Bayesian Microwave Breast Imaging

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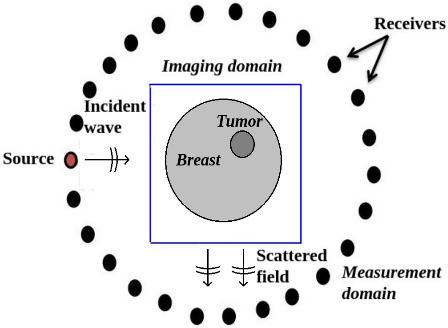
Summary

Motivation

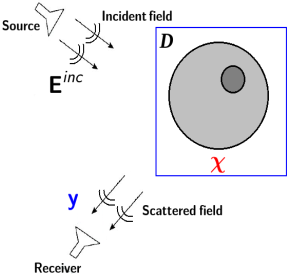
- Breast cancer ♀
  - Most commonly diagnosed cancer
  - Second cause of cancer death for women → Need to early stage cancer detection
- Detection techniques
  - X-ray mammography: Ionizing radiation, costly, high false-positive rate, low sensitivity
  - Ultrasound and MRI: High spatial resolution but luck of functional information
  - Microwave imaging: non negligible dielectric contrast between cancerous and normal healthy breast tissues, non-ionizing nature, easy accessibility → Promising alternative for breast tumor detection

Microwave imaging



- Different source positions
- Several receivers and frequencies in the microwave range
- Retrieve unknown target (electrical permittivity and conductivity) from measurements of the scattered field, the incident wave being known
- Non-linear ill-posed inverse problem

Domain integral representation



- Observation equation

$$y(\mathbf{r}) = k_1^2 \int_{\mathcal{D}} G(\mathbf{r}, \mathbf{r}')\, \chi(\mathbf{r}') E(\mathbf{r})\, \mathrm{d}\mathbf{r}'$$

- Coupling equation

$$E(\mathbf{r}) = E^{inc}(\mathbf{r}) + k_1^2 \int_{\mathcal{D}} G(\mathbf{r}, \mathbf{r}')\, \chi(\mathbf{r}') E(\mathbf{r})\, \mathrm{d}\mathbf{r}'$$

- $E(\mathbf{r})$: total field in the object
- $\chi(\mathbf{r}) = (k(\mathbf{r})^2 - k_1^2)/k_1^2$: normalized contrast function
- $k(\mathbf{r})^2 = \omega^2 \epsilon_0 \mu_0 \boxed{\epsilon_r(\mathbf{r})} + i\omega\mu_0 \boxed{\sigma(\mathbf{r})}$, k_1 = propagation constant of the outside medium
- $G(\mathbf{r}, \mathbf{r}')$: Green's function

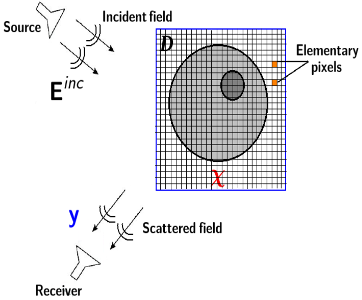
Discretized model

- Test domain $\mathcal{D}$ partitioned into $N_{\mathcal{D}}$ elementary pixels
- Coupling equation rewritten in terms of contrast sources: $w(\mathbf{r}') = \chi(\mathbf{r}')E(\mathbf{r}')$
- Method of moments [Harrington, 1993]

$\Rightarrow$ bilinear discret model:

$$\begin{cases} \mathbf{y} = \mathbf{G}^o \ \mathbf{w} + \epsilon \ (observation) \\ \mathbf{w} = \chi \ \mathbf{E}^{inc} + \chi \ \mathbf{G}^c \ \mathbf{w} + \xi \ (coupling) \end{cases}$$

ϵ, ξ: model and measurements errors



Bayesian framework

$$\begin{cases} \mathbf{y} = \mathbf{G}^o\ \mathbf{w} + \epsilon\ (observation) \\ \mathbf{w} = \chi\ \mathbf{E}^{inc} + \chi\ \mathbf{G}^c\ \mathbf{w} + \xi\ (coupling) \end{cases}$$

Let consider the first equation:

$$\mathbf{y} = \mathbf{G}^o\mathbf{w} + \epsilon, \quad p(\epsilon) = \mathcal{N}(\epsilon|0, \sigma_\epsilon^2\mathbf{I})$$

Bayes rule:

$$p(\mathbf{y}|\mathbf{w}) = \frac{p(\mathbf{y}|\mathbf{w})p(\mathbf{w})}{p(\mathbf{y})}$$

$$\begin{cases} p(\mathbf{y}|\mathbf{w}) = \mathcal{N}(\mathbf{y}|\mathbf{G}^o\mathbf{w}, \sigma_\epsilon^2\mathbf{I}) \\ p(\mathbf{w}) = \mathcal{N}(\mathbf{w}|0, \sigma_w^2\mathbf{I}) \end{cases} \rightarrow p(\mathbf{w}|\mathbf{y}) = \mathcal{N}(\mathbf{w}|\widehat{\mathbf{w}}, \widehat{\mathbf{\Sigma}})$$

$$\begin{cases} \widehat{\mathbf{w}} = [\mathbf{G}^{o^T}\mathbf{G}^o + \lambda\mathbf{I}]^{-1}\mathbf{G}^{o^T}\mathbf{y} \\ \widehat{\mathbf{\Sigma}} = [\mathbf{G}^o\mathbf{G}^o + \lambda\mathbf{I}]^{-1} \end{cases}$$

We can do the same with the second equation.

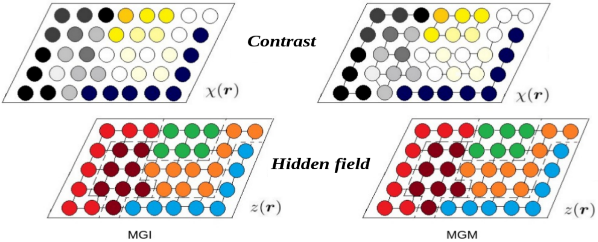
Bayesian framework

$$p(\chi, \mathbf{w}, \Theta | \mathbf{y}; \mathcal{M}) = \frac{p(\mathbf{y}|\mathbf{w}, \Theta; \mathcal{M})\, p(\mathbf{w}|\chi, \Theta; \mathcal{M})\, p(\chi|\Theta; \mathcal{M})\, p(\Theta|\mathcal{M})}{p(\mathbf{y}|\mathcal{M})}$$

- Θ: hyper-parameters
- $p(\mathbf{y}|\mathbf{w}, \Theta; \mathcal{M})$ (observation equation)
- $p(\mathbf{w}|\chi, \Theta; \mathcal{M})$ (coupling equation)
- $p(\chi|\Theta; \mathcal{M})$: *a priori* on the object
- $p(\Theta|\mathcal{M})$: a priori information on the hyperparameters
- $p(\chi, \mathbf{w}, \Theta|\mathbf{y}; \mathcal{M})$: *a posteriori*

Gauss-Markov-Potts prior

- Objects composed of a finite number K of materials distributed into homogenous and compact regions



$$p(\chi(\mathbf{r})|z(\mathbf{r})) = \mathcal{N}(m_k(\mathbf{r}), v_k(\mathbf{r})), \quad p(\mathbf{z}|\lambda) = \textit{Potts model}$$

- Mixture of independant Gaussian laws (MGI): pixels of the same class are assumed independant
- Gauss-Markov mixture model (MGM): a high correlation between pixels of the same class by means of a Markov field

Gauss-Markov-Potts prior

- **Homogeneity of a class:**

$$p(\chi(\mathbf{r})|z(\mathbf{r})) = \mathcal{N}(m_k(\mathbf{r}), v_k(\mathbf{r})), \qquad k = 1, \ldots, K,$$

- $v_k(\mathbf{r}) = v_k \quad \forall \mathbf{r} \in \mathcal{R}_k$

- $m_k(\mathbf{r}) = \begin{cases} m_k & \text{if } \mathcal{C}(\mathbf{r}) = 1 \\ \dfrac{1}{N_\mathcal{V}} \displaystyle\sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \chi(\mathbf{r}') & \text{if } \mathcal{C}(\mathbf{r}) = 0, \end{cases}$

$\mathcal{C}(\mathbf{r}) = 1 - \prod_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \delta(z(\mathbf{r}') - z(\mathbf{r}))$, $\mathcal{V}(\mathbf{r})$ is a neighborhood of $\mathbf{r}$ made of the four nearest pixels and $N_\mathcal{V} = card(\mathcal{V})$.

- **Compactness of the regions (Potts):**

$$p(\mathbf{z}|\lambda) \propto \exp\left[\sum_{\mathbf{r} \in \mathcal{D}} \left(\Phi(z(\mathbf{r})) + \frac{\lambda}{2} \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \delta\left(z(\mathbf{r}) - z(\mathbf{r}')\right)\right)\right]$$

Probabilistic modeling

- Model and measurement errors:

$$\epsilon \sim \mathcal{N}(\mathbf{0}, v_\epsilon \mathbf{I}), \quad \xi \sim \mathcal{N}(\mathbf{0}, v_\xi \mathbf{I})$$

- Hyperparameters:

$$p(m_k) = \mathcal{N}(\mu, \tau), \quad p(v_k) = \mathcal{IG}(\eta, \phi)$$
$$p(v_\epsilon) = \mathcal{IG}(\eta_\epsilon, \phi_\epsilon), \quad p(v_\xi) = \mathcal{IG}(\eta_\xi, \phi_\xi)$$

- *A posteriori*:

$$\begin{aligned} p\left(\chi, \mathbf{w}, \mathbf{z}, \Theta | \mathbf{y}\right) \propto\ & p\left(\mathbf{y}|\mathbf{w}, v_\epsilon\right) p\left(\mathbf{w}|\chi, v_\xi\right) p\left(\chi|\mathbf{z}, \mathbf{m}, \mathbf{v}\right) \\ \times\ & p\left(\mathbf{z}|\lambda\right) p\left(\mathbf{m}|\mu, \tau\right) p\left(\mathbf{v}|\eta, \phi\right) \\ \times\ & p\left(v_\epsilon|\eta_\epsilon, \phi_\epsilon\right) p\left(v_\xi|\eta_\xi, \phi_\xi\right) \end{aligned}$$

with $\Theta = \{\mathbf{m}, \mathbf{v}, v_\epsilon, v_\xi\}$.

Bayesian inversion techniques

$$
\begin{aligned}
p\left(\chi, \mathbf{w}, \mathbf{z}, \Theta | \mathbf{y}\right) \quad \propto \quad & p\left(\mathbf{y} | \mathbf{w}, v_{\epsilon}\right) p\left(\mathbf{w} | \chi, v_{\xi}\right) p\left(\chi | \mathbf{z}, \mathbf{m}, \mathbf{v}\right) \\
\times \quad & p\left(\mathbf{z} | \lambda\right) p\left(\mathbf{m} | \mu, \tau\right) p\left(\mathbf{v} | \eta, \phi\right) \\
\times \quad & p\left(v_{\epsilon} | \eta_{\epsilon}, \phi_{\epsilon}\right) p\left(v_{\xi} | \eta_{\xi}, \phi_{\xi}\right)
\end{aligned}
$$

- Joint Maximum A Posteriori (JMAP) (intractable solutions in non-linear cases) [Gharsalli et al., 2014]
  $\rightarrow$ needs to approximate the joint *a posteriori* distribution
- Monte-Carlo Markov Chain sampling (MCMC) (costly) [Féron, 2006]
- Variational Bayesian Approach (VBA) (less costly alternative) [Ayasso et al., 2012]

Variational Bayesian Approach (VBA) [Mackay, 2003]

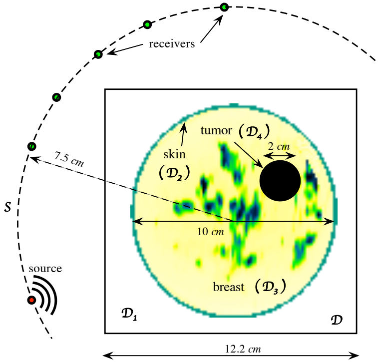
Aim:

Approximating $p(\chi, \mathbf{w}, \mathbf{z}, \Theta | \mathbf{y}; \mathcal{M})$ by a separable law:

$$q(\chi, \mathbf{w}, \mathbf{z}, \Theta) = q_1(\chi) q_2(\mathbf{w}) q_3(\mathbf{z}) q_4(\Theta)$$

- Criterion: $\mathrm{KL}(\mathbf{q} : \mathbf{p}) = \int \mathbf{q} \ln \frac{\mathbf{q}}{\mathbf{p}} = \left\langle \frac{\mathbf{q}}{\mathbf{p}} \right\rangle_{\mathbf{q}} = \mathcal{F}(\mathbf{q}) + \ln p(\mathbf{y}|\mathcal{M})$
- Free negative energy:
$$\mathcal{F}(\mathbf{q}) = \left\langle \ln \frac{p\left(\mathbf{u}, \mathbf{y}|\mathcal{M}\right)}{q\left(\mathbf{u}\right)} \right\rangle_{\mathbf{q}}, \quad \mathbf{u} = \{\chi, \mathbf{w}, \mathbf{z}, \Theta\}$$
- Minimize $\mathrm{KL}(\mathbf{q} : \mathbf{p}) \Leftrightarrow$ Maximize $\mathcal{F}(\mathbf{q})$
- Find: $\mathbf{q}^{opt} = \arg\max_{\mathbf{q}} \mathcal{F}(\mathbf{q})$
- $q(\mathbf{u}_i) \propto \exp\left\{ \langle \log(p(\mathbf{u}, \mathbf{y})) \rangle_{\prod_{j \neq i} q(\mathbf{u}_j)} \right\}$

Measurement configuration: application to breast cancer detection

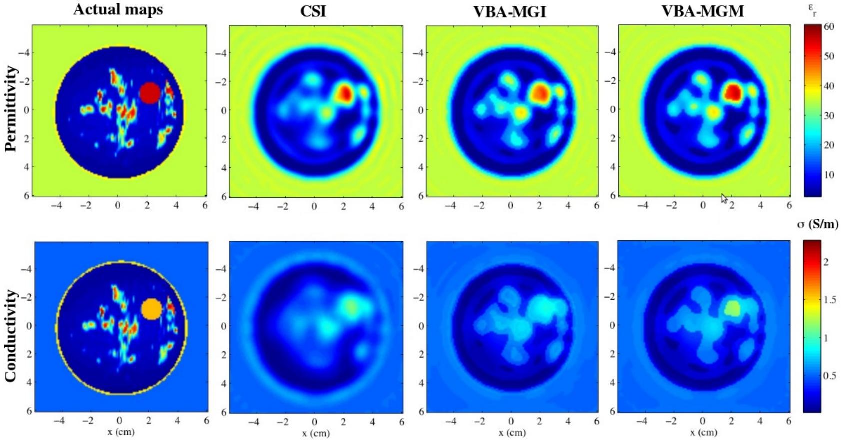


- Breast model built up from MRI scan [Zastrow and Hagness, 2008]
- 64 sources
- 64 receivers
- 6 frequencies in the band 0.5 – 3 GHz

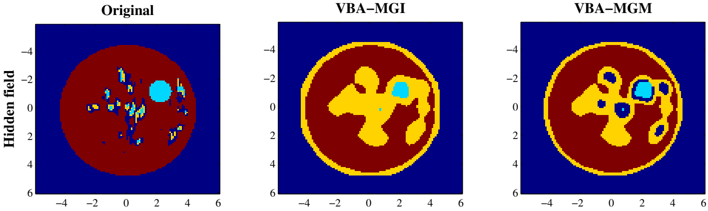
$\mathcal{D} = N_x \times N_y$	D1 $(\epsilon_1, \sigma_1(S/m))$	D2 $(\epsilon_2, \sigma_2(S/m))$	D3 $(\epsilon_3, \sigma_3(S/m))$	D4 $(\epsilon_3, \sigma_3(S/m))$
120×120	(10, 0.5)	(6.12, 0.11)	([2.46, 60.6], [0.01, 2.28])	(55.3, 1.57)

Reconstruction

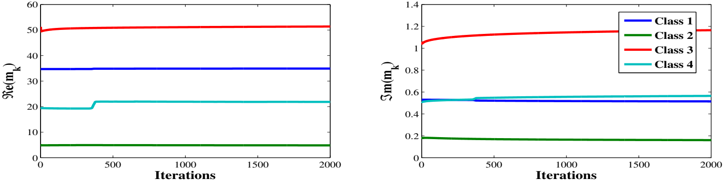
CSI: Contrast Source Inversion, VBA: Variational Bayesian Approach,
MGI: Independent Gaussian mixture, MGM: Gauss-Markov mixture



Reconstruction



- 3267 and 3029 pixels have undergone a change of class during the iterative process



Conclusions :

- Bayesian approach to microwave imaging: complexity of breast imaging compared to previous applications [Ayasso, 2010]
- Better resolution with VBA than with CSI
- Better identication of heterogeneous areas with MGM than with MGI prior
- Risk of hiding small details of interest

Prospects

- Accelerate the convergence by introducing a gradient step
- Account for breast tissue frequency dispersivity by means of Debye models
- Test on laboratory controlled experimental data involving breast phantoms
- Study the false alarm rate of the method $\rightarrow$ need of a reference database

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Thanks!

Questions?