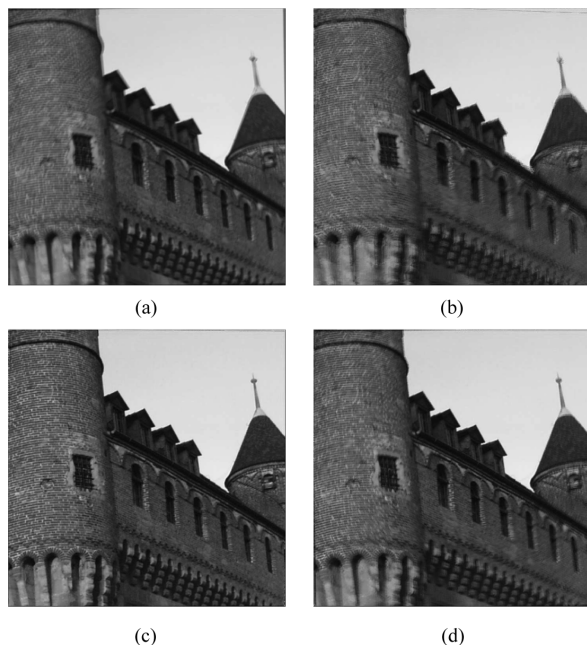


Figure 10 (a) Reconstructed HR image using Marcel et al. method; (b) reconstructed HR image using Lucchese et al. method; (c) reconstructed HR image using Keren et al. method and (d) reconstructed HR image using Vandewalle et al method



7 Conclusions

In this paper, we have presented three methods for SR image reconstruction. The proposed method is based on the Bayesian estimation approach with an MGMPP, which provides an HR image and its segmentation. This image restoration registration technique is then applied to SR imaging to reconstruct an HR image from a set of an LR images. This algorithm is compared with some other methods in simulations and practical experiments. Simulations and experimental results show that the proposed methods are more robust than the other methods in reconstructing the HR image with a very small error.

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