

# Detection - Estimation

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A Graduated Course

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# Contents

<b>1</b>	<b>Introduction</b>	<b>9</b>
1.1	Basic definitions . . . . .	11
1.2	Summary of notations . . . . .	14
<b>2</b>	<b>Basic concepts of binary hypothesis testing</b>	<b>15</b>
2.1	Binary hypothesis testing . . . . .	15
2.2	Bayesian binary hypothesis testing . . . . .	16
2.3	Minimax binary hypothesis testing . . . . .	22
2.4	Neyman-Pearson hypothesis testing . . . . .	23
<b>3</b>	<b>Basic concepts of general hypothesis testing</b>	<b>25</b>
3.1	A general $M$ -ary hypothesis testing problem . . . . .	25
3.1.1	Deterministic or probabilistic decision rules . . . . .	26
3.1.2	Conditional, A priori and Joint Probability Distributions . . . . .	27
3.1.3	Probabilities of false and correct detection . . . . .	28
3.1.4	Penalty or costs coefficients . . . . .	28
3.1.5	Conditional and Bayes risks . . . . .	28
3.1.6	Bayesian and non Bayesian hypothesis testing . . . . .	29
3.1.7	Admissible decision rules and stopping rule . . . . .	29
3.1.8	Classification of simple hypothesis testing schemes . . . . .	30
3.2	Composite hypothesis . . . . .	32
3.2.1	Penalty or cost functions . . . . .	32
3.2.2	Case of binary hypothesis testing . . . . .	33
3.2.3	Classification of hypothesis testing schemes for a parametrically known stochastic process . . . . .	33
3.3	Classification of parameter estimation schemes . . . . .	36
3.4	Summary of notations and abbreviations . . . . .	40
<b>4</b>	<b>Bayesian hypothesis testing</b>	<b>43</b>
4.1	Introduction . . . . .	43
4.2	Optimization problem . . . . .	44
4.3	Examples . . . . .	47
4.3.1	Radar applications . . . . .	47
4.3.2	Detection of a known signal in an additive noise . . . . .	49
4.4	Binary channel transmission . . . . .	52

<b>5</b>	<b>Signal detection and structure of optimal detectors</b>	<b>55</b>
5.1	Bayesian composite hypothesis testing . . . . .	55
5.1.1	Case of binary composite hypothesis testing . . . . .	56
5.2	Uniform most powerful (UMP) test . . . . .	57
5.3	Locally most powerful (LMP) test . . . . .	57
5.4	Maximum likelihood test . . . . .	58
5.5	Examples of signal detection schemes . . . . .	59
5.5.1	Case of Gaussian noise . . . . .	59
5.5.2	Laplacian noise . . . . .	60
5.5.3	Locally optimal detectors . . . . .	61
5.6	Detection of signals with unknown parameter . . . . .	63
5.7	sequential detection . . . . .	68
5.8	Robust detection . . . . .	74
<b>6</b>	<b>Elements of parameter estimation</b>	<b>75</b>
6.1	Bayesian parameter estimation . . . . .	75
6.1.1	Minimum-Mean-Squared-Error . . . . .	76
6.1.2	Minimum-Mean-Absolute-Error . . . . .	76
6.1.3	Maximum <i>A Posteriori</i> (MAP) estimation . . . . .	77
6.2	Other cost functions and related estimators . . . . .	82
6.3	Examples of posterior calculation . . . . .	83
6.4	Estimation of vector parameters . . . . .	84
6.4.1	Minimum-Mean-Squared-Error . . . . .	84
6.4.2	Minimum-Mean-Absolute-Error . . . . .	84
6.4.3	Marginal Maximum <i>A Posteriori</i> (MAP) estimation . . . . .	85
6.4.4	Maximum <i>A Posteriori</i> (MAP) estimation . . . . .	85
6.4.5	Estimation of a Gaussian vector parameter from jointly Gaussian observation . . . . .	86
6.4.6	Case of linear models . . . . .	87
6.5	Examples . . . . .	88
6.5.1	curve fitting . . . . .	88
<b>7</b>	<b>Elements of signal estimation</b>	<b>101</b>
7.1	Introduction . . . . .	101
7.2	Kalman filtering : General linear case . . . . .	103
7.3	Examples . . . . .	107
7.3.1	1D case: . . . . .	107
7.3.2	Track-While-Scan (TWS) Radar . . . . .	108
7.3.3	Track-While-Scan (TWS) Radar with dependent acceleration se- quences . . . . .	108
7.4	Fast Kalman filter equations . . . . .	110
7.5	Kalman filter equations for signal deconvolution . . . . .	112
7.5.1	AR, MA and ARMA Models . . . . .	116

<b>8</b>	<b>Some complements to Bayesian estimation</b>	<b>117</b>
8.1	Choice of a prior law in the Bayesian estimation . . . . .	117
8.1.1	Invariance principles . . . . .	117
8.2	Conjugate priors . . . . .	120
8.3	Non informative priors based on Fisher information . . . . .	126
<b>9</b>	<b>Linear Estimation</b>	<b>129</b>
9.1	Introduction . . . . .	129
9.2	One step prediction . . . . .	131
9.3	Levinson algorithm . . . . .	131
9.4	Vector observation case . . . . .	132
9.5	Wiener-Kolmogorov filtering . . . . .	133
9.5.1	Non causal Wiener-Kolmogorov . . . . .	133
9.6	Causal Wiener-Kolmogorov . . . . .	135
9.7	Rational spectra . . . . .	139
<b>A</b>	<b>Annexes</b>	<b>143</b>
A.1	Summary of Bayesian inference . . . . .	144
A.2	Summary of probability distributions . . . . .	169
<b>A</b>	<b>Exercises</b>	<b>179</b>
A.1	Exercise 1: Signal detection and parameter estimation . . . . .	180
A.2	Exercise 2: Discrete deconvolution . . . . .	182



# Preface

During my visit at the Electrical Engineering Department of the Notre Dame University, I had to teach a course on Detection-Estimation.

I could not find really a complete and convenient textbook to cover all the materials that an Electrical Engineer have to know on the subject. Some books are too mathematical, some others are too oriented on the applications and not enough rigorous on the mathematics.

The purpose of these notes is to fill this gap: to introduce the reader to the basic theory of hypothesis testing and estimation theory using the main probability and statistical tools and also to give him the basic theory of signal detection and estimation as used in practical applications of electrical engineering.

The contents of these notes are mainly covered by two books:

- Detection and estimation, by D. Kazakos and P. Papantoni-Kazakos, and
- An introduction to signal detection and estimation, by H. Vincent Poor.

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