Bayesian sparse sources separation

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General source separation problem

$$\boldsymbol{g}(t) = \boldsymbol{A}\boldsymbol{f}(t) + \boldsymbol{\epsilon}(t), \quad t \in [1, \cdots, T]$$

$$\boldsymbol{g}(\boldsymbol{r}) = \boldsymbol{A}\boldsymbol{f}(\boldsymbol{r}) + \boldsymbol{\epsilon}(\boldsymbol{r}), \quad \boldsymbol{r} = (x, y) \in R^2$$

- $\boldsymbol{f}$ unknown sources
- $\boldsymbol{A}$ mixing matrix, $\boldsymbol{a}_{*j}$ steering vectors
- $\boldsymbol{g}$ observed signals
- $\boldsymbol{\epsilon}$ represents the errors of modeling and measurement

$$\boldsymbol{g} = \boldsymbol{A}\boldsymbol{f} \longrightarrow g_i = \sum_j a_{ij} f_j \longrightarrow \boldsymbol{g} = \sum_j \boldsymbol{a}_{*j} f_j$$

$$\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} f_1 & 0 & f_2 & 0 \\ 0 & f_1 & 0 & f_2 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{bmatrix}$$

$$\boldsymbol{g} = \boldsymbol{A}\boldsymbol{f} = \boldsymbol{F}\boldsymbol{a} \quad \text{with} \quad \boldsymbol{F} = \boldsymbol{f} \odot \boldsymbol{I}, \quad \boldsymbol{a} = \text{vec}(\boldsymbol{A})$$

- $\boldsymbol{A}$ known, estimation of $\boldsymbol{f}$: $\quad \boldsymbol{g} = \boldsymbol{A}\boldsymbol{f} + \boldsymbol{\epsilon}$
- $\boldsymbol{f}$ known, estimation of $\boldsymbol{A}$: $\quad \boldsymbol{g} = \boldsymbol{F}\boldsymbol{a} + \boldsymbol{\epsilon}$
- Joint estimation of $\boldsymbol{f}$ and $\boldsymbol{A}$: $\quad \boldsymbol{g} = \boldsymbol{A}\boldsymbol{f} + \boldsymbol{\epsilon} = \boldsymbol{F}\boldsymbol{a} + \boldsymbol{\epsilon}$

General Bayesian source separation problem

$$p(\boldsymbol{f}, \boldsymbol{A}|\boldsymbol{g}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3) = \frac{p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{A}, \boldsymbol{\theta}_1)\, p(\boldsymbol{f}|\boldsymbol{\theta}_2)\, p(\boldsymbol{A}|\boldsymbol{\theta}_3)}{p(\boldsymbol{g}|\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3)}$$

- $p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{A}, \boldsymbol{\theta}_1)$ likelihood
- $p(\boldsymbol{f}|\boldsymbol{\theta}_2)$ and $p(\boldsymbol{A}|\boldsymbol{\theta}_3)$ priors
- $p(\boldsymbol{f}, \boldsymbol{A}|\boldsymbol{g}, \boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3)$ joint posterior
- $\boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \boldsymbol{\theta}_3)$ hyper-parameters

Two approaches:

- Estimate first $\boldsymbol{A}$ and then use it for estimating $\boldsymbol{f}$
- Joint estimation

In real application, we also have to estimate $\boldsymbol{\theta}$:

$$p(\boldsymbol{f}, \boldsymbol{A}, \boldsymbol{\theta}|\boldsymbol{g}) = \frac{p(\boldsymbol{g}|\boldsymbol{f}, \boldsymbol{A}, \boldsymbol{\theta}_1)\, p(\boldsymbol{f}|\boldsymbol{\theta}_2)\, p(\boldsymbol{A}|\boldsymbol{\theta}_3)\, p(\boldsymbol{\theta})}{p(\boldsymbol{g})}$$

Bayesian inference for sources f when A is known

$$g = Af + \epsilon$$

- Prior knowledge on ϵ:

$$\epsilon \sim \mathcal{N}(\epsilon|0, v_\epsilon I) \longrightarrow p(g|f, A) = \mathcal{N}(g|Af, v_\epsilon I) \propto \exp\left\{\frac{1}{2v_\epsilon}\|g - Af\|^2\right\}$$

- Simple prior models for f: $p(f|\alpha) \propto \exp\{-\alpha\Omega(f)\}$
- Expression of the posterior law:

$$p(f|g, A) \propto p(g|f, A)\, p(f) \propto \exp\{-J(f)\}$$

with $$J(f) = \frac{1}{2v_\epsilon}\|g - Af\|^2 + \alpha\Omega(f)$$

- Link between MAP estimation and regularization

$$\boxed{p(f|\theta, g)} \longrightarrow \boxed{\text{Optimization of } J(f) = \tfrac{1}{2v_\epsilon}\|g - Af\|^2 + \alpha\Omega(f)} \longrightarrow \widehat{f}$$

MAP estimation with sparsity enforcing priors

- Gaussian: $\Omega(\boldsymbol{f}) = \|\boldsymbol{f}\|^2 = \sum_j |f_j|^2$

$$J(\boldsymbol{f}) = \frac{1}{2v_\epsilon}\|\boldsymbol{g} - \boldsymbol{A}\boldsymbol{f}\|^2 + \alpha\|\boldsymbol{f}\|^2 \longrightarrow \widehat{\boldsymbol{f}} = [\boldsymbol{A}'\boldsymbol{A} + \lambda \boldsymbol{I}]^{-1}\boldsymbol{A}'\boldsymbol{g}$$

- Generalized Gaussian:

$$\Omega(\boldsymbol{f}) = \gamma \sum_j |f_j|^\beta)$$

- Student-t model:

$$\Omega(\boldsymbol{f}) = \frac{\nu+1}{2}\sum_j \log\left(1 + f_j^2/\nu\right)$$

- Elastic Net model:

$$\Omega(\boldsymbol{f}) = \sum_j \left[\gamma_1 |f_j| + \gamma_2 f_j^2\right]$$

For an extended list of such sparsity enforcing priors see:
A. Mohammad-Djafari, "Bayesian approach with prior models which enforce sparsity in signal and image processing," EURASIP Journal on Advances in Signal Processing, vol. Special issue on Sparse Signal Processing, 2012.

Estimation of $\boldsymbol{A}$ when the sources $\boldsymbol{f}$ are known

Source separation is a bilinear model:

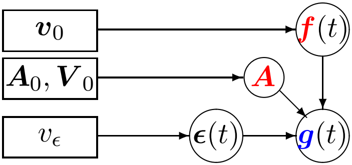
$$\boldsymbol{g} = \boldsymbol{A}\boldsymbol{f} = \boldsymbol{F}\boldsymbol{a} = \boldsymbol{A}\boldsymbol{f}$$

$$\begin{bmatrix} g_1 \\ g_2 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} f_1 \\ f_2 \end{bmatrix} = \begin{bmatrix} f_1 & 0 & f_2 & 0 \\ 0 & f_1 & 0 & f_2 \end{bmatrix} \begin{bmatrix} a_{11} \\ a_{21} \\ a_{12} \\ a_{22} \end{bmatrix}$$

$$\boldsymbol{F} = \boldsymbol{f} \odot \boldsymbol{I}, \quad \boldsymbol{a} = \text{vec}(\boldsymbol{A})$$

- Problem is more ill-posed.
- We need absolutely to impose constraintes on elements or the structure of $\boldsymbol{A}$, for example:
  - Positivity of the elements
  - Toeplitz or TBT structure
  - Symmetry $\quad p(\boldsymbol{A}) \propto \exp\left\{-\alpha \|\boldsymbol{I} - \boldsymbol{A}'\boldsymbol{A}|^2\right\}$
  - Sparsity $\quad p(\boldsymbol{A}) \propto \exp\left\{-\alpha \sum_{i,j} |\boldsymbol{A}_{ij}|\right\}$
- The same Bayesian approach then can be applied.

General case: Joint Estimation of $\boldsymbol{A}$ and $\boldsymbol{f}$



$$p(f_j(t)|v_{0j}) = \mathcal{N}(0, v_{0j})$$
$$p(\boldsymbol{f}(t)|\boldsymbol{v}_0) \propto \exp\left\{-\frac{1}{2}\sum_j f_j^2(t)/v_{0j}\right\}$$
$$p(\boldsymbol{A}_{ij}|\boldsymbol{A}_{0ij}, \boldsymbol{V}_{0ij}) = \mathcal{N}(\boldsymbol{A}_{0ij}, \boldsymbol{V}_{0ij})$$
$$p(\boldsymbol{A}|\boldsymbol{A}_0, \boldsymbol{V}_0) = \mathcal{N}(\boldsymbol{A}_0, \boldsymbol{V}_0)$$
$$p(\boldsymbol{g}(t)|\boldsymbol{A}, \boldsymbol{f}(t), v_\epsilon) = \mathcal{N}(\boldsymbol{A}\boldsymbol{f}(t), v_\epsilon \boldsymbol{I})$$

$$\begin{aligned} p(\boldsymbol{f}_{1..T}, \boldsymbol{A}|\boldsymbol{g}_{1..T}) &\propto p(\boldsymbol{g}_{1..T}|\boldsymbol{A}, \boldsymbol{f}_{1..T}, v_\epsilon)\, p(\boldsymbol{f}_{1..T})\, p(\boldsymbol{A}|\boldsymbol{A}_0, \boldsymbol{V}_0) \\ &\propto \prod_t p(\boldsymbol{g}(t)|\boldsymbol{A}, \boldsymbol{f}(t), v_\epsilon)\, p(\boldsymbol{f}(t)|\boldsymbol{v}_0)\, p(\boldsymbol{A}|\boldsymbol{A}_0, \boldsymbol{V}_0) \end{aligned}$$

$$p(\boldsymbol{f}(t)|\boldsymbol{g}_{1..T}, \boldsymbol{A}, v_\epsilon, \boldsymbol{v}_0) = \mathcal{N}(\widehat{\boldsymbol{f}}(t), \widehat{\boldsymbol{\Sigma}})$$

$$p(\boldsymbol{A}|\boldsymbol{g}_{1..T}, \boldsymbol{f}_{1..T}, v_\epsilon, \boldsymbol{A}_0, \boldsymbol{V}_0) = \mathcal{N}(\widehat{\boldsymbol{A}}, \widehat{\boldsymbol{V}})$$

Two approaches:

- Alternate joint MAP (JMAP) estimation
- Bayesian Variational Approximation

Joint Estimation of $\boldsymbol{A}$ and $\boldsymbol{f}$: Alternate JMAP

Let do some simplification:

$$\boldsymbol{v}_0 = [v_f, .., v_f]', \quad \text{All sources a priori same variance } v_f$$
$$\boldsymbol{v}_\epsilon = [v_\epsilon, .., v_\epsilon]', \quad \text{All noise terms a priori same variance } v_\epsilon$$
$$\boldsymbol{A}_0 = 0, \quad \boldsymbol{V}_0 = v_a \boldsymbol{I}$$

$$p(\boldsymbol{f}(t)|\boldsymbol{g}(t), \boldsymbol{A}, v_\epsilon, \boldsymbol{v}_0) = \mathcal{N}(\widehat{\boldsymbol{f}}(t), \widehat{\boldsymbol{\Sigma}})$$
$$\begin{cases} \widehat{\boldsymbol{\Sigma}} = (\boldsymbol{A}'\boldsymbol{A} + \lambda_f \boldsymbol{I})^{-1} \\ \widehat{\boldsymbol{f}}(t) = (\boldsymbol{A}'\boldsymbol{A} + \lambda_f \boldsymbol{I})^{-1} \boldsymbol{A}'\boldsymbol{g}(t), \quad \lambda_f = v_\epsilon / v_f \end{cases}$$

$$p(\boldsymbol{A}|\boldsymbol{g}(t), \boldsymbol{f}(t), v_\epsilon, \boldsymbol{A}_0, \boldsymbol{V}_0) = \mathcal{N}(\widehat{\boldsymbol{A}}, \widehat{\boldsymbol{V}})$$
$$\begin{cases} \widehat{\boldsymbol{V}} = (\boldsymbol{F}'\boldsymbol{F} + \lambda_f \boldsymbol{I})^{-1} \\ \widehat{\boldsymbol{A}} = \sum_t \boldsymbol{g}(t)\boldsymbol{f}'(t) \left(\sum_t \boldsymbol{f}(t)\boldsymbol{f}'(t) + \lambda_a \boldsymbol{I}\right)^{-1}, \quad \lambda_a = v_\epsilon / v_a \end{cases}$$

Joint Estimation of $\boldsymbol{A}$ and $\boldsymbol{f}$: Alternate JMAP

$$
\begin{aligned}
p(\boldsymbol{f}_{1..T},\boldsymbol{A}|\boldsymbol{g}_{1..T}) &\propto p(\boldsymbol{g}_{1..T}|\boldsymbol{A},\boldsymbol{f}_{1..T},v_\epsilon)\, p(\boldsymbol{f}_{1..T})\, p(\boldsymbol{A}|\boldsymbol{A}_0,\boldsymbol{V}_0)\\
&\propto \prod_t p(\boldsymbol{g}(t)|\boldsymbol{A},\boldsymbol{f}(t),v_\epsilon)\, p(\boldsymbol{f}(t)|\boldsymbol{z}(t))\, p(\boldsymbol{A}|\boldsymbol{A}_0,\boldsymbol{V}_0)
\end{aligned}
$$

Joint MAP: Alternate optimization

$$
\left\{
\begin{array}{ll}
\widehat{\boldsymbol{f}}(t) = (\widehat{\boldsymbol{A}}'\widehat{\boldsymbol{A}} + \lambda_f \boldsymbol{I})^{-1}\widehat{\boldsymbol{A}}'\boldsymbol{g}(t), & \lambda_f = v_\epsilon/v_f \\
\widehat{\boldsymbol{A}} = \sum_t \boldsymbol{g}(t)\widehat{\boldsymbol{f}}'(t)\left(\sum_t \widehat{\boldsymbol{f}}(t)\widehat{\boldsymbol{f}}'(t) + \lambda_a \boldsymbol{I}\right)^{-1} & \lambda_a = v_\epsilon/v_a
\end{array}
\right.
$$

Alternate optimization Algorithm:

$$
\begin{array}{ccccc}
\boldsymbol{A}^{(0)} \longrightarrow \widehat{\boldsymbol{A}} \longrightarrow & \boxed{\left(\widehat{\boldsymbol{A}}'\widehat{\boldsymbol{A}} + \lambda_f \boldsymbol{I}\right)^{-1}\widehat{\boldsymbol{A}}'\boldsymbol{g}} & \longrightarrow \widehat{\boldsymbol{f}}(t) \\
\uparrow & & \downarrow \\
\widehat{\boldsymbol{A}} \longleftarrow & \boxed{\sum_t \boldsymbol{g}(t)\widehat{\boldsymbol{f}}'(t)\left(\sum_t \widehat{\boldsymbol{f}}(t)\widehat{\boldsymbol{f}}'(t) + \lambda_a \boldsymbol{I}\right)^{-1}} & \longleftarrow \widehat{\boldsymbol{f}}(t)
\end{array}
$$

Variational Bayesian Approximation

Can we do better? Yes, VBA is a good solution.

- Main idea: Approximate a joint pdf $p(\boldsymbol{x})$ difficult to handle by a simpler one (for example a separable one $q(\boldsymbol{x}) = \prod_j q_j(x_j)$)
- Criterion: minimize

$$\mathsf{KL}(q|p) = \int q \ln \frac{q}{p} = \left\langle \ln \frac{q}{p} \right\rangle_q = \sum_j H(q_j) - < \ln p(\boldsymbol{x}) >_q$$

- Solution: $q_j(x_j) \propto \exp\left\{- < \ln p(\boldsymbol{x}) >_{q_{-j}}\right\}$
- In our case: Approximate $p(\boldsymbol{f}, \boldsymbol{A}|\boldsymbol{g})$ by a separable one $q(\boldsymbol{f}, \boldsymbol{A}) = q_1(\boldsymbol{f})q_2(\boldsymbol{A})$
- Solution obtained by alternate optimization:

$$\left\{ \begin{array}{l} q_1(\boldsymbol{f}) \propto \exp\left\{- < \ln p(\boldsymbol{f}, \boldsymbol{A}|\boldsymbol{g}) >_{q_2(\boldsymbol{A})}\right\} \\ q_2(\boldsymbol{A}) \propto \exp\left\{- < \ln p(\boldsymbol{f}, \boldsymbol{A}|\boldsymbol{g}) >_{q_1(\boldsymbol{f})}\right\} \end{array} \right.$$

Joint Estimation: Variational Bayesian Approximation

$$p(\boldsymbol{f}_{1..T}, \boldsymbol{A}|\boldsymbol{g}_{1..T}) \longrightarrow q_1(\boldsymbol{f}_{1..T}|\widetilde{\boldsymbol{A}}, \boldsymbol{g}_{1..T})\; q_2(\boldsymbol{A}|\widetilde{\boldsymbol{f}}_{1..T}, \boldsymbol{g}_{1..T})$$

$$q_1(\boldsymbol{f}(t)|\boldsymbol{g}(t), \boldsymbol{A}, v_\epsilon, \boldsymbol{v}_0) = \mathcal{N}(\widehat{\boldsymbol{f}}(t), \widehat{\boldsymbol{\Sigma}})$$

$$\begin{cases} \widehat{\boldsymbol{\Sigma}} = (\boldsymbol{A}'\boldsymbol{A} + \lambda_f \widehat{\boldsymbol{V}})^{-1} \\ \widehat{\boldsymbol{f}}(t) = (\boldsymbol{A}'\boldsymbol{A} + \lambda_f \widehat{\boldsymbol{V}})^{-1} \boldsymbol{A}'\boldsymbol{g}(t), \quad \lambda_f = v_\epsilon / v_f \end{cases}$$

$$q_2(\boldsymbol{A}|\boldsymbol{g}(t), \boldsymbol{f}(t), v_\epsilon, \boldsymbol{A}_0, \boldsymbol{V}_0) = \mathcal{N}(\widehat{\boldsymbol{A}}, \widehat{\boldsymbol{V}})$$

$$\begin{cases} \widehat{\boldsymbol{V}} = (\boldsymbol{F}'\boldsymbol{F} + \lambda_f \widehat{\boldsymbol{\Sigma}})^{-1} \\ \widehat{\boldsymbol{A}} = \sum_t \boldsymbol{g}(t)\boldsymbol{f}'(t) \left(\sum_t \boldsymbol{f}(t)\boldsymbol{f}'(t) + \lambda_a \widehat{\boldsymbol{\Sigma}}\right)^{-1}, \quad \lambda_a = v_\epsilon / v_a \end{cases}$$

$$\boldsymbol{A}^{(0)} \longrightarrow \widehat{\boldsymbol{A}} \longrightarrow \qquad \boldsymbol{V}^{(0)} \longrightarrow \widehat{\boldsymbol{V}} \longrightarrow$$

$$\boxed{\begin{array}{l} \widehat{\boldsymbol{f}}(t) = \left(\widehat{\boldsymbol{A}}'\widehat{\boldsymbol{A}} + \lambda_f \widehat{\boldsymbol{V}}\right)^{-1} \widehat{\boldsymbol{A}}'\boldsymbol{g}(t) \\ \widehat{\boldsymbol{\Sigma}} = (\boldsymbol{A}'\boldsymbol{A} + \lambda_f \widehat{\boldsymbol{V}})^{-1} \end{array}} \longrightarrow \widehat{\boldsymbol{f}}(t),\ \widehat{\boldsymbol{\Sigma}}$$

$$\Uparrow \qquad\qquad \Downarrow$$

$$\widehat{\boldsymbol{A}},\ \widehat{\boldsymbol{V}} \longleftarrow \boxed{\begin{array}{l} \widehat{\boldsymbol{A}} = \sum_t \boldsymbol{g}(t)\widehat{\boldsymbol{f}}'(t) \left(\sum_t \widehat{\boldsymbol{f}}(t)\widehat{\boldsymbol{f}}'(t) + \lambda_a \widehat{\boldsymbol{\Sigma}}\right)^{-1} \\ \widehat{\boldsymbol{V}} = (\boldsymbol{F}'\boldsymbol{F} + \lambda_f \widehat{\boldsymbol{\Sigma}})^{-1} \end{array}} \longleftarrow \widehat{\boldsymbol{f}}(t),\ \widehat{\boldsymbol{\Sigma}}$$

Bayesian Sparse Sources Separation

Three main steps:

- Assigning priors (sparsity enforcing):
  - Simple priors: $p(\boldsymbol{f})$ and $p(\boldsymbol{A})$
  - Hierarchical priors: $p(\boldsymbol{f}|\boldsymbol{z})\,p(\boldsymbol{z})$ and $p(\boldsymbol{A}|\boldsymbol{q})\,p(\boldsymbol{q})$
- Obtaining the expressions of $p(\boldsymbol{f},\boldsymbol{A},\boldsymbol{\theta}|\boldsymbol{g})$ or $p(\boldsymbol{f},\boldsymbol{A},\boldsymbol{z},\boldsymbol{q},\boldsymbol{\theta}|\boldsymbol{g})$
- Doing the computations:
  - Joint optimization of $p(\boldsymbol{f},\boldsymbol{A},\boldsymbol{\theta}|\boldsymbol{g})$;
  - MCMC Gibbs sampling methods which need generation of samples from the conditionals $p(\boldsymbol{f}|\boldsymbol{A},\boldsymbol{\theta},\boldsymbol{g})$, $p(\boldsymbol{A}|\boldsymbol{f},\boldsymbol{\theta},\boldsymbol{g})$ and $p(\boldsymbol{\theta}|\boldsymbol{f},\boldsymbol{A},\boldsymbol{g})$;
  - Bayesian Variational Approximation (BVA) methods which approximate $p(\boldsymbol{f},\boldsymbol{A},\boldsymbol{\theta}|\boldsymbol{g})$ by a separable one

$$q(\boldsymbol{f},\boldsymbol{A},\boldsymbol{\theta}|\boldsymbol{g}) = q_1(\boldsymbol{f}|\widetilde{\boldsymbol{A}},\widetilde{\boldsymbol{\theta}},\boldsymbol{g})\; q_2(\boldsymbol{A}|\widetilde{\boldsymbol{f}},\widetilde{\boldsymbol{\theta}},\boldsymbol{g})\; q_3(\boldsymbol{\theta}|\widetilde{\boldsymbol{f}},\widetilde{\boldsymbol{A}},\boldsymbol{g})$$

  and then using them for the estimation.

Conclusions

- General source separation problem
  - Estimation of $\boldsymbol{f}$ when $\boldsymbol{A}$ is known
  - Estimation of $\boldsymbol{A}$ when the sources $\boldsymbol{f}$ are known
  - Joint estimation of the sources $\boldsymbol{f}$ and the mixing matrix $\boldsymbol{A}$
- Priors which enforce sparsity:
  - Generalized Gaussian, Student-t, Elastic nets, ...
  - Scaled Gaussian Mixture, Mixture of Gaussians or Gammas, Bernoulli-Gaussian
- Computational tools:
  - Alternate optimization of JMAP criterion
  - MCMC
  - Variational Bayesian Approximation
- Advanced Bayesian methods: Non-Gaussian, Dependent and nonstationnary signals and images.
- Some domaines of applications
  - Acoustic Source localization, Radar and SAR imaging, Spectrometry, Cosmic Microwave Background, Sattelite Image separation, Hyperspectral image processing

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