Bayesian Approach for Data and Image Fusion

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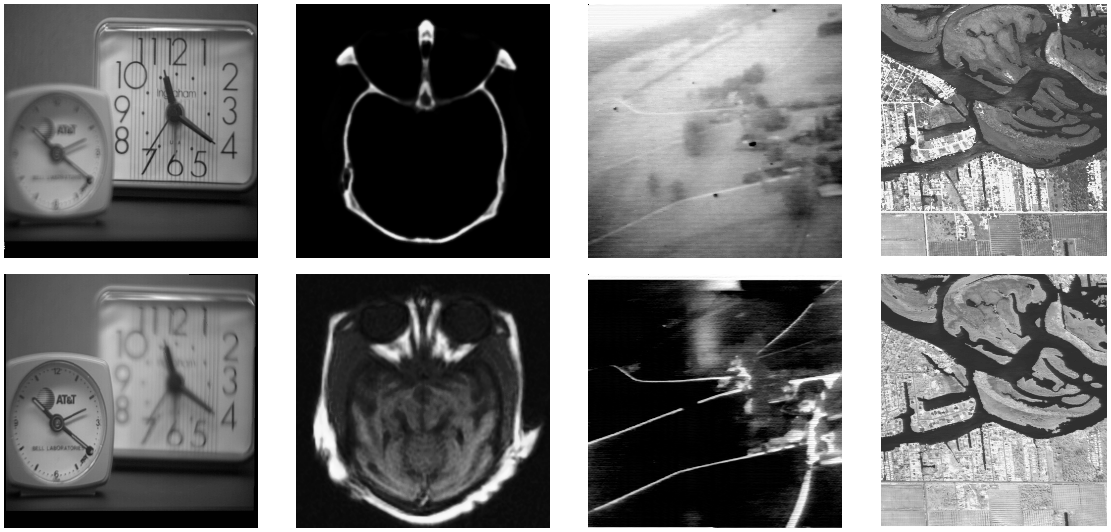
1 Examples of simple image fusion problems

1.1 Fusion of registered images



2 Basics of the Bayesian approach

- Data: D, Unknown: X, Model: $D = \mathcal{M}(\mathcal{X})$
- Likelihood $P(D|X)$
- A priori $P(X)$
- A posteriori $P(X|D) = \dfrac{P(D|X)\,P(X)}{P(D)} \propto P(D|X)\,P(X)$
- Summaries:

  Mean $=\bar{X} = \mathrm{E}\left\{X|D\right\}$,
  Mode $=\arg\max_X \left\{P(X|D\right\}$,
  Median $=\tilde{X}$ such that $P(X < \tilde{X}) = P(X > \tilde{X})$,
  Variance $=V = \mathrm{E}\left\{(X - \bar{X})^2\right\}$
  ...
  Covariances $=V_{i,j} = \mathrm{E}\left\{(X_i - \bar{X}_i)(X_j - \bar{X}_j)\right\}$,
  ...

Decision theory:

- Cost function: $C(\widehat{X}, X)$,
- Expected cost function: $\bar{C}(Z) = \mathrm{E}_{X|D}\left\{C(Z, X)\right\}$
- Best estimate $\widehat{X} = \arg\min_Z \left\{C(Z)\right\}$
- Examples:

$$C(Z, X) = (Z - X)^2 \longrightarrow \widehat{X} = \bar{X} = \mathrm{E}\left\{X|D\right\}$$

$$C(Z, X) = |Z - X| \longrightarrow \widehat{X} = \tilde{X} = \mathrm{Median}(X|D)$$

$$C(Z, X) = \delta(Z - X) = \begin{cases} 1 & \text{if } Z = X \\ 0 & \text{elsewhere} \end{cases} \longrightarrow \widehat{X} = \arg\max_X \left\{P(X|D\right\}$$

A very simple modeling

$$g_i(x, y) = f(x, y) + \epsilon_i(x, y), \quad i = 1, \ldots, M$$

- Hypothesis on $\epsilon_i \longrightarrow \quad p(g_i|f)$
- Hypothesis on $f \longrightarrow \quad p(f)$
- Bayesian MAP estimation:

$$\widehat{f} = \arg\max_f \left\{p(f|g_1, \ldots, g_M)\right\} = \arg\min_f \left\{J(f) = -\ln p(f|g_1, \ldots, g_M)\right\}$$

— Case of Generalized Exponential priors:

$$J(f) = \sum_i \frac{1}{\sigma_i^2}\|g_i - f\|^\beta + \frac{1}{\sigma_0^2}\|f - f_0\|^\alpha \quad \text{with} \quad \|f\|^\alpha = \iint |f(x, y)|^\alpha \,\mathrm{d}x \,\mathrm{d}y$$

— Case of Gaussian priors ($\alpha = \beta = 2$):

$$\nabla J = 0 \longrightarrow \widehat{f} = \frac{1}{\sum_i \lambda_i}\left(\sum_i \lambda_i g_i + \lambda_0 f_0\right), \quad \lambda_i = \frac{1}{\sigma_i^2}, \quad i = 0, \ldots, M$$

$$\lambda_0 = 0, \lambda_1 = \ldots = \lambda_M = 1 \longrightarrow \widehat{f} = \frac{1}{M}\sum_i g_i \quad \text{mean}$$

— Case of Exponential ($\alpha = 0, \beta = 1$): $\widehat{f}$ =median$\{g_i, i = 1, \ldots, M\}$

A more realistic modeling

$$g_i(x,y) = h_i f(x,y) + \epsilon_i(x,y), \quad i=1,\ldots,M \longrightarrow \boldsymbol{g} = \boldsymbol{h}\, f + \boldsymbol{\epsilon}$$

- MAP estimation: $(\widehat{f}, \widehat{\boldsymbol{h}}) = \arg\max_{(f,\boldsymbol{h})} \{p(f, \boldsymbol{h}|\boldsymbol{g})\}$
- An iterative algorithem: $\begin{cases} \widehat{\boldsymbol{h}} = \arg\max_{\boldsymbol{h}} \left\{p(\widehat{f}, \boldsymbol{h}|\boldsymbol{g})\right\} \\ \widehat{f} = \arg\max_f \left\{p(f, \widehat{\boldsymbol{h}}|\boldsymbol{g})\right\} \end{cases}$

  but may not converge to the right solution.
- A two step algorithem: $\begin{cases} \widehat{\boldsymbol{h}} = \arg\max_{\boldsymbol{h}} \{p(\boldsymbol{h}|\boldsymbol{g})\} \\ \widehat{f} = \arg\max_f \left\{p(f|\widehat{\boldsymbol{h}}, \boldsymbol{g})\right\} \end{cases}$

  but needs integration $p(\boldsymbol{h}|\boldsymbol{g}) = \int p(f, \boldsymbol{h}|\boldsymbol{g})\, \mathrm{d}f$

  which may not always be possible $\longrightarrow$ EM algorithem.

- EM algorithem (an iterative algorithem for computing $\widehat{\boldsymbol{h}}$):

$$\begin{cases} Q(\boldsymbol{h}, \widehat{\boldsymbol{h}}) = \mathrm{E}\{\ln p(f, \boldsymbol{h}|\boldsymbol{g})\} = \int \ln p(f, \boldsymbol{h}|\boldsymbol{g}) p(f|\boldsymbol{g}, \widehat{\boldsymbol{h}})\, \mathrm{d}f \\ \widehat{\boldsymbol{h}} = \arg\max_{\boldsymbol{h}} \left\{Q(\boldsymbol{h}, \widehat{\boldsymbol{h}})\right\} \end{cases}$$

- Gaussian case:

$$p(f, \boldsymbol{h}|\boldsymbol{g}) \propto \exp\{-J(f, \boldsymbol{h})\} \quad \text{with} \quad J(f, \boldsymbol{h}) = \sum_i \frac{1}{\sigma_i^2} \|g_i - h_i f\|^2 + \frac{1}{\sigma_0^2} \|Df\|^2$$

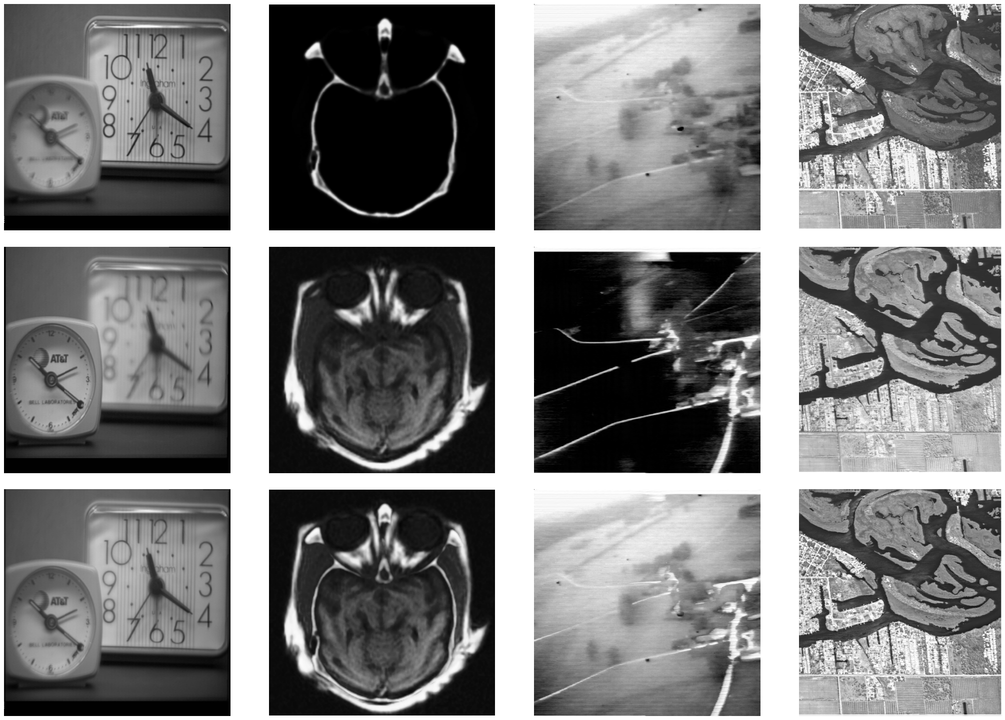
$$\boldsymbol{g}|\boldsymbol{h} \sim \mathcal{N}(\boldsymbol{0}, \Sigma_g) \quad \text{with} \quad \Sigma_g = \boldsymbol{h} \Sigma_f \boldsymbol{h}^t + \Sigma_\epsilon$$

$$p(\boldsymbol{h}|\boldsymbol{g}) \propto p(\boldsymbol{g}|\boldsymbol{h})\, p(\boldsymbol{h})$$

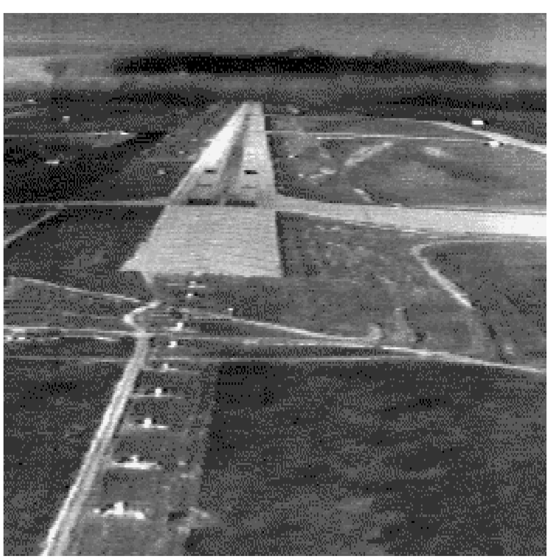
- Particular case: $p(\boldsymbol{h})$ uniform, $\Sigma_f = \boldsymbol{I}$ and $\Sigma_\epsilon = \sigma^2 \boldsymbol{I} \longrightarrow$ FA, PCA ($\sigma^2 = 0$):

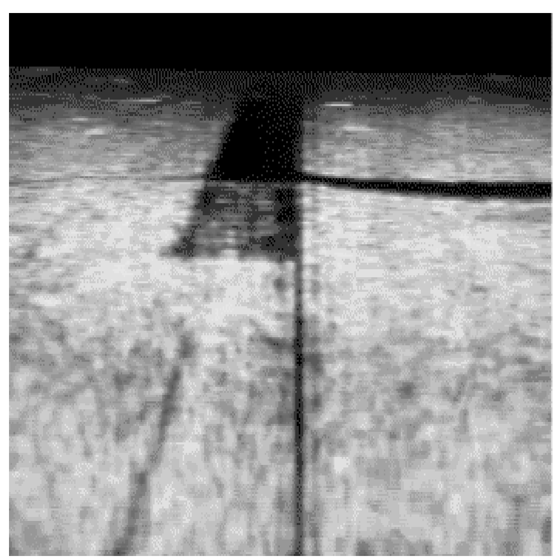
— Estimation of Σ_g with $(\Sigma_g)_{i,j} = < g_i, g_j >$

— λ_i = eigenvalues of Σ_g and $h_i = \begin{cases} \lambda_i - \sigma^2 & \lambda_i > \sigma^2 \\ 0 & \lambda_i < \sigma^2 \end{cases}$



3 Fusion of unregistered images





Registration and fusion of images

$$g_i(x,y) = h_i f\left(x + u(x,y,\boldsymbol{\theta}), y + v(x,y,\boldsymbol{\theta})\right) + \epsilon_i(x,y), \quad i = 1,\ldots,M$$

Two global models (rigid body scene):

- Rotation, Translation, ...:

$$\begin{bmatrix} u \\ v \end{bmatrix} = \begin{bmatrix} u_0 \\ v_0 \end{bmatrix} + \begin{bmatrix} \cos\theta & \sin\theta \\ \sin\theta & \cos\theta \end{bmatrix} \begin{bmatrix} \alpha x \\ \beta y \end{bmatrix} \longrightarrow \boldsymbol{\theta} = (u_0, v_0, \theta, \alpha, \beta)$$

- Projective model of a 3D scene on a plane:

$$\begin{cases} u = \theta_1 + \theta_3 x + \theta_5 y + \theta_7 x^2 + \theta_8 xy \\ v = \theta_2 + \theta_4 x + \theta_6 y + \theta_7 xy + \theta_8 y^2 \end{cases}$$

  Example: $\theta_7 = \theta_8 = 0 \longrightarrow$ Affine transformation

- MAP estimation: $(\widehat{f}, \widehat{\boldsymbol{h}}, \widehat{\boldsymbol{\theta}}) = \arg\max_{(f,\boldsymbol{h},\boldsymbol{\theta})} \{p(f,\boldsymbol{h},\boldsymbol{\theta}|\boldsymbol{g}\}$
- Main difficulty: Optimisation with respect to $\boldsymbol{\theta}$ which needs a global optimization algorithem.

- An iterative algorithem:

$$\begin{cases} \widehat{\boldsymbol{\theta}} = \arg\max_{\boldsymbol{\theta}} \left\{p(\widehat{f}, \widehat{\boldsymbol{h}}, \boldsymbol{\theta}|\boldsymbol{g})\right\} \\ \widehat{\boldsymbol{h}} = \arg\max_{\boldsymbol{h}} \left\{p(\widehat{f}, \boldsymbol{h}, \widehat{\boldsymbol{\theta}}|\boldsymbol{g})\right\} \\ \widehat{f} = \arg\max_{f} \left\{p(f, \widehat{\boldsymbol{h}}, \widehat{\boldsymbol{\theta}}|\boldsymbol{g})\right\} \end{cases}$$

  but may not converge.

- A two step algorithem:

$$\begin{cases} (\widehat{\boldsymbol{h}}, \widehat{\boldsymbol{\theta}}) = \arg\max_{(\boldsymbol{h},\boldsymbol{\theta})} \{p(\boldsymbol{h},\boldsymbol{\theta}|\boldsymbol{g})\} \\ \widehat{f} = \arg\max_{f} \left\{p(f|\widehat{\boldsymbol{h}}, \boldsymbol{g})\right\} \end{cases}$$

  but needs integration $p(\boldsymbol{h},\boldsymbol{\theta}|\boldsymbol{g}) = \int p(f,\boldsymbol{h},\boldsymbol{\theta}|\boldsymbol{g})\,\mathrm{d}f$

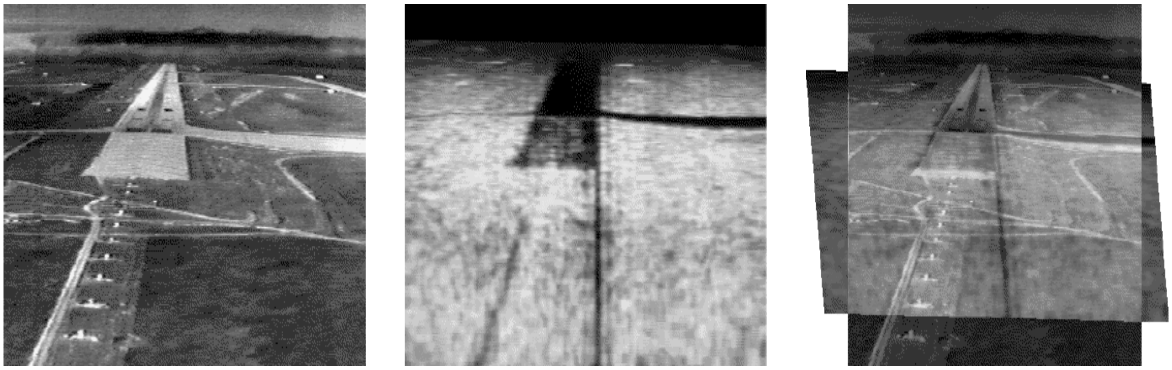
  which may not always be possible $\longrightarrow$ EM algorithem.

- EM algorithem $\boldsymbol{\phi} = (\boldsymbol{h}, \boldsymbol{\theta})$:

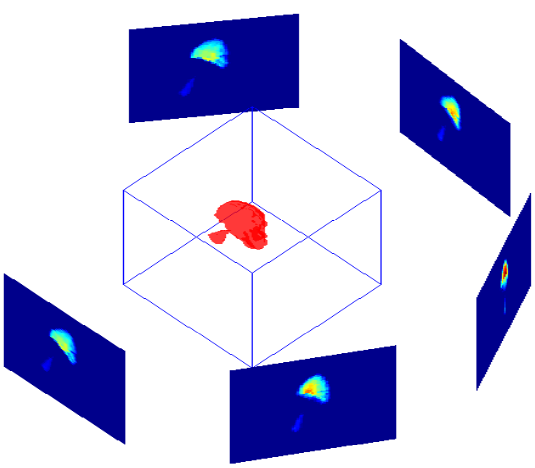
$$\begin{cases} Q(\boldsymbol{\phi}, \widehat{\boldsymbol{\phi}}) = \mathrm{E}\{\ln p(f,\boldsymbol{\phi}|\boldsymbol{g})\} = \int \ln p(f,\boldsymbol{\phi}|\boldsymbol{g}) p(f|\boldsymbol{g},\widehat{\boldsymbol{\phi}})\,\mathrm{d}f \\ \widehat{\boldsymbol{\phi}} = \arg\max_{\boldsymbol{h}} \left\{Q(\boldsymbol{\phi}, \widehat{\boldsymbol{\phi}})\right\} \end{cases}$$

- Main difficulty: Optimisation with respect to $\boldsymbol{\theta}$ which needs a global optimization algorithem $\longrightarrow$ Pyramidal multiresolution approaches

Registration and fusion of images



4 Compted tomography as an image fusion

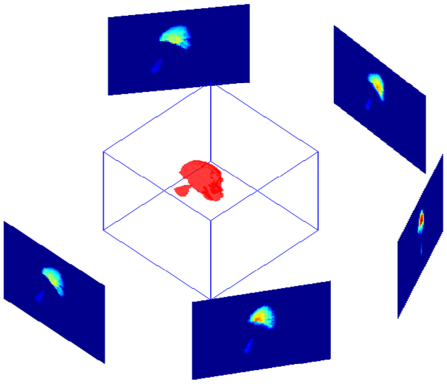


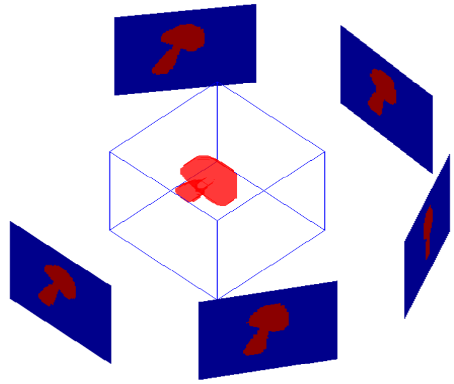
$$g_i(x,y) = \int_{L_{\phi_i,x,y}} f(\vec{r})\,\mathrm{d}l + \epsilon_i(x,y),$$

$$g_i(x,y) = [\mathcal{H}_i f(\vec{r})](x,y) + \epsilon_i(x,y), \quad i = 1,\ldots,M$$

$$\boldsymbol{g}_i = \boldsymbol{H}_i \boldsymbol{f} + \boldsymbol{\epsilon}_i, \quad i = 1,\ldots,M$$

5 Image fusion problems in computer vision





$$f(\vec{r}) = \begin{cases} 1 & \text{if } \vec{r} \in \mathcal{C} \\ 0 & \text{elsewhere} \end{cases}$$

$$g_i(x,y) = [\mathcal{H}_i f(\vec{r})](x,y) + \epsilon_i(x,y),$$

$$\boldsymbol{g}_i = \boldsymbol{H}_i \boldsymbol{f} + \boldsymbol{\epsilon}_i = \boldsymbol{H}_i(\mathcal{C}) + \boldsymbol{\epsilon}_i, \quad i = 1, \ldots, M$$

$$\boldsymbol{g}_i = \boldsymbol{H}_i \boldsymbol{f} + \boldsymbol{\epsilon}_i, \quad i = 1, \ldots, M$$

Hypotheses:

- $\boldsymbol{\epsilon}_i$ i.i.d $\sim \mathcal{N}(0, \boldsymbol{R}_{\epsilon_i})$

$$p(\boldsymbol{g}_i|\boldsymbol{f}) = \mathcal{N}(\boldsymbol{H}_i \boldsymbol{f}, \boldsymbol{R}_{\epsilon_i}) \quad \text{with} \quad \boldsymbol{R}_{\epsilon_i} = \sigma_i^2 \boldsymbol{I}$$

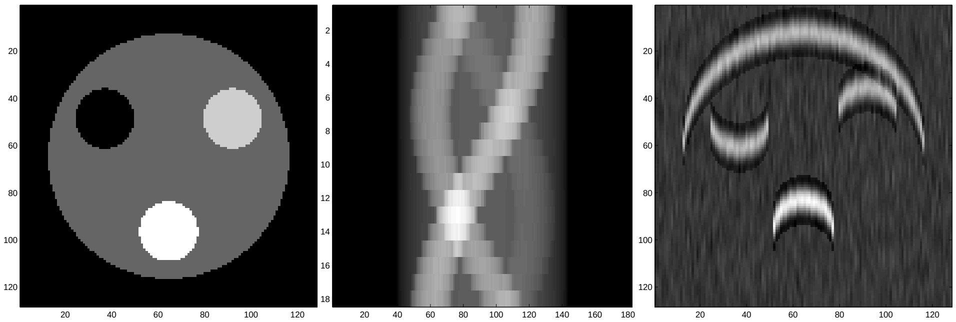
$$\begin{array}{lcl} p(\boldsymbol{g}_i|\boldsymbol{f};\sigma_i^2) & \propto & \exp\{-\frac{1}{2\sigma_i^2}||\boldsymbol{g}_i - \boldsymbol{H}_i\boldsymbol{f}||^2\} \\ p(\boldsymbol{f}|\sigma_f^2) & \propto & \exp\{-\frac{1}{2\sigma_f^2}\phi(\boldsymbol{f})\} \\ p(\boldsymbol{f}|\{\boldsymbol{g}_i, \sigma_i^2, i=1,\ldots,m\}, \sigma_f^2) & \propto & \exp\{-J(\boldsymbol{f})\} \end{array}$$

$$J(\boldsymbol{f}) = \sum_i \frac{1}{2\sigma_i^2}||\boldsymbol{g}_i - \boldsymbol{H}_i\boldsymbol{f}||^2 + \frac{1}{2\sigma_f^2}\phi(\boldsymbol{f})$$

- MAP estimate $\widehat{\boldsymbol{f}} = \arg\min_{\boldsymbol{f}} \{J(\boldsymbol{f})\}$
- When $\phi(\boldsymbol{f})$ quadratic (Gaussian model), $\widehat{\boldsymbol{f}}$ is a linear function of the data $\boldsymbol{g}_i$
- When $\phi(\boldsymbol{f})$ non-quadratic but convexe (for example Generalized Gaussian model), $\widehat{\boldsymbol{f}}$ is a nonlinear function of the data $\boldsymbol{g}_i$, but can be computed easily.

6 Data fusion in computed tomography

6.1 X-rays and ultrasounds data

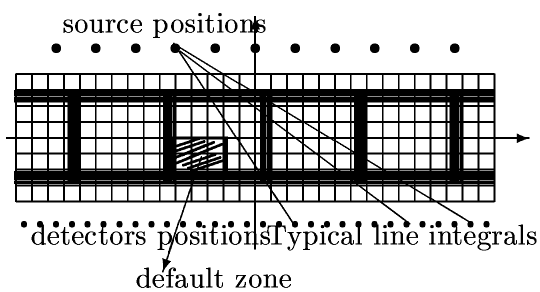


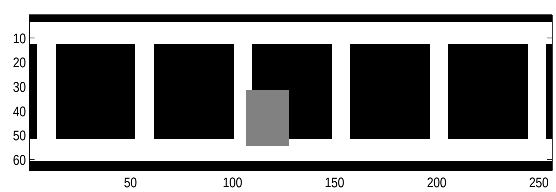
Object	X-rays	Ultrasounds
$\boldsymbol{o} = (\boldsymbol{x}, \boldsymbol{r})$	$\boldsymbol{y} = \boldsymbol{H}_1 \boldsymbol{x} + \boldsymbol{\epsilon}_1$	$\boldsymbol{z} = \boldsymbol{H}_2 \boldsymbol{r} + \boldsymbol{\epsilon}_2$

$\boldsymbol{x}$: material density,

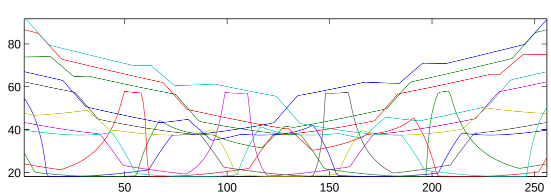
$\boldsymbol{r}$: reflectivity related to change of material density

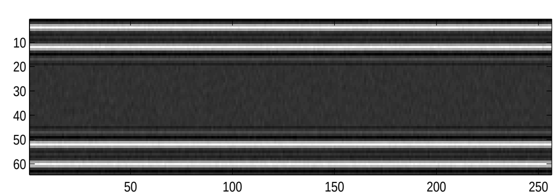
Data fusion in computed tomography for CND





Object $\boldsymbol{x}$





X rays $\quad \boldsymbol{y} = \boldsymbol{H}_1 \boldsymbol{x} + \boldsymbol{\epsilon}_1$

Ultrasounds $\quad \boldsymbol{z} = \boldsymbol{H}_2 \boldsymbol{r} + \boldsymbol{\epsilon}_2$

Bayesian approach and hierarchical modeling

$$\begin{cases} \boldsymbol{y} = \boldsymbol{H}_1\boldsymbol{x} + \boldsymbol{\epsilon}_1 \\ \boldsymbol{z} = \boldsymbol{H}_2\boldsymbol{r} + \boldsymbol{\epsilon}_2 \end{cases} \qquad \text{Data: } (\boldsymbol{y}, \boldsymbol{z}) \qquad \text{Unknowns: } \boldsymbol{o} = (\boldsymbol{x}, \boldsymbol{r})$$

$$p(\boldsymbol{o}) = p(\boldsymbol{x}, \boldsymbol{r}) = p(\boldsymbol{x}|\boldsymbol{r})\, p(\boldsymbol{r})$$

$$p(\boldsymbol{x}, \boldsymbol{r}|\boldsymbol{y}, \boldsymbol{z}) \propto p(\boldsymbol{y}, \boldsymbol{z}|\boldsymbol{x}, \boldsymbol{r})\, p(\boldsymbol{x}, \boldsymbol{r}) = p(\boldsymbol{y}, \boldsymbol{z}|\boldsymbol{x}, \boldsymbol{r})\, p(\boldsymbol{x}|\boldsymbol{r})\, p(\boldsymbol{r})$$

- Conditional independence of $\boldsymbol{y}$ and $\boldsymbol{z}$ and Gaussian process for $\boldsymbol{\epsilon}_1$ and $\boldsymbol{\epsilon}_2$

$$p(\boldsymbol{y}, \boldsymbol{z}|\boldsymbol{x}, \boldsymbol{r}) = p(\boldsymbol{y}|\boldsymbol{x})\, p(\boldsymbol{z}|\boldsymbol{r})$$

$$\begin{aligned} p(\boldsymbol{y}|\boldsymbol{x}; \sigma_1^2) &\propto \exp\{-\tfrac{1}{2\sigma_1^2}||\boldsymbol{y} - \boldsymbol{H}_1\boldsymbol{x}||^2\} \\ p(\boldsymbol{z}|\boldsymbol{r}; \sigma_2^2) &\propto \exp\{-\tfrac{1}{2\sigma_2^2}||\boldsymbol{z} - \boldsymbol{H}_2\boldsymbol{r}||^2\} \end{aligned}$$

- Markovian model for $\boldsymbol{x}|\boldsymbol{r}$: $\quad p(\boldsymbol{x}|\boldsymbol{r}) \propto \exp\{-\alpha\phi(\boldsymbol{x}|\boldsymbol{r})\}$
- A Generalized Gaussian model for $\boldsymbol{r}$: $\quad p(\boldsymbol{r}) \propto \exp\{-\beta\psi(\boldsymbol{r})\}$.

$$p(\boldsymbol{x}, \boldsymbol{r}|\boldsymbol{y}, \boldsymbol{z}) \propto \exp\{-J(\boldsymbol{x}, \boldsymbol{r})\}$$

with $J(\boldsymbol{x}, \boldsymbol{r}) = \dfrac{1}{2\sigma_1^2}||\boldsymbol{y} - \boldsymbol{H}_1\boldsymbol{x}||^2 + \dfrac{1}{2\sigma_2^2}||\boldsymbol{z} - \boldsymbol{H}_2\boldsymbol{r}||^2 + \alpha\,\phi(\boldsymbol{x}|\boldsymbol{r}) \,+\, \beta\,\psi(\boldsymbol{r})$.

- Joint MAP estimation (JMAP):

$$(\widehat{\boldsymbol{x}}, \widehat{\boldsymbol{r}}) = \arg\max_{(\boldsymbol{x}, \boldsymbol{r})} \{p(\boldsymbol{x}, \boldsymbol{r}|\boldsymbol{y}, \boldsymbol{z})\}$$

$$\begin{cases} \widehat{\boldsymbol{r}}^{(k)} = \arg\max_{\boldsymbol{r}} \left\{p(\widehat{\boldsymbol{x}}^{(k-1)}, \boldsymbol{r}|\boldsymbol{y}, \boldsymbol{z})\right\} \\ \widehat{\boldsymbol{x}}^{(k+1)} = \arg\max_{\boldsymbol{x}} \left\{p(\boldsymbol{x}, \widehat{\boldsymbol{r}}^{(k)}|\boldsymbol{y}, \boldsymbol{z})\right\} \end{cases}$$

- Integrate out $\boldsymbol{r}$ and estimate directly $\boldsymbol{x}$:

$$\widehat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} \{p(\boldsymbol{x}|\boldsymbol{y}, \boldsymbol{z})\}$$

where

$$p(\boldsymbol{x}|\boldsymbol{z}, \boldsymbol{y}) = \int p(\boldsymbol{x}, \boldsymbol{r}|\boldsymbol{z}, \boldsymbol{y})\, \mathrm{d}\boldsymbol{r}$$

- First estimate $\boldsymbol{r}$ using $\boldsymbol{z}$ and $\boldsymbol{y}$ then use it to estimate $\boldsymbol{x}$:

$$\begin{cases} \widehat{\boldsymbol{r}} = \arg\max_{\boldsymbol{r}} \{p(\boldsymbol{r}|\boldsymbol{z},\boldsymbol{y})\} \\ \widehat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} \{p(\boldsymbol{x}|\boldsymbol{y},\widehat{\boldsymbol{r}})\} \end{cases}$$

where $\quad p(\boldsymbol{r}|\boldsymbol{z},\boldsymbol{y}) = \int p(\boldsymbol{x},\boldsymbol{r}|\boldsymbol{z},\boldsymbol{y})\,\mathrm{d}\boldsymbol{x} \quad$ and $\quad p(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{r}) \propto p(\boldsymbol{y}|\boldsymbol{x})p(\boldsymbol{x}|\boldsymbol{r})p(\boldsymbol{r})$.

- First estimate $\boldsymbol{r}$ using only $\boldsymbol{z}$ and then use it to estimate $\boldsymbol{x}$:

$$\begin{cases} \widehat{\boldsymbol{r}} = \arg\max_{\boldsymbol{r}} \{p(\boldsymbol{r}|\boldsymbol{z})\} \\ \widehat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} \{p(\boldsymbol{x}|\boldsymbol{y},\widehat{\boldsymbol{r}})\} \end{cases}$$

where $\quad p(\boldsymbol{r}|\boldsymbol{z}) \propto p(\boldsymbol{z}|\boldsymbol{r})p(\boldsymbol{z}) \quad$ and $p(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{r}) \propto p(\boldsymbol{y}|\boldsymbol{x})p(\boldsymbol{x}|\boldsymbol{r})p(\boldsymbol{r})$.

- Estimation of $\boldsymbol{r}$ using only $\boldsymbol{z}$: $\quad \boldsymbol{z} = \boldsymbol{H}_2\boldsymbol{r} + \boldsymbol{\epsilon}_2$

$$p(\boldsymbol{r}) \propto \exp\{-\alpha \sum_j |r_j|^\beta\} \quad \text{with} \quad 1 \leq \beta \leq 2,$$

$$\widehat{\boldsymbol{r}} = \arg\max_{\boldsymbol{r}} \{p(\boldsymbol{r}|\boldsymbol{z})\} = \arg\min_{\boldsymbol{r}} \{J_1(\boldsymbol{r}|\boldsymbol{z})\}$$

with

$$J_1(\boldsymbol{r}|\boldsymbol{z}) = ||\boldsymbol{z} - \boldsymbol{H}_2\boldsymbol{r}||^2 + \lambda_1 \sum_j |r_j|^\beta.$$

- Estimation of $\boldsymbol{x}$ using $\widehat{\boldsymbol{r}}$ and $\boldsymbol{y}$: $\quad \boldsymbol{y} = \boldsymbol{H}_1\boldsymbol{x} + \boldsymbol{\epsilon}_1$

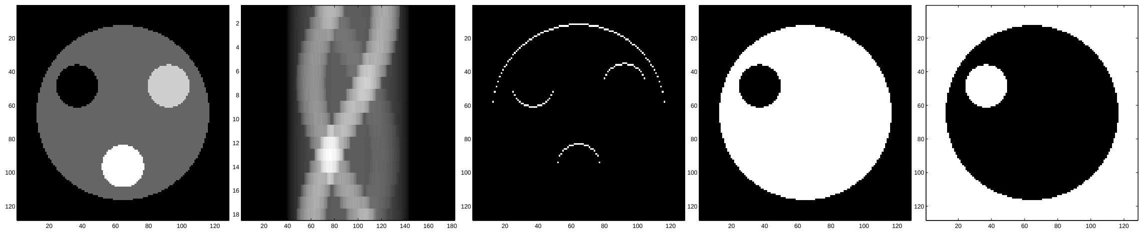
$$p(\boldsymbol{x}|\boldsymbol{r}) \propto \exp\{-q(r_j)\,\phi(x_j - x_{j-1})\} \quad \text{with} \quad q_j = 1 - \frac{|r_j|}{\max(|r_j|)}$$

$$\widehat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} \{p(\boldsymbol{x}|\boldsymbol{y},\widehat{\boldsymbol{q}})\} = \arg\min_{\boldsymbol{x}} \{J_2(\boldsymbol{x}|\boldsymbol{y};\widehat{\boldsymbol{q}})\}$$

with

$$J_2(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{q}) = ||\boldsymbol{y} - \boldsymbol{H}_1\boldsymbol{x}||^2 + \lambda_2 \sum_j (1-q_j)\phi(x_{j+1} - x_j).$$

7 Fusion of radiographic and geometrical data



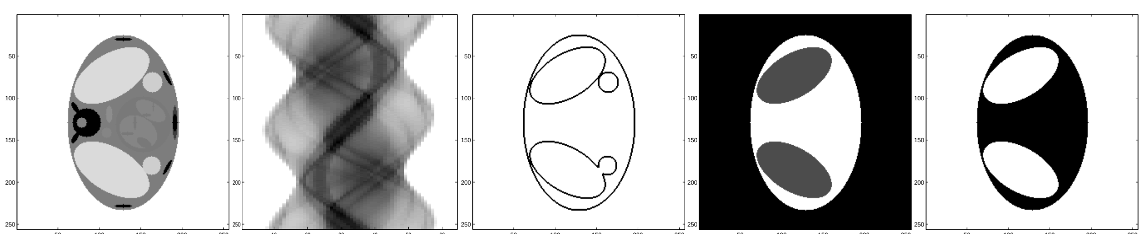
object $\boldsymbol{x}$ sinogram data $\boldsymbol{y}$ borders $\boldsymbol{q}$ regions $\boldsymbol{s}$ reliability $\boldsymbol{\mu}$

$$\boldsymbol{y} = \boldsymbol{H}_1\boldsymbol{x} + \boldsymbol{\epsilon}_1 \longrightarrow p(\boldsymbol{y}|\boldsymbol{x}) \propto \exp\{||\boldsymbol{y} - \boldsymbol{H}_1\boldsymbol{x}||^2\}$$

$$p(\boldsymbol{x}|\boldsymbol{q}) \propto \exp\{-\lambda_1 \sum_j (1-q_j)\,\phi_1(x_j - s_j)\}, \quad p(\boldsymbol{x}|\boldsymbol{s},\boldsymbol{\mu}) \propto \exp\{-\lambda_2 \sum_j \mu_j\,\phi_2(x_j - s_j)\}$$

$$\phi(u) = \{2\ln(cosh(u)), \quad 2\sqrt{1+u^2}-2\} \text{ or } \{\min(u^2,1), \quad u^2/(1+u^2), \quad \ln(1+u^2)\}$$

Fusion of radiographic and anatomical data in medical imaging



object $\boldsymbol{x}$ sinogram data $\boldsymbol{y}$ borders $\boldsymbol{q}$ regions $\boldsymbol{s}$ reliability $\boldsymbol{\mu}$

$$\boldsymbol{y} = \boldsymbol{H}_1\boldsymbol{x} + \boldsymbol{\epsilon}_1 \longrightarrow p(\boldsymbol{y}|\boldsymbol{x}) \propto \exp\{||\boldsymbol{y} - \boldsymbol{H}_1\boldsymbol{x}||^2\}$$

$$p(\boldsymbol{x}|\boldsymbol{q}) \propto \exp\{-\lambda_1 \sum_j (1-q_j)\,\phi_1(x_j - s_j)\}, \quad p(\boldsymbol{x}|\boldsymbol{s},\boldsymbol{\mu}) \propto \exp\{-\lambda_2 \sum_j \mu_j\,\phi_2(x_j - s_j)\}$$

$$\phi(u) = \{2\ln(cosh(u)), \quad 2\sqrt{1+u^2}-2\} \text{ or } \{\min(u^2,1), \quad u^2/(1+u^2), \quad \ln(1+u^2)\}$$

Proposed method

Given the data $\boldsymbol{y}$ and partial knowledge of $\boldsymbol{q}$ and $\boldsymbol{s}$ estimate $\boldsymbol{x}$ by

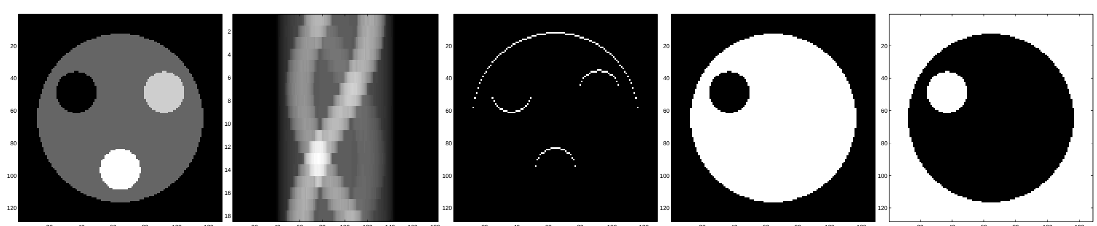
$$\widehat{\boldsymbol{x}} = \arg\max_{\boldsymbol{x}} \{p(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{q},\boldsymbol{s},\boldsymbol{\mu})\} = \arg\min_{\boldsymbol{x}} \{J(\boldsymbol{x}) = -\ln p(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{q},\boldsymbol{s},\boldsymbol{\mu})\}$$

$$J(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{q},\boldsymbol{s},\boldsymbol{\mu}) = ||\boldsymbol{y} - \boldsymbol{H}_1\boldsymbol{x}||^2 + \lambda_1 \sum_j (1-q_j)\,\phi_1(x_{j+1}-x_j) + \lambda_2 \sum_j \mu_j\,\phi_2(x_j - s_j)$$

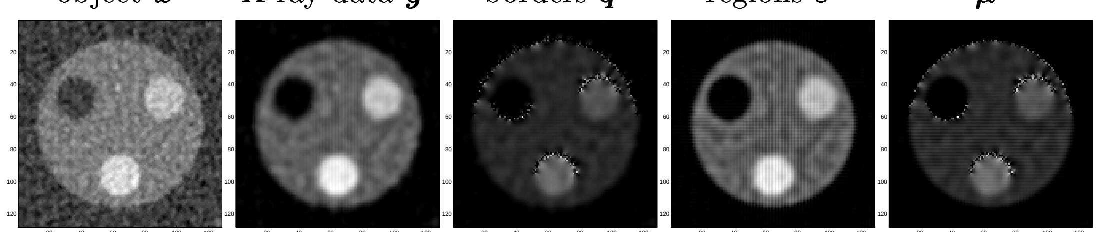
Algorithem:

1. Initialize $\boldsymbol{q} = \boldsymbol{q}^{(0)}$ and $\boldsymbol{s} = \boldsymbol{s}^{(0)}$ (and $\boldsymbol{\mu} = \boldsymbol{\mu}^{(0)}$)
2. Compute $\boldsymbol{x}$ by optimizing $J(\boldsymbol{x}|\boldsymbol{y},\boldsymbol{q},\boldsymbol{s},\boldsymbol{\mu})$
3. Estimate new values for $\boldsymbol{q}$ and $\boldsymbol{s}$ (and $\boldsymbol{\mu}$) from $\boldsymbol{x}$ and $\boldsymbol{q}^{(0)}$ and $\boldsymbol{s}^{(0)}$ (and $\boldsymbol{\mu}^{(0)}$)
4. Return to 2. until convergency.

Fusion of radiographic and ultrasound data in CND

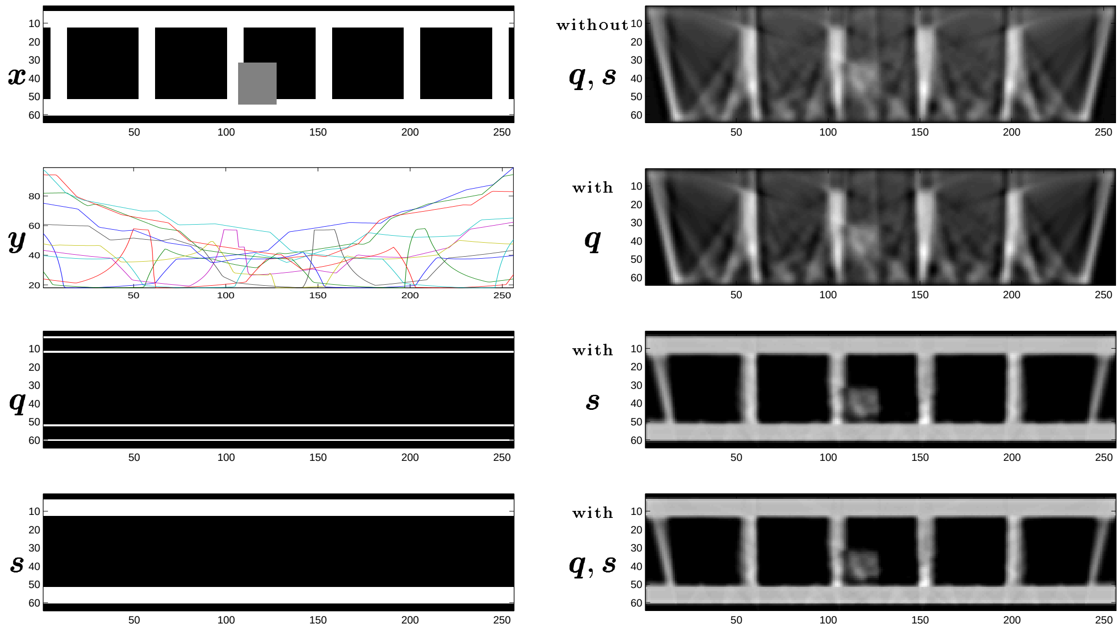


object $\boldsymbol{x}$ | X ray data $\boldsymbol{y}$ | borders $\boldsymbol{q}$ | regions $\boldsymbol{s}$ | $\boldsymbol{\mu}$

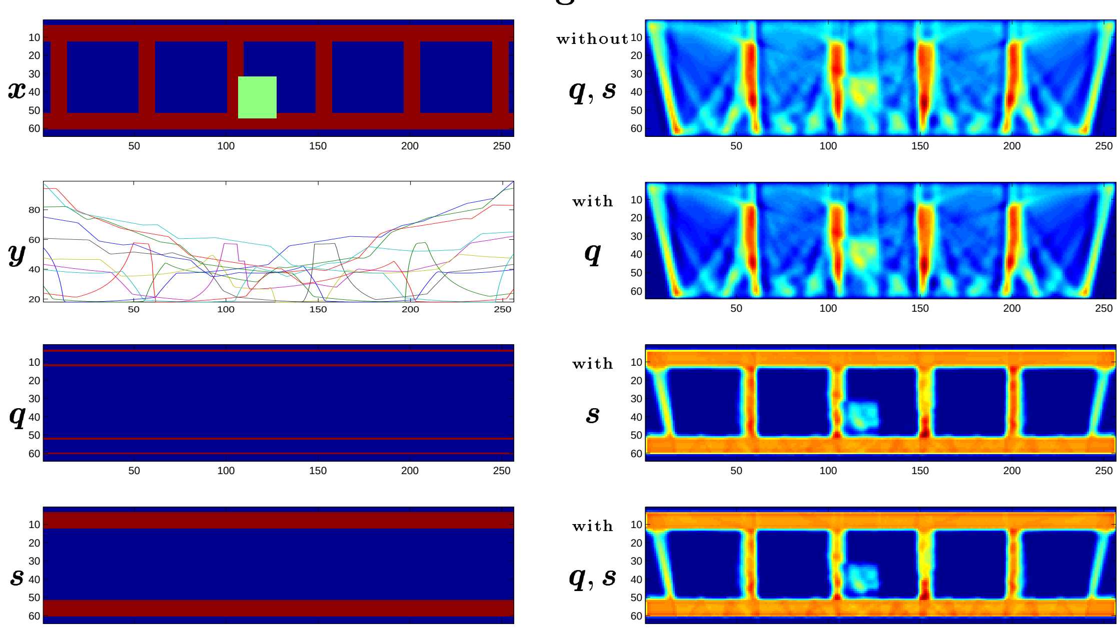


Filtered BP | Quad. regul. | known $\boldsymbol{q}$ | known $\boldsymbol{s}$ | known $\boldsymbol{q}$ and $\boldsymbol{s}$

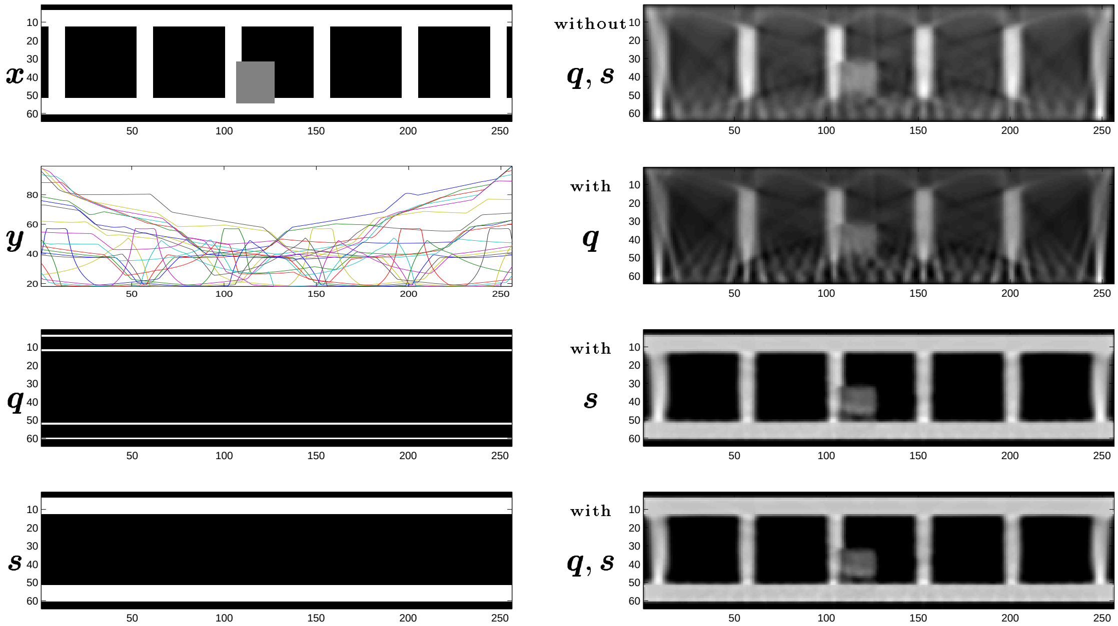
Reconstruction with exact regions and borders data



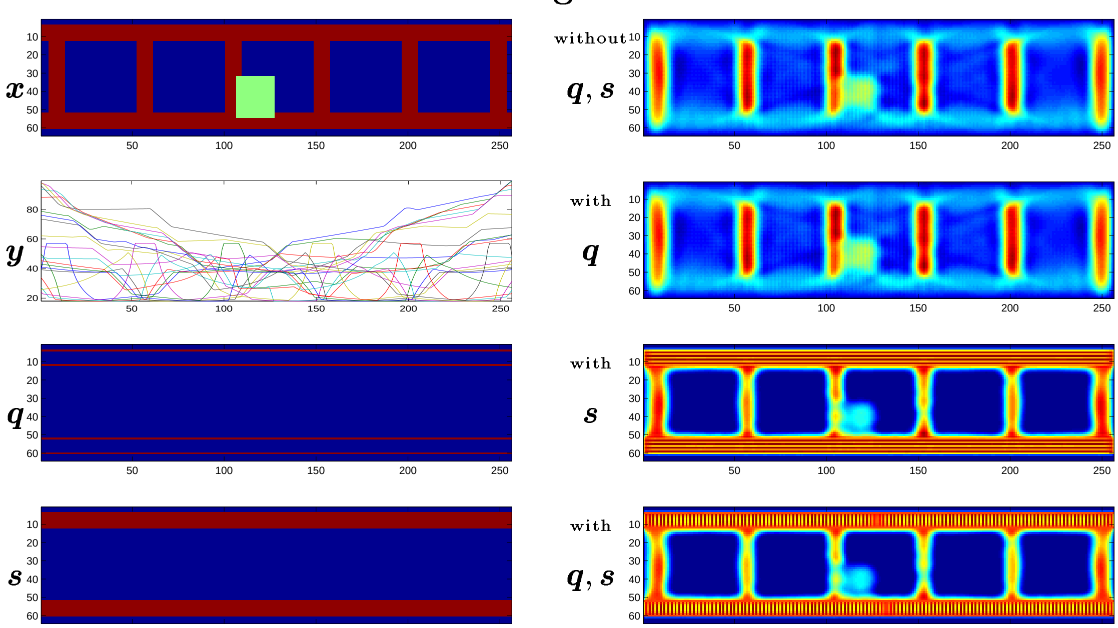
Reconstruction with exact regions and borders data



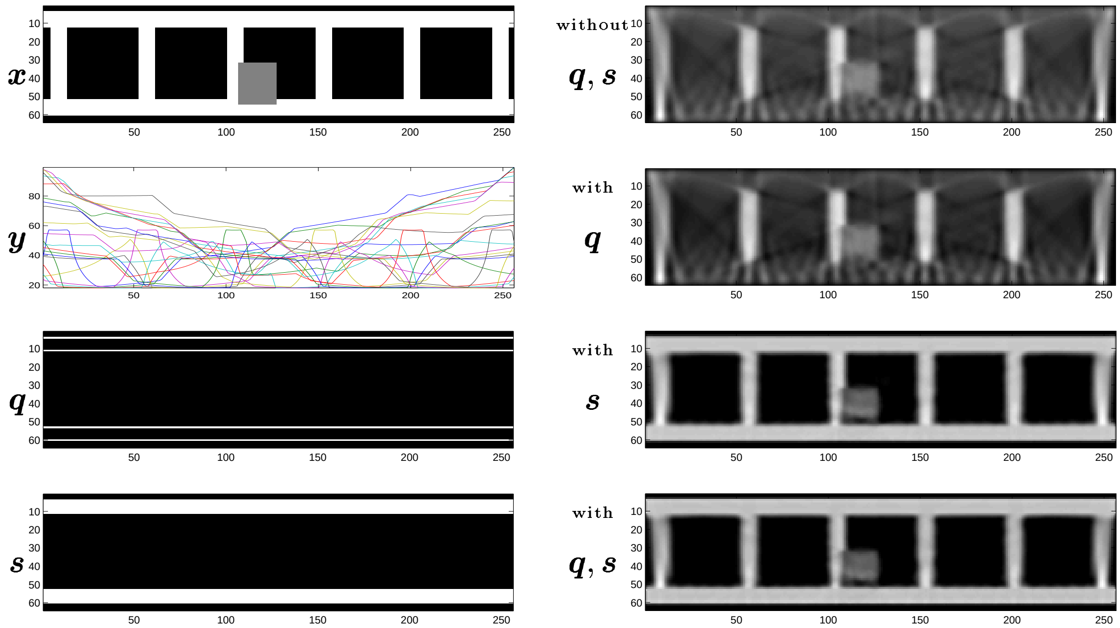
Reconstruction with exact regions and borders data



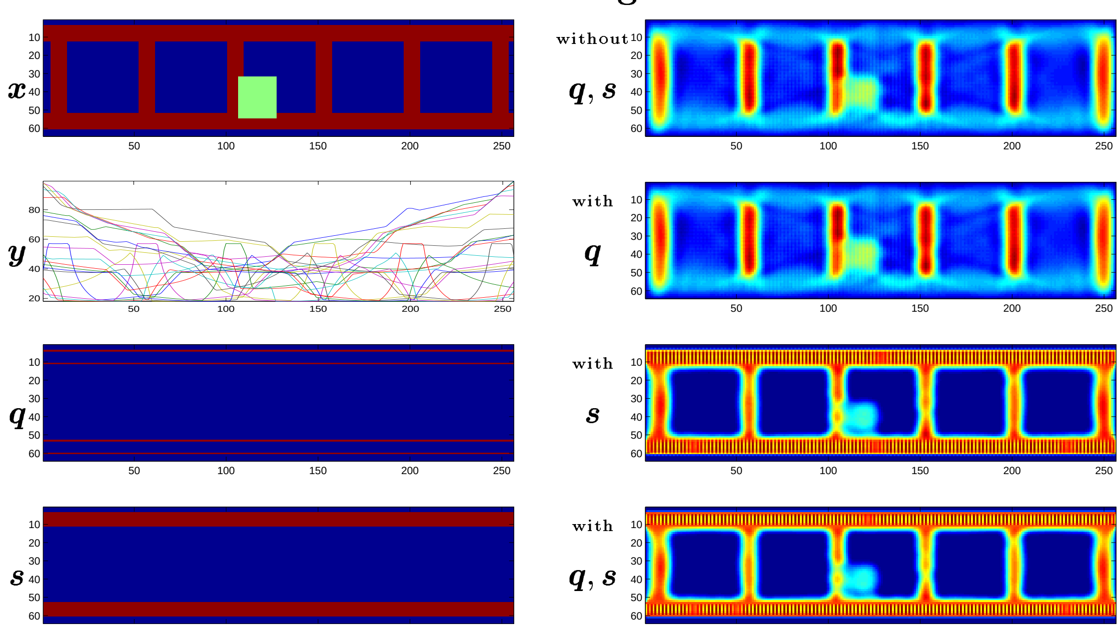
Reconstruction with exact regions and borders data



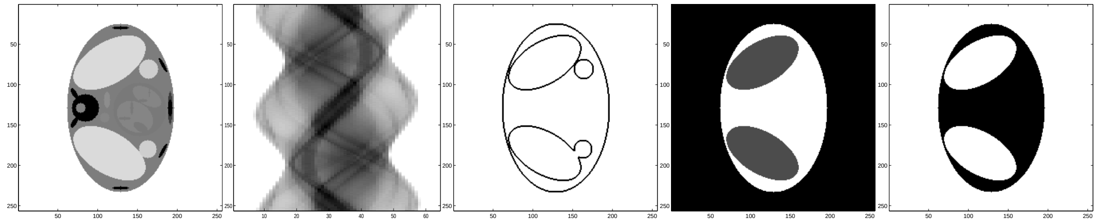
Reconstruction with errors in regions and borders data



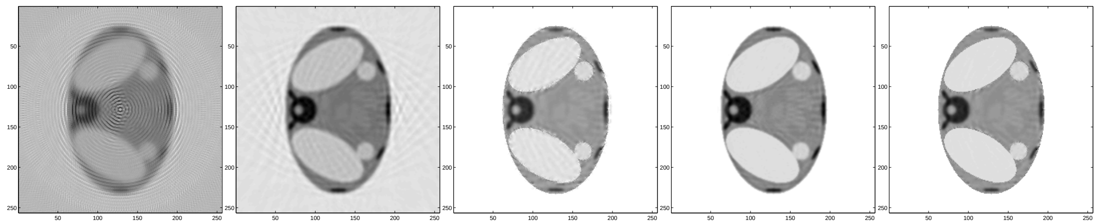
Reconstruction with errors in regions and borders data



Fusion of radiographic and anatomical data in medical imaging

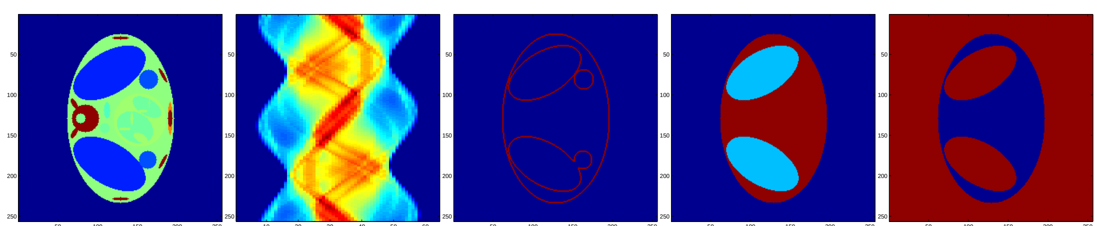


object $\boldsymbol{x}$ | X ray data $\boldsymbol{y}$ | borders $\boldsymbol{q}$ | regions $\boldsymbol{s}$ | $\boldsymbol{\mu}$

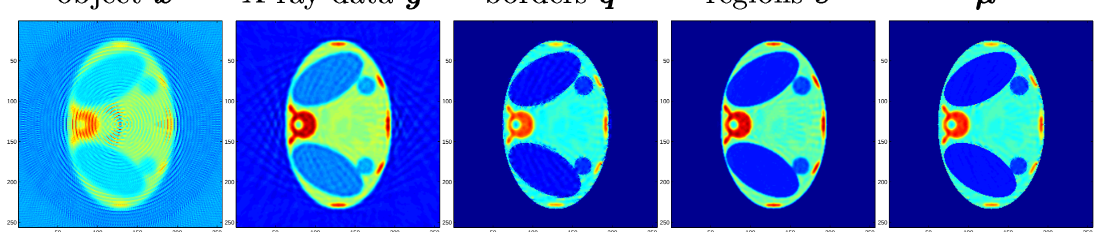


Filtered BP | Quad. regul. | known $\boldsymbol{q}$ | known $\boldsymbol{s}$ | known $\boldsymbol{q}$ and $\boldsymbol{s}$

Fusion of radiographic and anatomical data in medical imaging

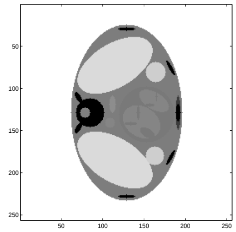


object $\boldsymbol{x}$ | X ray data $\boldsymbol{y}$ | borders $\boldsymbol{q}$ | regions $\boldsymbol{s}$ | $\boldsymbol{\mu}$

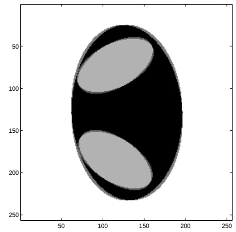


Filtered BP | Quad. regul. | known $\boldsymbol{q}$ | known $\boldsymbol{s}$ | known $\boldsymbol{q}$ and $\boldsymbol{s}$

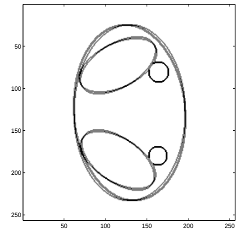
Reconstruction with errors in regions and borders data



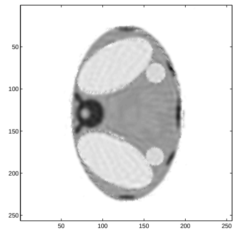
object



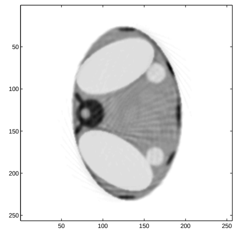
Errors in regions



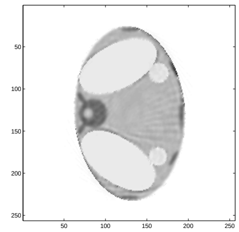
Errors in borders



known borders $\boldsymbol{q}$

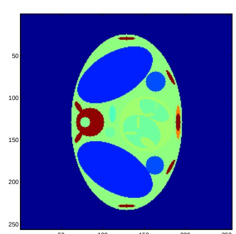


known regions $\boldsymbol{s}$

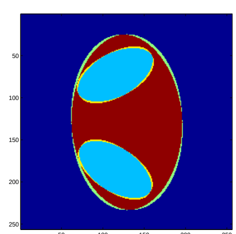


known $\boldsymbol{q}$ and $\boldsymbol{s}$

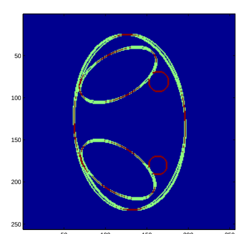
Reconstruction with errors in regions and borders data



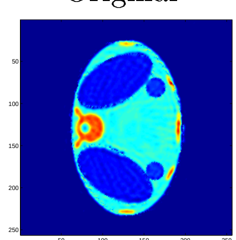
Original



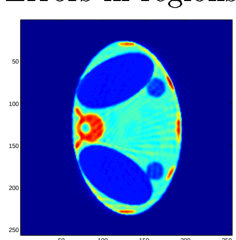
Errors in regions



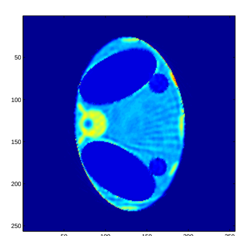
Errors in borders



known borders $\boldsymbol{q}$

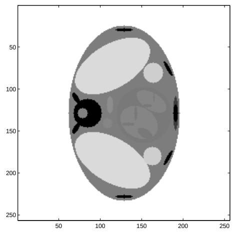


known regions $\boldsymbol{s}$

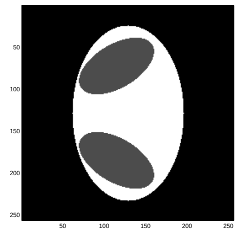


known $\boldsymbol{q}$ and $\boldsymbol{s}$

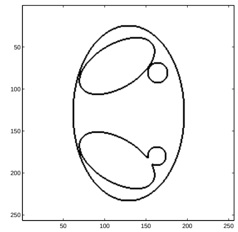
Joint estimation of $\boldsymbol{x}$ and $\boldsymbol{q}$



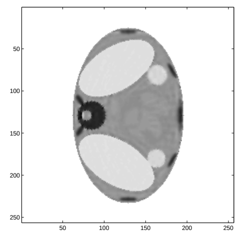
Original



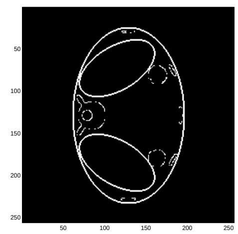
known regions $\boldsymbol{s}^{(0)}$



known borders $\boldsymbol{q}^{(0)}$

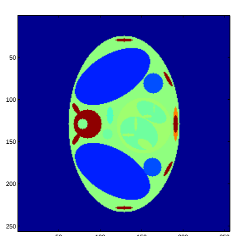


Estimated $\boldsymbol{x}$

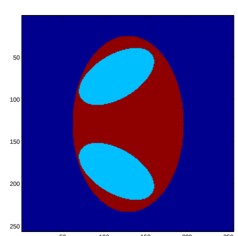


Estimated $\boldsymbol{q}$

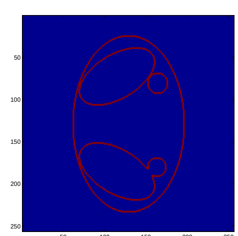
Joint estimation of $\boldsymbol{x}$ and $\boldsymbol{q}$



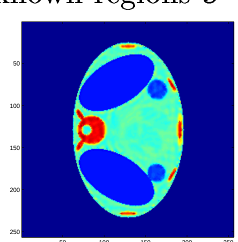
Original



known regions $\boldsymbol{s}^{(0)}$



known borders $\boldsymbol{q}^{(0)}$



Estimated $\boldsymbol{x}$

Estimated $\boldsymbol{q}$

8 Conclusions

- Bayesian inference is an appropriate framework for data fusion.
- Hierarchical Markov modeling is an appropriate tool for linking different unknowns (properties of an object).
- MAP estimation leads to optimization where the criteria are driven from likelihoods and priors.
- ...
- In our numerical experiments with anatomical data fusion in CT, the nowledge of regions brings more than knowledge of borders.

Future work

Algorithem:

1. Initialize $\boldsymbol{q} = \boldsymbol{q}^{(0)}$ and $\boldsymbol{s} = \boldsymbol{s}^{(0)}$ (and $\boldsymbol{\mu} = \boldsymbol{\mu}^{(0)}$)
2. Compute $\boldsymbol{x}$ by optimizing $J(\boldsymbol{x}|\boldsymbol{y}, \boldsymbol{q}, \boldsymbol{s}, \boldsymbol{\mu})$
3. Estimate new values for $\boldsymbol{q}$ and $\boldsymbol{s}$ (and $\boldsymbol{\mu}$) from $\boldsymbol{x}$ and $\boldsymbol{q}^{(0)}$ and $\boldsymbol{s}^{(0)}$ (and $\boldsymbol{\mu}^{(0)}$)
4. Return to 2. until convergency.

- Better joint estimation via MCMC algorithems

$$p(\boldsymbol{x}, \boldsymbol{q}, \boldsymbol{s}, \boldsymbol{\mu}|\boldsymbol{y}) \propto p(\boldsymbol{y}|\boldsymbol{x})\, p(\boldsymbol{x}|\boldsymbol{q})\, p(\boldsymbol{q})\, p(\boldsymbol{x}|\boldsymbol{s}, \boldsymbol{\mu})\, p(\boldsymbol{s})\, p(\boldsymbol{\mu})$$

$$\begin{aligned}
\widehat{\boldsymbol{x}} &\sim p(\boldsymbol{x}|\boldsymbol{y}, \widehat{\boldsymbol{q}}, \widehat{\boldsymbol{s}}, \widehat{\boldsymbol{\mu}}) \\
\widehat{\boldsymbol{q}} &\sim p(\boldsymbol{q}|\boldsymbol{y}, \widehat{\boldsymbol{x}}, \widehat{\boldsymbol{s}}, \widehat{\boldsymbol{\mu}}) \\
\widehat{\boldsymbol{s}} &\sim p(\boldsymbol{s}|\boldsymbol{y}, \widehat{\boldsymbol{x}}, \widehat{\boldsymbol{q}}, \widehat{\boldsymbol{\mu}}) \\
\widehat{\boldsymbol{\mu}} &\sim p(\boldsymbol{\mu}|\boldsymbol{y}, \widehat{\boldsymbol{x}}, \widehat{\boldsymbol{s}}, \widehat{\boldsymbol{q}})
\end{aligned}$$