Chapitre 10

Tableaux récapitulatifs

10.1 Inférence bayésienne

Modèle des observations:

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad \boldsymbol{z}_n = \{x_1, \cdots, x_n\}, \quad \boldsymbol{z}_{n+1} = \{x_1, \cdots, x_n, x_{n+1}\},$

$p(x_i|\boldsymbol{\theta})$

Vraisemblance et statistiques suffisantes :

$$l(\boldsymbol{\theta}|\boldsymbol{z}) = \prod_{i=1}^{n} p(x_i|\boldsymbol{\theta}) = p(\boldsymbol{x}|\boldsymbol{\theta})$$

$$l(\boldsymbol{\theta}|\boldsymbol{z}) = l(\boldsymbol{\theta}|\boldsymbol{t}(\boldsymbol{z}))$$

$$p(\boldsymbol{t}(\boldsymbol{z})|\theta) = \frac{l(\boldsymbol{\theta}|\boldsymbol{t}(\boldsymbol{z}))}{\int l(\boldsymbol{\theta}|\boldsymbol{t}(\boldsymbol{z}))\,\mathrm{d}\boldsymbol{\theta}}$$

Inférence avec une loi *a priori* quelconque :

$p(\boldsymbol{\theta})$

$$p(x_i) = \int p(x_i|\boldsymbol{\theta})p(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta} \quad \text{ou} \quad p(x) = \int p(x|\boldsymbol{\theta})p(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$$

$$p(\boldsymbol{t}(\boldsymbol{z})) = \int p(\boldsymbol{t}(\boldsymbol{z})|\boldsymbol{\theta})p(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$$

$$p(\boldsymbol{z}, \boldsymbol{\theta}) = p(\boldsymbol{z}|\boldsymbol{\theta})\,p(\boldsymbol{\theta})$$

$$p(\boldsymbol{z}) = \int p(\boldsymbol{z}|\boldsymbol{\theta})\,p(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}, \qquad \text{prior predictive}$$

$$p(\boldsymbol{\theta}|\boldsymbol{z}) = \frac{p(\boldsymbol{z}|\boldsymbol{\theta})\,p(\theta)}{p(\boldsymbol{z})}$$

$$p(x|\boldsymbol{z}) = \frac{p(\boldsymbol{z}, x)}{p(\boldsymbol{z})} = \frac{p(\boldsymbol{z}_{n+1})}{p(\boldsymbol{z}_n)}, \qquad \text{posterior predictive}$$

Inférence avec les lois *a priori* conjuguées :

$$p(\boldsymbol{\theta}|\boldsymbol{\tau}_0) = \frac{p(\boldsymbol{t} = \boldsymbol{\tau}_0|\boldsymbol{\theta})}{\int p(\boldsymbol{t} = \boldsymbol{\tau}_0|\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}} = f(\boldsymbol{\theta}, \boldsymbol{\tau}_0)$$

$$p(\boldsymbol{\theta}|\boldsymbol{z}) = f(\boldsymbol{\theta}, \boldsymbol{\tau}), \quad \text{avec} \quad \boldsymbol{\tau} = g(\boldsymbol{\tau}_0, n)$$

Inférence avec les lois conjuguées et famille exponentielles généralisées:

si $\quad p(x_i|\boldsymbol{\theta}) = f(x_i)g(\boldsymbol{\theta}) \exp\left[\sum_{k=1}^{K} c_k\phi_k(\boldsymbol{\theta})h_k(x_i)\right]$

alors

$$t_k(\boldsymbol{x}) = \sum_{j=1}^{n} h_k(x_j), k = 1, \cdots, K$$

$$p(\boldsymbol{\theta}|\boldsymbol{\tau}_0) = \frac{1}{Z(\boldsymbol{\tau})}[g(\boldsymbol{\theta})]^{\tau_0} \exp\left[\sum_{k=1}^{K} \tau_k\phi_k(\boldsymbol{\theta})\right]$$

et la loi *a posteriori* associée est

$$p(\boldsymbol{\theta}|\boldsymbol{x},\boldsymbol{\tau}) = [g(\boldsymbol{\theta})]^{n+\tau_0} \frac{\prod_{j=1}^{n} f(x_j)}{Z(\boldsymbol{\tau})} \exp\left[\sum_{k=1}^{K} c_k\phi_k(\boldsymbol{\theta})\left(\tau_k + t_k(\boldsymbol{x})\right)\right].$$

Inférence avec les lois conjuguées et famille exponentielles naturelles:

si $\quad p(x_i|\boldsymbol{\theta}) = a(x_i) \exp\left[\boldsymbol{\theta}^t\boldsymbol{x} - b(\boldsymbol{\theta})\right]$

alors

$$t(\boldsymbol{x}) = \sum_{j=1}^{n} x_j, k = 1, \cdots, K$$

$$p(\boldsymbol{\theta}|\boldsymbol{\tau}_0) = c(\boldsymbol{\theta}) \exp\left[\boldsymbol{\tau}_0^t\boldsymbol{\theta} - d(\boldsymbol{\tau}_0)\right]$$

et la loi *a posteriori* associée est

$$p(\boldsymbol{\theta}|\boldsymbol{x},\boldsymbol{\tau}_0) = c(\boldsymbol{\theta}) \exp\left[\boldsymbol{\tau}_n^t\boldsymbol{\theta} - d(\boldsymbol{\tau}_n)\right] \quad \text{avec} \quad \boldsymbol{\tau}_n = \boldsymbol{\tau}_0 + \bar{\boldsymbol{x}}$$

où $\quad \bar{\boldsymbol{x}} = \frac{1}{n}\sum_{j=1}^{n} \boldsymbol{x}_j,$

et la loi prédictive associée est

$$p(\boldsymbol{y}|\boldsymbol{x},\boldsymbol{\tau}_0) = p(\boldsymbol{y}|\boldsymbol{t}_n(\boldsymbol{x}),\boldsymbol{\tau}_0) = c(\boldsymbol{\theta}) \exp\left[\boldsymbol{\tau}_n^t\boldsymbol{\theta} - d(\boldsymbol{\tau}_n)\right] \quad \text{avec} \quad \boldsymbol{\tau}_n = \boldsymbol{\tau}_0 + \bar{\boldsymbol{x}}$$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\theta})$

$$p(x_i) = \int p(x_i|\boldsymbol{\theta})\pi(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$$

We note $\quad p(x) = \int p(x|\boldsymbol{\theta})\pi(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$

$$p(\boldsymbol{t}(\boldsymbol{z})) = \int p(\boldsymbol{t}(\boldsymbol{z})|\boldsymbol{\theta})\pi(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$$

$$p(\boldsymbol{z},\boldsymbol{\theta}) = p(\boldsymbol{z}|\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})$$

$$p(\boldsymbol{z}) = \int p(\boldsymbol{z}|\boldsymbol{\theta})\,\pi(\boldsymbol{\theta})\,\mathrm{d}\boldsymbol{\theta}$$

$$p(\boldsymbol{\theta}|\boldsymbol{z}) = \frac{p(\boldsymbol{z}|\boldsymbol{\theta})\,p(\theta)}{p(\boldsymbol{z})}$$

$$p(x|\boldsymbol{z}) = \frac{p(\boldsymbol{z},x)}{p(\boldsymbol{z})}$$

Modèle de Bernouilli :

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i \in \{0,1\}, \quad r = \sum x_i$: nombre des 1, $\quad n-r$: nombre des 0

$p(x_i|\theta) = \mathbf{Ber}(x_i|\theta), \quad 0 < \theta < 1$

Vraisemblance et statistiques suffisantes :

$$l(\theta|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{Ber}(x_i|\theta) = \theta^{\sum x_i}(1-\theta)^{n-\sum x_i} = \theta^r(1-\theta)^{n-r}$$

$$t(\boldsymbol{z}) = r = \sum_{i=1}^{n} x_i, \qquad l(\theta|r) = \theta^r(1-\theta)^{1-r}$$

$$p(r|\theta) = \mathbf{Bin}(r|\theta, n)$$

Inférence avec les lois *a priori* conjuguées :

$$p(\theta) = \mathbf{Bet}(\theta|\alpha, \beta)$$

$$p(x) = \mathbf{BinBet}(x|\alpha, \beta, 1)$$

$$p(r) = \mathbf{BinBet}(r|\alpha, \beta, n)$$

$$p(\theta|\boldsymbol{z}) = \mathbf{Bet}(\theta|\alpha + r, \beta + n - r), \qquad \mathrm{E}\left[\theta|\boldsymbol{z}\right] = \frac{\alpha + r}{\beta + n - r}$$

$$p(x|\boldsymbol{z}) = \mathbf{BinBet}(x|\alpha + r, \beta + n - r, 1), \qquad \mathrm{E}\left[x|\boldsymbol{z}\right] = \frac{\alpha + r}{\beta + n - r}$$

Inférence avec les lois *a priori* de référence :

$$\pi(\theta) = \mathbf{Bet}(\theta|\frac{1}{2}, \frac{1}{2})$$

$$\pi(x) = \mathbf{BinBet}(x|\frac{1}{2}, \frac{1}{2}, 1)$$

$$\pi(r) = \mathbf{BinBet}(r|\frac{1}{2}, \frac{1}{2}, n)$$

$$\pi(\theta|\boldsymbol{z}) = \mathbf{Bet}(\theta|\frac{1}{2} + r, \frac{1}{2} + n - r)$$

$$\pi(x|\boldsymbol{z}) = \mathbf{BinBet}(x|\frac{1}{2} + r, \frac{1}{2} + n - r, 1)$$

Modèle Binomiale :

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i = 0, 1, 2, \cdots, m$

$p(x_i|\theta, m) = \mathbf{Bin}(x_i|\theta, m), \quad 0 < \theta < 1, \quad m = 0, 1, 2, \cdots$

Vraisemblance et statistiques suffisantes :

$$l(\theta|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{Bin}(x_i|\theta, m)$$

$$t(\boldsymbol{z}) = r = \sum_{i=1}^{n} x_i$$

$p(r|\theta) = \mathbf{Bin}(r|\theta, nm)$

Inférence avec les lois *a priori* conjuguées :

$p(\theta) = \mathbf{Bet}(\theta|\alpha, \beta)$

$p(x) = \mathbf{BinBet}(x|\alpha, \beta, m)$

$p(r) = \mathbf{BinBet}(r|\alpha, \beta, nm)$

$$p(\theta|\boldsymbol{z}) = \mathbf{Bet}(\theta|\alpha + r, \beta + n - r), \qquad \mathrm{E}\left[\theta|\boldsymbol{z}\right] = \frac{\alpha + r}{\beta + n - r}$$

$p(x|\boldsymbol{z}) = \mathbf{BinBet}(x|\alpha + r, \beta + n - r, m)$

Inférence avec les lois *a priori* de référence :

$$\pi(\theta) = \mathbf{Bet}(\theta|\frac{1}{2}, \frac{1}{2})$$

$$\pi(x) = \mathbf{BinBet}(x|\frac{1}{2}, \frac{1}{2}, 1)$$

$$\pi(r) = \mathbf{BinBet}(r|\frac{1}{2}, \frac{1}{2}, n)$$

$$\pi(\theta|\boldsymbol{z}) = \mathbf{Bet}(\theta|\frac{1}{2} + r, \frac{1}{2} + n - r)$$

$$\pi(x|\boldsymbol{z}) = \mathbf{BinBet}(x|\frac{1}{2} + r, \frac{1}{2} + n - r, m)$$

Modèle de Poisson :

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i = 0, 1, 2, \cdots$

$p(x_i|\lambda) = \mathbf{Pn}(x_i|\lambda), \quad \lambda \geq 0$

Vraisemblance et statistiques suffisantes :

$$l(\lambda|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{Pn}(x_i|\lambda)$$

$$t(\boldsymbol{z}) = r = \sum_{i=1}^{n} x_i$$

$$p(r|\lambda) = \mathbf{Pn}(r|n\lambda)$$

Inférence avec les lois *a priori* conjuguées :

$p(\lambda) = \mathbf{Gam}(\lambda|\alpha, \beta)$

$p(x) = \mathbf{PnGam}(x|\alpha, \beta, 1)$

$p(r) = \mathbf{PnGam}(r|\alpha, \beta, n)$

$$p(\lambda|\boldsymbol{z}) = \mathbf{Gam}(\lambda|\alpha + r, \beta + n), \qquad \mathrm{E}[\lambda|\boldsymbol{z}] = \frac{\alpha + r}{\beta + n}$$

$p(x|\boldsymbol{z}) = \mathbf{PnGam}(x|\alpha + r, \beta + n, 1)$

Inférence avec les lois *a priori* de référence :

$$\pi(\lambda) \propto \lambda^{-1/2} = \mathbf{Gam}(\lambda|\frac{1}{2}, 0)$$

$$\pi(x) = \mathbf{PnGam}(x|\frac{1}{2}, 0, 1)$$

$$\pi(r) = \mathbf{PnGam}(r|\frac{1}{2}, 0, n)$$

$$\pi(\lambda|\boldsymbol{z}) = \mathbf{Gam}(\lambda|\frac{1}{2} + r, n)$$

$$\pi(x|\boldsymbol{z}) = \mathbf{PnGam}(x|\frac{1}{2} + r, n, 1)$$

Modèle de Binomiale négative :

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i = 0, 1, 2, \cdots$

$p(x_i|\theta, r) = \mathbf{NegBin}(x_i|\theta, r), \quad 0 < \theta < 0,\ r = 1, 2, \cdots$

Vraisemblance et statistiques suffisantes :

$$l(\theta|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{NegBin}(x_i|\theta, r)$$

$$t(\boldsymbol{z}) = s = \sum_{i=1}^{n} x_i$$

$p(s|\theta) = \mathbf{NegBin}(s|\theta, nr)$

Inférence avec les lois *a priori* conjuguées :

$p(\theta) = \mathbf{Bet}(\theta|\alpha, \beta)$

$p(x) = \mathbf{NegBinBet}(x|\alpha, \beta, r)$

$p(s) = \mathbf{NegBinBet}(s|\alpha, \beta, nr)$

$$p(\theta|\boldsymbol{z}) = \mathbf{Bet}(\theta|\alpha + nr, \beta + s), \qquad \mathrm{E}[\theta|\boldsymbol{z}] = \frac{\alpha + nr}{\beta + s}$$

$p(x|\boldsymbol{z}) = \mathbf{NegBinBet}(x|\alpha + nr, \beta + s, nr)$

Inférence avec les lois *a priori* de référence :

$$\pi(\theta) \propto \theta^{-1}(1-\theta)^{-1/2} = \mathbf{Bet}(\theta|0, \frac{1}{2})$$

$$\pi(x) = \mathbf{NegBinBet}(x|0, \frac{1}{2}, r)$$

$$\pi(s) = \mathbf{NegBinBet}(s|0, \frac{1}{2}, nr)$$

$$\pi(\theta|\boldsymbol{z}) = \mathbf{Bet}(\theta|nr, s + \frac{1}{2})$$

$$\pi(x|\boldsymbol{z}) = \mathbf{NegBinBet}(x|nr, s + \frac{1}{2}, nr)$$

Modèle Exponentiel :

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad 0 < x_i < \infty$

$p(x_i|\lambda) = \mathbf{Ex}(x_i|\lambda), \quad \lambda > 0$

Vraisemblance et statistiques suffisantes :

$$l(\lambda|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{Ex}(x_i|\lambda)$$

$$t(\boldsymbol{z}) = t = \sum_{i=1}^{n} x_i$$

$p(t|\lambda) = \mathbf{Gam}(t|n, \lambda)$

Inférence avec les lois *a priori* conjuguées :

$p(\lambda) = \mathbf{Gam}(\lambda|\alpha, \beta)$

$p(x) = \mathbf{GamGam}(x|\alpha, \beta, 1)$

$p(t) = \mathbf{GamGam}(t|\alpha, \beta, n)$

$$p(\lambda|\boldsymbol{z}) = \mathbf{Gam}(\lambda|\alpha + n, \beta + t) \qquad \mathrm{E}\left[\lambda|\boldsymbol{z}\right] = \frac{\alpha + n}{\beta + t}$$

$p(x|\boldsymbol{z}) = \mathbf{GamGam}(x|\alpha + n, \beta + t, 1)$

Inférence avec les lois *a priori* de référence :

$\pi(\lambda) \propto \lambda^{-1} = \mathbf{Gam}(\lambda|0, 0)$

$\pi(x) = \mathbf{GamGam}(x|0, 0, 1)$

$\pi(t) = \mathbf{GamGam}(t|0, 0, n)$

$\pi(\lambda|\boldsymbol{z}) = \mathbf{Gam}(\lambda|n, t)$

$\pi(x|\boldsymbol{z}) = \mathbf{GamGam}(x|n, t, 1)$

Modèle Uniforme :

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad 0 < x_i < \theta$

$p(x_i|\theta) = \mathbf{Uni}(x_i|0, \theta), \quad \theta > 0$

Vraisemblance et statistiques suffisantes :

$$l(\theta|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{Uni}(x_i|0, \theta)$$

$t(\boldsymbol{z}) = t = \max\{x_1, \cdots, x_n\}$

$p(t|\theta) = \mathbf{IPar}(t|n, \theta^{-1})$

Inférence avec les lois *a priori* conjuguées :

$p(\theta) = \mathbf{Par}(\theta|\alpha, \beta)$

$$p(x) = \begin{cases} \frac{\alpha}{\alpha+1}\mathbf{Uni}(x|0, \beta), & \textbf{si} \quad x \leq \beta, \\ \frac{1}{\alpha+1}\mathbf{Par}(x|\alpha, \beta), & \textbf{si} \quad x > \beta \end{cases}$$

$$p(t) = \begin{cases} \frac{\alpha}{\alpha+n}\mathbf{IPar}(t|n, \beta^{-1}), & \textbf{si} \quad t \leq \beta, \\ \frac{n}{\alpha+n}\mathbf{Par}(t|\alpha, \beta), & \textbf{si} \quad x > \beta \end{cases}$$

$p(\theta|\boldsymbol{z}) = \mathbf{Par}(\theta|\alpha + n, \beta_n), \quad \beta_n = \max\{\beta, t\}$

$$p(x|\boldsymbol{z}) = \begin{cases} \frac{\alpha+n}{\alpha+n+1}\mathbf{Uni}(x|0, \beta_n), & \textbf{si} \quad t \leq \beta_n, \\ \frac{1}{\alpha+n+1}\mathbf{Par}(x|\alpha, \beta_n), & \textbf{si} \quad x > \beta_n \end{cases}$$

Inférence avec les lois *a priori* de référence :

$\pi(\theta) \propto \theta^{-1} = \mathbf{Par}(\theta|0, 0)$

$\pi(\theta|\boldsymbol{z}) = \mathbf{Par}(\theta|n, t)$

$$\pi(x|\boldsymbol{z}) = \begin{cases} \frac{n}{n+1}\mathbf{Uni}(x|0, t), & \textbf{si} \quad x \leq t, \\ \dfrac{1}{n+1}\mathbf{Par}(x|n, t), & \textbf{si} \quad x > t \end{cases}$$

Modèle Normale avec la précision $\lambda = \frac{1}{\sigma^2} > 0$ connue (estimation de μ) :

$$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i \in \mathbb{R}, \quad x_i = \mu + b_i, \quad b_i \sim \mathbf{N}(b_i|0, \lambda)$$
$$p(x_i|\mu, \lambda) = \mathbf{N}(x_i|\mu, \lambda), \quad \mu \in \mathbb{R}$$

Vraisemblance et statistiques suffisantes :

$$l(\mu|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}(x_i|\mu, \lambda)$$
$$t(\boldsymbol{z}) = \bar{x} = \frac{1}{n} \sum_{i=1}^{n} x_i$$
$$p(\bar{x}|\mu, \lambda) = \mathbf{N}(\bar{x}|\mu, n\lambda)$$

Inférence avec les lois *a priori* conjuguées :

$$p(\mu) = \mathbf{N}(\mu|\mu_0, \lambda_0)$$
$$p(x) = \mathbf{N}\left(x|\mu_0, \frac{\lambda\lambda_0}{\lambda + \lambda_0}\right)$$
$$p(\bar{x}) = \mathbf{N}\left(\bar{x}|\mu_0, \frac{n\lambda\lambda_0}{\lambda_n}\right), \quad \lambda_n = \lambda_0 + n\lambda$$
$$p(\mu|\boldsymbol{z}) = \mathbf{N}\left(\mu|\mu_n, \lambda_n\right), \quad \mu_n = \frac{\lambda_0\mu_0 + n\lambda\bar{x}}{\lambda_n}$$
$$p(x|\boldsymbol{z}) = \mathbf{N}\left(x|\mu_n, \frac{\lambda\lambda_n}{\lambda+\lambda_n}\right)$$

Inférence avec les lois *a priori* de référence :

$$\pi(\mu) = \textbf{constant}$$
$$\pi(\mu|\boldsymbol{z}) = \mathbf{N}(\mu|\bar{x}, n\lambda)$$
$$\pi(x|\boldsymbol{z}) = \mathbf{N}\left(x|\bar{x}, \frac{n\lambda}{n+1}\right)$$

Inférence avec autres lois *a priori* :

$$\pi(\mu) = \mathbf{St}(\mu|0, \tau^2, \alpha) = \pi_1(\mu|\rho)\pi_2(\rho|\alpha)$$

avec

$$\pi_1(\mu|\rho) = \mathbf{N}(\mu|0, \tau^2\rho),$$
$$\pi_2(\rho|\alpha) = \mathbf{IGam}(\rho|\alpha/2, \alpha/2),$$
$$\pi(\mu|\boldsymbol{z}, \rho) = \mathbf{N}\left(\mu|\frac{1}{1+\tau^2\rho}\bar{x}, \frac{\tau^2\rho}{1+\tau^2\rho}\right)$$
$$\pi(\rho|\boldsymbol{z}) \propto (1+\tau^2\rho)^{-1/2} \exp\left[\frac{-1}{2(1+\tau^2\rho)}\boldsymbol{x}^t\boldsymbol{x}\right] \pi_2(\rho)$$

Modèle Normale avec la variance $\sigma^2 > 0$ connue (estimation de μ) :

$$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i \in \mathbb{R}, \quad x_i = \mu + b_i, \quad b_i \sim \mathbf{N}(b_i|0, \sigma^2)$$
$$p(x_i|\mu, \sigma^2) = \mathbf{N}(x_i|\mu, \sigma^2), \quad \mu \in \mathbb{R}$$

Vraisemblance et statistiques suffisantes :

$$l(\mu|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}(x_i|\mu, \sigma^2)$$
$$t(\boldsymbol{z}) = \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i$$
$$p(\bar{x}|\mu, \sigma^2) = \mathbf{N}(\bar{x}|\mu, \frac{1}{n}\sigma^2)$$

Inférence avec les lois *a priori* conjuguées :

$$p(\mu) = \mathbf{N}(\mu|\mu_0, \sigma_0^2)$$
$$p(x) = \mathbf{N}\left(x|\mu_0, \sigma_0^2 + \sigma^2\right)$$
$$p(x_1, \cdots, x_n) = \mathbf{N}_n\left(\boldsymbol{x}|\mu_0\mathbf{1}, \sigma^2\boldsymbol{I} + \sigma_0^2\mathbf{1}.\mathbf{1}^t\right)$$
$$p(\bar{x}) = \mathbf{N}\left(\bar{x}|\mu_0, \sigma_0^2 + \frac{1}{n}\sigma^2\right),$$
$$p(\mu|\boldsymbol{z}) = \mathbf{N}\left(\mu|\mu_n, \sigma_n^2\right), \quad \mu_n = \frac{\sigma_0^2\sigma^2}{n\sigma_0^2 + \sigma^2}\left(\frac{1}{\sigma_0^2}\mu_0 + \frac{n}{\sigma^2}\bar{x}\right), \quad \sigma_n^2 = \frac{\sigma_0^2\sigma^2}{n\sigma_0^2 + \sigma^2}$$
$$p(x|\boldsymbol{z}) = \mathbf{N}\left(x|\mu_n, \sigma^2 + \sigma_n^2\right)$$

Inférence avec les lois *a priori* de référence :

$$\pi(\mu) = \mathbf{constant}$$
$$\pi(\mu|\boldsymbol{z}) = \mathbf{N}(\mu|\bar{x}, \frac{1}{n}\sigma^2)$$
$$\pi(x|\boldsymbol{z}) = \mathbf{N}\left(x|\bar{x}, \frac{n+1}{n\sigma^2}\right)$$

Inférence avec autres lois *a priori* :

$$\pi(\mu) = \mathbf{St}(\mu|0, \tau^2, \alpha) = \pi_1(\mu|\rho)\pi_2(\rho|\alpha)$$

avec

$$\pi_1(\mu|\rho) = \mathbf{N}(\mu|0, \tau^2\rho),$$
$$\pi_2(\rho|\alpha) = \mathbf{IGam}(\rho|\alpha/2, \alpha/2),$$
$$\pi(\mu|\boldsymbol{z}, \rho) = \mathbf{N}\left(\mu|\frac{1}{1+\tau^2\rho}\bar{x}, \frac{\tau^2\rho}{1+\tau^2\rho}\right)$$
$$\pi(\rho|\boldsymbol{z}) \propto (1+\tau^2\rho)^{-1/2}\exp\left[\frac{-1}{2(1+\tau^2\rho)}\boldsymbol{x}^t\boldsymbol{x}\right]\ \pi_2(\rho)$$

Modèle Normale avec la moyenne $\mu \in \mathbb{R}$ connue (estimation de λ) :

$$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i \in \mathbb{R}, \quad x_i = \mu + b_i, \quad b_i \sim \mathbf{N}(b_i|0, \lambda)$$
$$p(x_i|\mu, \lambda) = \mathbf{N}(x_i|\mu, \lambda), \quad \lambda > 0$$

Vraisemblance et statistiques suffisantes :

$$l(\lambda|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}(x_i|\mu, \lambda)$$
$$t(\boldsymbol{z}) = t = \sum_{i=1}^{n} (x_i - \mu)^2$$
$$p(t|\mu, \lambda) = \mathbf{Gam}(t|\frac{n}{2}, \lambda/2), \quad p(\lambda t|\mu, \lambda) = \mathbf{Chi}^2(\lambda t|n)$$

Inférence avec les lois *a priori* conjuguées :

$$p(\lambda) = \mathbf{Gam}(\lambda|\alpha, \beta)$$
$$p(x) = \mathbf{St}\,(x|\mu, \alpha/\beta, 2\alpha)$$
$$p(t) = \mathbf{GamGam}\left(t|\alpha, 2\beta, \frac{n}{2}\right)$$
$$p(\lambda|\boldsymbol{z}) = \mathbf{Gam}\left(\lambda|\alpha + \frac{n}{2}, \beta + \frac{t}{2}\right)$$
$$p(x|\boldsymbol{z}) = \mathbf{St}\left(x|\mu, \frac{\alpha + \frac{n}{2}}{\beta + \frac{t}{2}}, 2\alpha + n\right)$$

Inférence avec les lois *a priori* de référence :

$$\pi(\lambda) \propto \lambda^{-1} = \mathbf{Gam}(\lambda|0, 0)$$
$$\pi(\lambda|\boldsymbol{z}) = \mathbf{Gam}(\lambda|\frac{n}{2}, \frac{t}{2})$$
$$\pi(x|\boldsymbol{z}) = \mathbf{St}\left(x|\mu, \frac{n}{t}, n\right)$$

Modèle Normale avec la moyenne $\mu \in \mathbb{R}$ connue (estimation de σ^2) :

$$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i \in \mathbb{R}, \quad x_i = \mu + b_i, \quad b_i \sim \mathbf{N}(b_i|0, \sigma^2)$$
$$p(x_i|\mu, \sigma^2) = \mathbf{N}(x_i|\mu, \sigma^2), \quad \sigma^2 > 0$$

Vraisemblance et statistiques suffisantes :

$$l(\sigma^2|\boldsymbol{z}) = \prod_{i=1}^n \mathbf{N}(x_i|\mu, \sigma^2)$$
$$t(\boldsymbol{z}) = t = \sum_{i=1}^n (x_i - \mu)^2$$
$$p(t|\mu, \sigma^2) = \mathbf{Gam}\left(t|\frac{n}{2}, \frac{\sigma^2}{2}\right), \quad p(\frac{t}{\sigma^2}|\mu, \sigma^2) = \mathbf{Chi}^2\left(\frac{t}{\sigma^2}|n\right)$$

Inférence avec les lois *a priori* conjuguées :

$$p(\sigma^2) = \mathbf{IGam}(\sigma^2|\alpha, \beta)$$
$$p(x) = \mathbf{St}\,(x|\mu, \alpha/\beta, 2\alpha)$$
$$p(t) = \mathbf{GamGam}\left(t|\alpha, 2\beta, \frac{n}{2}\right)$$
$$p(\sigma^2|\boldsymbol{z}) = \mathbf{IGam}\left(\sigma^2|\alpha + \frac{n}{2}, \beta + \frac{t}{2}\right)$$
$$p(x|\boldsymbol{z}) = \mathbf{St}\left(x|\mu, \frac{\alpha+\frac{n}{2}}{\beta+\frac{t}{2}}, 2\alpha + n\right)$$

Inférence avec les lois *a priori* de référence :

$$\pi(\sigma^2) \propto \frac{1}{\sigma^2} = \mathbf{IGam}(\sigma^2|0, 0)$$
$$\pi(\sigma^2|\boldsymbol{z}) = \mathbf{IGam}\left(\sigma^2|\frac{n}{2}, \frac{t}{2}\right)$$
$$\pi(x|\boldsymbol{z}) = \mathbf{St}\left(x|\mu, \frac{n}{t}, n\right)$$

Modèle Normale avec les deux paramètres (μ, λ) inconnus :

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i \in \mathbb{R}, \quad x_i = \mu + b_i, \quad b_i \sim \mathbf{N}(b_i|0, \lambda)$

$p(x_i|\mu, \lambda) = \mathbf{N}(x_i|\mu, \lambda), \quad \mu \in \mathbb{R},\ \lambda > 0$

Vraisemblance et statistiques suffisantes :

$$l(\mu, \lambda|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}(x_i|\mu, \lambda)$$

$$t(\boldsymbol{z}) = (\bar{x}, s), \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$$

$p(\bar{x}|\mu, \lambda) = \mathbf{N}(\bar{x}|\mu, n\lambda),$

$p(ns^2|\mu, \lambda) = \mathbf{Gam}(ns^2|(n-1)/2, \lambda/2), \quad p(\lambda ns^2|\mu, \lambda) = \mathbf{Chi}^2(\lambda ns^2|n-1)$

Inférence avec les lois *a priori* conjuguées :

$p(\mu, \lambda) = \mathbf{NGam}(\mu, \lambda|\mu_0, n_0, \alpha, \beta) = \mathbf{N}(\mu|\mu_0, n_0\lambda)\, \mathbf{Gam}(\lambda|\alpha, \beta)$

$$p(\mu) = \mathbf{St}\left(\mu|\mu_0, n_0\frac{\alpha}{\beta}, 2\alpha\right)$$

$p(\lambda) = \mathbf{Gam}(\lambda|\alpha, \beta)$

$$p(x) = \mathbf{St}\left(x|\mu_0, \frac{n_0}{n_0+1}\frac{\alpha}{\beta}, 2\alpha\right)$$

$$p(\bar{x}) = \mathbf{St}\left(\bar{x}|\mu_0, \frac{n_0 n}{n_0+n}\frac{\alpha}{\beta}, 2\alpha\right)$$

$$p(ns^2) = \mathbf{GamGam}\left(ns^2|\alpha, 2\beta, \frac{n-1}{2}\right)$$

$$p(\mu|\boldsymbol{z}) = \mathbf{St}\left(\mu|\mu_n, (n+n_0)(\alpha_n)\beta_n^{-1}, 2\alpha_n\right),$$

$\quad \alpha_n = \alpha + \frac{n}{2},$

$\quad \mu_n = \frac{n_0\mu_0 + n\bar{x}}{n_0+n},$

$\quad \beta_n = \beta + ns^2/2 + \frac{1}{2}\frac{n_0 n}{n_0+n}(\mu_0 - \bar{x})^2$

$p(\lambda|\boldsymbol{z}) = \mathbf{Gam}\left(\lambda|\alpha_n, \beta_n\right)$

$$p(x|\boldsymbol{z}) = \mathbf{St}\left(x|\mu_n, \frac{n+n_0}{n+n_0+1}\frac{\alpha_n}{\beta_n}, 2\alpha_n\right)$$

Inférence avec les lois *a priori* de référence :

$\pi(\mu, \lambda) = \pi(\lambda, \mu) \propto \lambda^{-1}, \quad n > 1$

$\pi(\mu|\boldsymbol{z}) = \mathbf{St}(\mu|\bar{x}, (n-1)s^2, n-1)$

$\pi(\lambda|\boldsymbol{z}) = \mathbf{Gam}(\lambda|(n-1)/2, ns^2/2)$

$$\pi(x|\boldsymbol{z}) = \mathbf{St}\left(x|\bar{x}, \frac{n-1}{n+1}s^{-2}, n-1\right)$$

Modèle Normale avec les deux paramètres (μ, σ^2) **inconnus :**

$\boldsymbol{z} = \{x_1, \cdots, x_n\}, \quad x_i \in \mathbb{R}, \quad x_i = \mu + b_i, \quad b_i \sim \mathbf{N}(b_i|0, \sigma^2)$

$p(x_i|\mu, \sigma^2) = \mathbf{N}(x_i|\mu, \sigma^2), \quad \mu \in \mathbb{R}, \, \sigma^2 > 0$

Vraisemblance et statistiques suffisantes :

$$l(\mu, \sigma^2|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}(x_i|\mu, \sigma^2)$$

$$t(\boldsymbol{z}) = (\bar{x}, s), \quad \bar{x} = \frac{1}{n}\sum_{i=1}^{n} x_i, \quad s^2 = \sum_{i=1}^{n}(x_i - \bar{x})^2$$

$$p(\bar{x}|\mu, \sigma^2) = \mathbf{N}(\bar{x}|\mu, \frac{1}{n} sigma^2),$$

$$p(ns^2|\mu, \sigma^2) = \mathbf{Gam}(ns^2|(n-1)/2, \tfrac{\sigma^2}{2}), \quad p(\sigma^2 ns^2|\mu, \sigma^2) = \mathbf{Chi}^2(\sigma^2 ns^2|n-1)$$

Inférence avec les lois *a priori* conjuguées :

$$p(\mu, \sigma^2) = \mathbf{NIGam}(\mu, \sigma^2|\mu_0, n_0, \alpha, \beta) = \mathbf{N}(\mu|\mu_0, n_0\sigma^2)\, \mathbf{IGam}(\sigma^2|\alpha, \beta)$$

$$p(\mu) = \mathbf{St}\left(\mu|\mu_0, n_0\frac{\alpha}{\beta}, 2\alpha\right)$$

$$p(\sigma^2) = \mathbf{IGam}(\sigma^2|\alpha, \beta)$$

$$p(x) = \mathbf{St}\left(x|\mu_0, \frac{n_0}{n_0+1}\frac{\alpha}{\beta}, 2\alpha\right)$$

$$p(\bar{x}) = \mathbf{St}\left(\bar{x}|\mu_0, \frac{n_0 n}{n_0+n}\frac{\alpha}{\beta}, 2\alpha\right)$$

$$p(ns^2) = \mathbf{GamGam}\left(ns^2|\alpha, 2\beta, \frac{n-1}{2}\right)$$

$$p(\mu|\boldsymbol{z}) = \mathbf{St}\left(\mu|\mu_n, (n+n_0)(\alpha_n)\beta_n^{-1}, 2\alpha_n\right),$$

$$\alpha_n = \alpha + \tfrac{n}{2},$$

$$\mu_n = \tfrac{n_0\mu_0 + n\bar{x}}{n_0+n},$$

$$\beta_n = \beta + ns^2/2 + \tfrac{1}{2}\tfrac{n_0 n}{n_0+n}(\mu_0 - \bar{x})^2$$

$$p(\sigma^2|\boldsymbol{z}) = \mathbf{IGam}\left(\sigma^2|\alpha_n, \beta_n\right)$$

$$p(x|\boldsymbol{z}) = \mathbf{St}\left(x|\mu_n, \tfrac{n+n_0}{n+n_0+1}\tfrac{\alpha_n}{\beta_n}, 2\alpha_n\right)$$

Inférence avec les lois *a priori* de référence :

$$\pi(\mu, \sigma^2) = \pi(\sigma^2, \mu) \propto \frac{1}{\sigma^2}, \quad n > 1$$

$$\pi(\mu|\boldsymbol{z}) = \mathbf{St}(\mu|\bar{x}, (n-1)s^2, n-1)$$

$$\pi(\sigma^2|\boldsymbol{z}) = \mathbf{IGam}(\sigma^2|(n-1)/2, ns^2/2)$$

$$\pi(x|\boldsymbol{z}) = \mathbf{St}\left(x|\bar{x}, \frac{n-1}{n+1}s^{-2}, n-1\right)$$

Modèle Multinomiale :

$$\boldsymbol{z} = \{r_1, \cdots, r_k, n\}, \quad r_i = 0, 1, 2, \cdots, \quad \sum_{i=1}^{k} r_i \leq n,$$

$p(r_i|\theta_i, n) = \mathbf{Bin}(r_i|\theta_i, n),$

$p(\boldsymbol{z}|\boldsymbol{\theta}, n) = \mathbf{Mu}_k(\boldsymbol{z}|\boldsymbol{\theta}, n), \quad 0 < \theta_i < 1, \quad \sum_{i=1}^{k} \theta_i \leq 1$

Vraisemblance et statistiques suffisantes :

$l(\theta|\boldsymbol{z}) = \mathbf{Mu}_k(\boldsymbol{z}|\boldsymbol{\theta}, n)$

$t(\boldsymbol{z}) = (\boldsymbol{r}, n), \quad \boldsymbol{r} = \{r_1, \cdots, r_k\}$

$p(\boldsymbol{r}|\boldsymbol{\theta}) = \mathbf{Mu}_k(\boldsymbol{r}|\boldsymbol{\theta}, n)$

Inférence avec les lois *a priori* conjuguées :

$p(\boldsymbol{\theta}) = \mathbf{Di}_k(\boldsymbol{\theta}|\boldsymbol{\alpha}), \quad \boldsymbol{\alpha} = \{\alpha_1, \cdots, \alpha_{k+1}\}$

$p(\boldsymbol{r}) = \mathbf{Mu}_k(\boldsymbol{r}|\boldsymbol{\alpha}, n)$

$$p(\boldsymbol{\theta}|\boldsymbol{z}) = \mathbf{Di}_k\left(\boldsymbol{\theta}|\alpha_1 + r_1, \cdots, \alpha_k + r_k, \alpha_{k+1} + n - \sum_{i=1}^{k} r_k\right)$$

$$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{Di}_k\left(\boldsymbol{\theta}|\alpha_1 + r_1, \cdots, \alpha_k + r_k, \alpha_{k+1} + n - \sum_{i=1}^{k} r_k\right)$$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\theta}) \propto ??$

$\pi(\boldsymbol{\theta}|\boldsymbol{z}) = ??$

$\pi(\boldsymbol{x}|\boldsymbol{z}) = ??$

Modèle Normale multivariée avec la matrice de précision $\boldsymbol{\Lambda}$ connue (estimation de $\boldsymbol{\mu}$) :

$\boldsymbol{z} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n\}, \quad \boldsymbol{x}_i \in \mathbb{R}^k, \quad \boldsymbol{x}_i = \boldsymbol{\mu} + \boldsymbol{b}_i, \quad \boldsymbol{b}_i \sim \mathbf{N}_k(\boldsymbol{b}_i|\mathbf{0}, \boldsymbol{\Lambda})$

$p(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Lambda}) = \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Lambda}), \quad \boldsymbol{\mu} \in \mathbb{R}^k$, $\boldsymbol{\Lambda}$ matrice d.p. de dimensions $k \times k$

Vraisemblance et statistiques suffisantes :

$$l(\boldsymbol{\mu}|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Lambda})$$

$$t(\boldsymbol{z}) = \bar{\boldsymbol{x}}, \quad \bar{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i,$$

$$p(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, \boldsymbol{\Lambda}) = \mathbf{N}_k(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, n\boldsymbol{\Lambda})$$

Inférence avec les lois *a priori* conjuguées :

$$p(\boldsymbol{\mu}) = \mathbf{N}_k(\boldsymbol{\mu}|\boldsymbol{\mu}_0, \boldsymbol{\Lambda}_0)$$

$$p(\boldsymbol{x}) = \mathbf{N}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_0, (\boldsymbol{\Lambda}_0\boldsymbol{\Lambda})\boldsymbol{\Lambda}_1^{-1}\right), \quad \boldsymbol{\Lambda}_1 = \boldsymbol{\Lambda}_0 + \boldsymbol{\Lambda}$$

$$p(\boldsymbol{\mu}|\boldsymbol{z}) = \mathbf{N}_k\left(\boldsymbol{\mu}|\boldsymbol{\mu}_n, \boldsymbol{\Lambda}_n\right)$$

$$\boldsymbol{\Lambda}_n = \boldsymbol{\Lambda}_0 + n\boldsymbol{\Lambda},$$

$$\boldsymbol{\mu}_n = \boldsymbol{\Lambda}_n^{-1}(\boldsymbol{\Lambda}_0\boldsymbol{\mu}_0 + n\boldsymbol{\Lambda}\bar{\boldsymbol{x}})$$

$$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{N}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_n, \boldsymbol{\Lambda}_n\right)$$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\mu}) =??$

$\pi(\boldsymbol{\mu}|\boldsymbol{z}) =??$

$\pi(\boldsymbol{x}|\boldsymbol{z}) =??$

Modèle Normale multivariée avec la matrice de covariance $\boldsymbol{\Sigma}$ connue (estimation de $\boldsymbol{\mu}$) :

$\boldsymbol{z} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n\}, \quad \boldsymbol{x}_i \in \mathbb{R}^k, \quad \boldsymbol{x}_i = \boldsymbol{\mu} + \boldsymbol{b}_i, \quad \boldsymbol{b}_i \sim \mathbf{N}_k(\boldsymbol{b}_i|\mathbf{0}, \boldsymbol{\Sigma})$

$p(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad \boldsymbol{\mu} \in \mathbb{R}^k$, $\boldsymbol{\Sigma}$ matrice d.p. de dimensions $k \times k$

Vraisemblance et statistiques suffisantes :

$$l(\boldsymbol{\mu}|\boldsymbol{z}) = \prod_{i=1}^n \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$t(\boldsymbol{z}) = \bar{\boldsymbol{x}}, \quad \bar{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^n \boldsymbol{x}_i,$$

$$p(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbf{N}_k(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, n\boldsymbol{\Sigma})$$

Inférence avec les lois *a priori* conjuguées :

$p(\boldsymbol{\mu}) = \mathbf{N}_k(\boldsymbol{\mu}|\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_0)$

$p(\boldsymbol{x}) = \mathbf{N}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_0, \boldsymbol{\Sigma}_1\right), \qquad \boldsymbol{\Sigma}_1 = \boldsymbol{\Sigma}_0 + \boldsymbol{\Sigma}$

$p(\boldsymbol{\mu}|\boldsymbol{z}) = \mathbf{N}_k\left(\boldsymbol{\mu}|\boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n\right)$

$\quad \boldsymbol{\Sigma}_n = \boldsymbol{\Sigma}_0 + \frac{1}{n}\boldsymbol{\Sigma},$

$\quad \boldsymbol{\mu}_n = \boldsymbol{\Sigma}_n^{-1}(\boldsymbol{\Sigma}_0\boldsymbol{\mu}_0 + n\boldsymbol{\Sigma}\bar{\boldsymbol{x}})$

$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{N}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_n, \boldsymbol{\Sigma}_n\right)$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\mu}) =??$

$\pi(\boldsymbol{\mu}|\boldsymbol{z}) =??$

$\pi(\boldsymbol{x}|\boldsymbol{z}) =??$

Modèle Normale multivariée avec la moyenne $\boldsymbol{\mu}$ connue (estimation de $\boldsymbol{\Lambda}$) :

$\boldsymbol{z} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n\}, \quad \boldsymbol{x}_i \in \mathbb{R}^k, \quad \boldsymbol{x}_i = \boldsymbol{\mu} + \boldsymbol{b}_i, \quad \boldsymbol{b}_i \sim \mathbf{N}_k(\boldsymbol{b}_i|\mathbf{0}, \boldsymbol{\Lambda})$

$p(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\lambda}) = \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\lambda}), \quad \boldsymbol{\mu} \in \mathbb{R}^k$, $\boldsymbol{\lambda}$ matrice d.p. de dimensions $k \times k$

Vraisemblance et statistiques suffisantes :

$$l(\boldsymbol{\lambda}|\boldsymbol{z}) = \prod_{i=1}^n \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\lambda})$$

$$t(\boldsymbol{z}) = \boldsymbol{S}, \quad \boldsymbol{S} = \sum_{i=1}^n (\boldsymbol{x}_i - \boldsymbol{\mu})(\boldsymbol{x}_i - \boldsymbol{\mu})^t$$

$$p(\boldsymbol{S}|\boldsymbol{\lambda}) = \mathbf{Wi}_k(\boldsymbol{S}|(n-1)/2, \boldsymbol{\lambda}/2),$$

Inférence avec les lois *a priori* conjuguées :

$$p(\boldsymbol{\lambda}) = \mathbf{Wi}_k(\boldsymbol{\lambda}|\alpha, \boldsymbol{\beta})$$

$$p(\boldsymbol{\lambda}) = \mathbf{Wi}_k(\boldsymbol{\lambda}|\alpha, \boldsymbol{\beta}) \quad ??$$

$$p(\boldsymbol{x}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_0, \frac{n_0}{n_0+1}(\alpha - \frac{k-1}{2})\boldsymbol{\beta}^{-1}, 2\alpha - k + 1\right)$$

$$p(\boldsymbol{\lambda}|\boldsymbol{z}) = \mathbf{Wi}_k\left(\boldsymbol{\lambda}|\alpha_n, \boldsymbol{\beta}_n\right)$$

$$\alpha_n = \alpha + \tfrac{n}{2} - \tfrac{k-1}{2},$$

$$\boldsymbol{\mu}_n = \tfrac{n_0\boldsymbol{\mu}_0 + n\bar{\boldsymbol{x}}}{n_0+n},$$

$$\boldsymbol{\beta}_n = \boldsymbol{\beta} + \tfrac{1}{2}\boldsymbol{S} + \tfrac{1}{2}\tfrac{n_0 n}{n_0+n}(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})^t$$

$$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_n, \tfrac{n+n_0}{n+n_0+1}\alpha_n\boldsymbol{\beta}_n^{-1}, 2\alpha_n\right)$$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\lambda}) = ??$

$\pi(\boldsymbol{\lambda}|\boldsymbol{z}) = ??$

$\pi(\boldsymbol{x}|\boldsymbol{z}) = ??$

Modèle Normale multivariée avec la moyenne $\boldsymbol{\mu}$ connue (estimation de $\boldsymbol{\Sigma}$) :

$\boldsymbol{z} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n\}, \quad \boldsymbol{x}_i \in \mathbb{R}^k, \quad \boldsymbol{x}_i = \boldsymbol{\mu} + \boldsymbol{b}_i, \quad \boldsymbol{b}_i \sim \mathbf{N}_k(\boldsymbol{b}_i|\mathbf{0}, \boldsymbol{\Sigma})$

$p(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad \boldsymbol{\mu} \in \mathbb{R}^k$, $\boldsymbol{\Sigma}$ matrice d.p. de dimensions $k \times k$

Vraisemblance et statistiques suffisantes :

$$l(\boldsymbol{\Sigma}|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$t(\boldsymbol{z}) = \boldsymbol{S}, \quad \boldsymbol{S} = \sum_{i=1}^{n} (\boldsymbol{x}_i - \boldsymbol{\mu})(\boldsymbol{x}_i - \boldsymbol{\mu})^t$$

$$p(\boldsymbol{S}|\boldsymbol{\Sigma}) = \mathbf{Wi}_k(\boldsymbol{S}|(n-1)/2, \boldsymbol{\Sigma}/2),$$

Inférence avec les lois *a priori* conjuguées :

$$p(\boldsymbol{\Sigma}) = \mathbf{IWi}_k(\boldsymbol{\Sigma}|\alpha, \boldsymbol{\beta})$$

$$p(\boldsymbol{x}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_0, \frac{n_0}{n_0+1}(\alpha - \frac{k-1}{2})\boldsymbol{\beta}^{-1}, 2\alpha - k + 1\right)$$

$$p(\boldsymbol{\Sigma}|\boldsymbol{z}) = \mathbf{IWi}_k\left(\boldsymbol{\Sigma}|\alpha_n, \boldsymbol{\beta}_n\right)$$

$$\alpha_n = \alpha + \tfrac{n}{2} - \tfrac{k-1}{2},$$

$$\boldsymbol{\mu}_n = \tfrac{n_0\boldsymbol{\mu}_0 + n\bar{\boldsymbol{x}}}{n_0+n},$$

$$\boldsymbol{\beta}_n = \boldsymbol{\beta} + \tfrac{1}{2}\boldsymbol{S} + \tfrac{1}{2}\tfrac{n_0 n}{n_0+n}(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})^t$$

$$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_n, \tfrac{n+n_0}{n+n_0+1}\alpha_n\boldsymbol{\beta}_n^{-1}, 2\alpha_n\right)$$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\Sigma})$ =??

$\pi(\boldsymbol{\Sigma}|\boldsymbol{z})$ =??

$\pi(\boldsymbol{x}|\boldsymbol{z})$ =??

Modèle Normale multivariée avec les deux paramètres $(\boldsymbol{\mu}, \boldsymbol{\Lambda})$ inconnus :

$\boldsymbol{z} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n\}, \quad \boldsymbol{x}_i \in \mathbb{R}^k,$

$\boldsymbol{x}_i = \boldsymbol{\mu} + \boldsymbol{b}_i, \quad \boldsymbol{b}_i \sim \mathbf{N}_k(\boldsymbol{b}_i|\mathbf{0}, \boldsymbol{\Lambda}) \quad \boldsymbol{\mu} \sim \mathbf{N}_k(\boldsymbol{\mu}|\boldsymbol{\mu}_0, n_0\boldsymbol{\Lambda}) \quad \boldsymbol{\Lambda} \sim \mathbf{Wi}_k(\boldsymbol{\Lambda}|\alpha, \boldsymbol{\beta})$

$p(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Lambda}) = \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Lambda}), \quad \boldsymbol{\mu} \in \mathbb{R}^k$, $\boldsymbol{\Lambda}$ matrice d.p. de dimensions $k \times k$

Vraisemblance et statistiques suffisantes :

$$l(\boldsymbol{\mu}, \boldsymbol{\Lambda}|\boldsymbol{z}) = \prod_{i=1}^{n} \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Lambda})$$

$$t(\boldsymbol{z}) = (\bar{\boldsymbol{x}}, \boldsymbol{S}), \quad \bar{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^{n} \boldsymbol{x}_i, \quad \boldsymbol{S} = \sum_{i=1}^{n} (\boldsymbol{x}_i - \bar{\boldsymbol{x}})(\boldsymbol{x}_i - \bar{\boldsymbol{x}})^t$$

$p(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, \boldsymbol{\Lambda}) = \mathbf{N}_k(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, n\boldsymbol{\Lambda})$

$p(\boldsymbol{S}|\boldsymbol{\Lambda}) = \mathbf{Wi}_k(\boldsymbol{S}|(n-1)/2, \boldsymbol{\Lambda}/2),$

Inférence avec les lois *a priori* conjuguées :

$p(\boldsymbol{\mu}, \boldsymbol{\Lambda}) = \mathbf{NWi}_k(\boldsymbol{\mu}, \boldsymbol{\Lambda}|\boldsymbol{\mu}_0, n_0, \alpha, \boldsymbol{\beta}) = \mathbf{N}_k(\boldsymbol{\mu}|\boldsymbol{\mu}_0, n_0\boldsymbol{\Lambda})\mathbf{Wi}_k(\boldsymbol{\Lambda}|\alpha, \boldsymbol{\beta})$

$p(\boldsymbol{\mu}) = \mathbf{St}_k\left(\boldsymbol{\mu}|\boldsymbol{\mu}_0, n_0\alpha\boldsymbol{\beta}^{-1}, 2\alpha\right) \quad ??$

$p(\boldsymbol{\Lambda}) = \mathbf{Wi}_k(\boldsymbol{\Lambda}|\alpha, \boldsymbol{\beta}) \quad ??$

$$p(\boldsymbol{x}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_0, \frac{n_0}{n_0+1}(\alpha - \frac{k-1}{2})\boldsymbol{\beta}^{-1}, 2\alpha - k + 1\right)$$

$p(\boldsymbol{\mu}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{\mu}|\boldsymbol{\mu}_n, (n+n_0)\alpha_n\boldsymbol{\beta}_n^{-1}, 2\alpha_n\right)$

$\quad \alpha_n = \alpha + \frac{n}{2} - \frac{k-1}{2},$

$\quad \boldsymbol{\mu}_n = \frac{n_0\boldsymbol{\mu}_0 + n\bar{\boldsymbol{x}}}{n_0+n},$

$\quad \boldsymbol{\beta}_n = \boldsymbol{\beta} + \frac{1}{2}\boldsymbol{S} + \frac{1}{2}\frac{n_0 n}{n_0+n}(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})^t$

$p(\boldsymbol{\Lambda}|\boldsymbol{z}) = \mathbf{Wi}_k\left(\boldsymbol{\Lambda}|\alpha_n, \boldsymbol{\beta}_n\right)$

$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_n, \frac{n+n_0}{n+n_0+1}\alpha_n\boldsymbol{\beta}_n^{-1}, 2\alpha_n\right)$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\mu}, \boldsymbol{\Lambda}) = ??$

$\pi(\boldsymbol{\mu}|\boldsymbol{z}) = ??$

$\pi(\boldsymbol{\Lambda}|\boldsymbol{z}) = ??$

$\pi(\boldsymbol{x}|\boldsymbol{z}) = ??$

Modèle Normale multivariée avec les deux paramètres $(\boldsymbol{\mu}, \boldsymbol{\Sigma})$ inconnus :

$\boldsymbol{z} = \{\boldsymbol{x}_1, \cdots, \boldsymbol{x}_n\}, \quad \boldsymbol{x}_i \in \mathbb{R}^k, \quad \boldsymbol{x}_i = \boldsymbol{\mu} + \boldsymbol{b}_i, \quad \boldsymbol{b}_i \sim \mathbf{N}_k(\boldsymbol{b}_i|\mathbf{0}, \boldsymbol{\Sigma})$

$p(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma}), \quad \boldsymbol{\mu} \in \mathbb{R}^k$, $\boldsymbol{\Sigma}$ matrice d.p. de dimensions $k \times k$

Vraisemblance et statistiques suffisantes :

$$l(\boldsymbol{\mu}, \boldsymbol{\Sigma}|\boldsymbol{z}) = \prod_{i=1}^n \mathbf{N}_k(\boldsymbol{x}_i|\boldsymbol{\mu}, \boldsymbol{\Sigma})$$

$$t(\boldsymbol{z}) = (\bar{\boldsymbol{x}}, \boldsymbol{S}), \quad \bar{\boldsymbol{x}} = \frac{1}{n}\sum_{i=1}^n \boldsymbol{x}_i, \quad \boldsymbol{S} = \sum_{i=1}^n (\boldsymbol{x}_i - \bar{\boldsymbol{x}})(\boldsymbol{x}_i - \bar{\boldsymbol{x}})^t$$

$p(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbf{N}_k(\bar{\boldsymbol{x}}|\boldsymbol{\mu}, n\boldsymbol{\Sigma})$

$p(\boldsymbol{S}|\boldsymbol{\Sigma}) = \mathbf{Wi}_k(\boldsymbol{S}|(n-1)/2, \boldsymbol{\Sigma}/2),$

Inférence avec les lois *a priori* conjuguées :

$p(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = \mathbf{NWi}_k(\boldsymbol{\mu}, \boldsymbol{\Sigma}|\boldsymbol{\mu}_0, n_0, \alpha, \boldsymbol{\beta}) = \mathbf{N}_k(\boldsymbol{\mu}|\boldsymbol{\mu}_0, n_0\boldsymbol{\Sigma})\mathbf{Wi}_k(\boldsymbol{\Sigma}|\alpha, \boldsymbol{\beta})$

$p(\boldsymbol{\mu}) = \mathbf{St}_k\left(\boldsymbol{\mu}|\boldsymbol{\mu}_0, n_0\alpha\boldsymbol{\beta}^{-1}, 2\alpha\right) \quad ??$

$p(\boldsymbol{\Sigma}) = \mathbf{Wi}_k(\boldsymbol{\Sigma}|\alpha, \boldsymbol{\beta}) \quad ??$

$$p(\boldsymbol{x}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_0, \frac{n_0}{n_0+1}(\alpha - \frac{k-1}{2})\boldsymbol{\beta}^{-1}, 2\alpha - k + 1\right)$$

$p(\boldsymbol{\mu}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{\mu}|\boldsymbol{\mu}_n, (n+n_0)\alpha_n\boldsymbol{\beta}_n^{-1}, 2\alpha_n\right)$

$\alpha_n = \alpha + \frac{n}{2} - \frac{k-1}{2},$

$\boldsymbol{\mu}_n = \frac{n_0\boldsymbol{\mu}_0 + n\bar{\boldsymbol{x}}}{n_0+n},$

$\boldsymbol{\beta}_n = \boldsymbol{\beta} + \frac{1}{2}\boldsymbol{S} + \frac{1}{2}\frac{n_0 n}{n_0+n}(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})(\boldsymbol{\mu}_0 - \bar{\boldsymbol{x}})^t$

$p(\boldsymbol{\Sigma}|\boldsymbol{z}) = \mathbf{Wi}_k(\boldsymbol{\Sigma}|\alpha_n, \boldsymbol{\beta}_n)$

$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{\mu}_n, \frac{n+n_0}{n+n_0+1}\alpha_n\boldsymbol{\beta}_n^{-1}, 2\alpha_n\right)$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\mu}, \boldsymbol{\Sigma}) = ??$

$\pi(\boldsymbol{\mu}|\boldsymbol{z}) = ??$

$\pi(\boldsymbol{\Sigma}|\boldsymbol{z}) = ??$

$\pi(\boldsymbol{x}|\boldsymbol{z}) = ??$

Régression linéaire :

$\boldsymbol{z} = (\boldsymbol{y}, \boldsymbol{X})$, $\boldsymbol{y} = \{y_1, \cdots, y_n\} \in \mathbb{R}^n$,

$\boldsymbol{x}_i = \{\boldsymbol{x}_{i_1}, \cdots, \boldsymbol{x}_{i_k}\} = \{x_{i1}, \cdots, x_{ik}\} \in \mathbb{R}^k$, $\boldsymbol{X} = (x_{i,j})$

$\boldsymbol{\theta} = \{\theta_1, \cdots, \theta_k\} \in \mathbb{R}^k$, $y_i = \boldsymbol{x}_i^t \boldsymbol{\theta} = \boldsymbol{\theta}^t \boldsymbol{x}_i$

$p(\boldsymbol{y}|\boldsymbol{X}, \boldsymbol{\theta}, \lambda) = \mathbf{N}_n(\boldsymbol{y}|\boldsymbol{X}\boldsymbol{\theta}, \lambda \boldsymbol{I}_n), \quad \boldsymbol{\theta} \in \mathbb{R}^k,\ \lambda > 0$

Vraisemblance et statistiques suffisantes :

$l(\boldsymbol{\theta}|\boldsymbol{z}) = \mathbf{N}_n(\boldsymbol{y}|\boldsymbol{X}\boldsymbol{\theta}, \lambda \boldsymbol{I}_n)$

$t(\boldsymbol{z}) = (\boldsymbol{X}^t\boldsymbol{X}, \boldsymbol{X}^t\boldsymbol{y})$

Inférence avec les lois *a priori* conjuguées :

$p(\boldsymbol{\theta}, \lambda) = \mathbf{NGam}_k(\boldsymbol{\theta}, \lambda|\boldsymbol{\theta}_0, \boldsymbol{\Lambda}_0, \alpha, \beta) = \mathbf{N}_k(\boldsymbol{\theta}|\boldsymbol{\theta}_0, \lambda\boldsymbol{\Lambda}_0)\mathbf{Gam}(\lambda|\alpha, \beta)$

$p(\boldsymbol{\theta}|\lambda) = \mathbf{N}_k(\boldsymbol{\theta}|\boldsymbol{\theta}_0, \lambda\boldsymbol{\Lambda}_0), \quad \mathrm{E}[\boldsymbol{\theta}|\lambda] = \boldsymbol{\theta}_0, \quad \mathrm{Var}[\boldsymbol{\theta}|\lambda] = (\lambda\boldsymbol{\Lambda}_0)^{-1}$

$p(\lambda|\alpha, \beta) = \mathbf{Gam}(\lambda|\alpha, \beta)$

$$p(\boldsymbol{\theta}) = \mathbf{St}_k\left(\boldsymbol{\theta}|\boldsymbol{\theta}_0, \frac{\alpha}{\beta}\boldsymbol{\Lambda}_0, 2\alpha\right), \quad \mathrm{E}[\boldsymbol{\theta}] = \boldsymbol{\theta}_0, \quad \mathrm{Var}[\boldsymbol{\theta}] = \frac{\alpha}{\alpha-2}\boldsymbol{\Lambda}_0^{-1}$$

$p(\lambda) = \mathbf{Gam}(\lambda|\alpha, \beta)$

$$p(y_i|\boldsymbol{x}_i) = \mathbf{St}\left(y_i|\boldsymbol{x}_i^t\boldsymbol{\theta}_0, \frac{\alpha}{\beta}f(\boldsymbol{x}_i), 2\alpha\right), \quad \text{avec} \quad f(\boldsymbol{x}_i) = 1 - \boldsymbol{x}_i^t(\boldsymbol{\Lambda}_0 + \boldsymbol{x}_i\boldsymbol{x}_i^t)^{-1}\boldsymbol{x}_i,$$

$p(\boldsymbol{\theta}, \lambda|\boldsymbol{z}) = \mathbf{NGam}_k(\boldsymbol{\theta}, \lambda|\boldsymbol{\theta}_n, \boldsymbol{\Lambda}_n, \alpha_n, \beta_n) = \mathbf{N}_k(\boldsymbol{\theta}|\boldsymbol{\theta}_n, \lambda\boldsymbol{\Lambda}_n)\mathbf{Gam}(\lambda|\alpha_n, \beta_n)$

$$p(\boldsymbol{\theta}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{\theta}|\boldsymbol{\theta}_n, (\boldsymbol{\Lambda}_0 + \boldsymbol{X}^t\boldsymbol{X})\frac{\alpha_n}{\beta_n}, 2\alpha_n\right)$$

$\alpha_n = \alpha + \frac{n}{2}$,

$\boldsymbol{\theta}_n = (\boldsymbol{\Lambda}_0 + \boldsymbol{X}^t\boldsymbol{X})^{-1}(\boldsymbol{\Lambda}_0\boldsymbol{\theta}_0 + \boldsymbol{X}^t\boldsymbol{y}) = (\boldsymbol{I} - \boldsymbol{\Lambda}_n)\boldsymbol{\theta}_0 + \boldsymbol{\Lambda}_n\widehat{\boldsymbol{\theta}}$,

$\beta_n = \beta + \frac{1}{2}(\boldsymbol{y} - \boldsymbol{X}^t\boldsymbol{\theta}_n)^t\boldsymbol{y} + \frac{1}{2}(\boldsymbol{\theta}_0 - \boldsymbol{\theta}_n)^t\boldsymbol{\Lambda}_0\boldsymbol{\theta}_0 = \beta + \frac{1}{2}\boldsymbol{y}^t\boldsymbol{y} + \frac{1}{2}\boldsymbol{\theta}_0\boldsymbol{\Lambda}_0\boldsymbol{\theta}_0 - \frac{1}{2}\boldsymbol{\theta}_n\boldsymbol{\Lambda}_n\boldsymbol{\theta}_n$

$\widehat{\boldsymbol{\theta}} = (\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X}^t\boldsymbol{y}, \quad \boldsymbol{\Lambda}_n = (\boldsymbol{\Lambda}_0 + \boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X}^t\boldsymbol{X}$

$\mathrm{E}[\boldsymbol{\theta}|\boldsymbol{z}] = \boldsymbol{\theta}_n, \quad \mathrm{Var}[\boldsymbol{\theta}|\boldsymbol{z}] = (\boldsymbol{\Lambda}_0 + \boldsymbol{X}^t\boldsymbol{X})^{-1}$

$p(\lambda|\boldsymbol{z}) = \mathbf{Gam}(\lambda|\alpha_n, \beta_n)$

$p(y_i|\boldsymbol{x}_i, \boldsymbol{z}) = \mathbf{St}\left(y_i|\boldsymbol{x}_i^t\boldsymbol{\theta}_n, f_n(\boldsymbol{x}_i)\frac{\alpha_n}{\beta_n}, 2\alpha_n\right)$

$f_n(\boldsymbol{x}_i) = 1 - \boldsymbol{x}_i^t(\boldsymbol{X}^t\boldsymbol{X} + \boldsymbol{\Lambda}_0 + \boldsymbol{x}_i\boldsymbol{x}_i^t)^{-1}\boldsymbol{x}_i,$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{\theta}, \lambda) = \pi(\lambda, \boldsymbol{\theta}) \propto \lambda^{-(k+1)/2}$

$$\pi(\boldsymbol{\theta}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{\theta}|\widehat{\boldsymbol{\theta}}_n, \frac{n-k}{2\widehat{\beta}_n}\boldsymbol{X}^t\boldsymbol{X}, n-k\right)$$

$\widehat{\boldsymbol{\theta}}_n = (\boldsymbol{X}^t\boldsymbol{X})^{-1}\boldsymbol{X}^t\boldsymbol{y}$,

$$\widehat{\beta}_n = \frac{1}{2}(\boldsymbol{y} - \boldsymbol{X}^t\widehat{\boldsymbol{\theta}}_n)^t(\boldsymbol{y} - \boldsymbol{X}^t\widehat{\boldsymbol{\theta}}_n)$$

$$\pi(\lambda|\boldsymbol{z}) = \mathbf{Gam}\left(\lambda|\frac{n-k}{2}, \widehat{\beta}_n\right)$$

$$p(y_i|\boldsymbol{x}_i, \boldsymbol{z}) = \mathbf{St}\left(y_i|\boldsymbol{x}_i^t\widehat{\boldsymbol{\theta}}_n, \frac{n-k}{2\widehat{\beta}_n}f_n(\boldsymbol{x}_i), n-k\right),$$

$f_n(\boldsymbol{x}_i) = 1 - \boldsymbol{x}_i^t(\boldsymbol{X}^t\boldsymbol{X} + \boldsymbol{x}_i\boldsymbol{x}_i^t)^{-1}\boldsymbol{x}_i,$

Problèmes inverses :

$\boldsymbol{z} = \boldsymbol{H}\boldsymbol{x} + \boldsymbol{b}$, $\boldsymbol{z} = \{y_1, \cdots, y_n\} \in \mathbb{R}^n$, $\boldsymbol{h}_i = \{h_{i_1}, \cdots, h_{i_k}\} \in \mathbb{R}^k$, $\boldsymbol{H} = (h_{i,j})$

$\boldsymbol{x} = \{x_1, \cdots, x_k\} \in \mathbb{R}^k$,

$p(\boldsymbol{z}|\boldsymbol{x}) = \mathbf{N}_n(\boldsymbol{z}|\boldsymbol{H}\boldsymbol{x}, \lambda \boldsymbol{I}_n), \quad \boldsymbol{x} \in \mathbb{R}^k, \ \lambda > 0$

Vraisemblance et statistiques suffisantes :

$l(\boldsymbol{\theta}|\boldsymbol{z}) = \mathbf{N}_n(\boldsymbol{z}|\boldsymbol{H}\boldsymbol{x}, \boldsymbol{I}_n)$

$t(\boldsymbol{z}) = (\boldsymbol{H}^t\boldsymbol{z}, \boldsymbol{H}^t\boldsymbol{x}^t\boldsymbol{x}\boldsymbol{H})$

Inférence avec les lois *a priori* conjuguées :

$p(\boldsymbol{x}, \lambda) = \mathbf{NGam}_k(\boldsymbol{x}, \lambda|\boldsymbol{x}_0, \boldsymbol{\Lambda}_0, \alpha, \beta) = \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{x}_0, \lambda\boldsymbol{\Lambda}_0)\mathbf{Gam}(\lambda|\alpha, \beta)$

$p(\boldsymbol{x}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{x}_0, \frac{\alpha}{\beta}\boldsymbol{\Lambda}_0, 2\alpha\right)$

$p(\lambda) = \mathbf{Gam}(\lambda|\alpha, \beta)$

$p(z_i|\boldsymbol{x}) = \mathbf{St}\left(z_i|\boldsymbol{x}^t\boldsymbol{x}_0, \frac{\alpha}{\beta}f(\boldsymbol{x}), 2\alpha\right)$

$\quad f(\boldsymbol{x}) = 1 - \boldsymbol{x}^t(\boldsymbol{\Lambda}_0 + \boldsymbol{x}^t\boldsymbol{x})^{-1}\boldsymbol{x}$,

$p(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{x}|\boldsymbol{x}_n, (\boldsymbol{\Lambda}_0 + \boldsymbol{H}^t\boldsymbol{H})\frac{\alpha_n}{\beta_n}, 2\alpha_n\right)$

$\quad \alpha_n = \alpha + \frac{n}{2}$,

$\quad \boldsymbol{x}_n = (\boldsymbol{\Lambda}_0 + \boldsymbol{H}^t\boldsymbol{H})^{-1}(\boldsymbol{\Lambda}_0\boldsymbol{x}_0 + \boldsymbol{H}^t\boldsymbol{z})$,

$\quad \beta_n = \beta + \frac{1}{2}(\boldsymbol{z} - \boldsymbol{H}^t\boldsymbol{x}_n)^t\boldsymbol{z} + \frac{1}{2}(\boldsymbol{x}_0 - \boldsymbol{x}_n)^t\boldsymbol{\Lambda}_0\boldsymbol{x}_0$

$p(\lambda|\boldsymbol{z}) = \mathbf{Gam}\,(\lambda|\alpha_n, \beta_n)$

Inférence avec les lois *a priori* de référence :

$\pi(\boldsymbol{x}, \lambda) = \pi(\lambda, \boldsymbol{x}) \propto \lambda^{-(k+1)/2}$

$\pi(\boldsymbol{x}|\boldsymbol{z}) = \mathbf{St}_k\left(\boldsymbol{x}|\widehat{\boldsymbol{x}}_n, \frac{n-k}{2\widehat{\beta}_n}\boldsymbol{H}^t\boldsymbol{H}, n-k\right)$

$\quad \widehat{\boldsymbol{x}}_n = (\boldsymbol{H}^t\boldsymbol{H})^{-1}\boldsymbol{H}^t\boldsymbol{z}$,

$\quad \widehat{\beta}_n = \frac{1}{2}(\boldsymbol{z} - \boldsymbol{H}^t\widehat{\boldsymbol{x}}_n)^t\boldsymbol{z}$

$\pi(\lambda|\boldsymbol{z}) = \mathbf{Gam}\left(\lambda|\frac{n-k}{2}, \widehat{\beta}_n\right)$

10.2 Lois de probabilités usuelles

10.2.1 Notations

Pour faciliter l'utilisation de ces tableaux, j'ai choisi une notation assez particulière qui est résumé dans les tableaux qui suivent.

Lois de probabibilté à une variable discrète

Bernoulli	**Ber** $(x\|\theta)$
Binomiale	**Bin** $(x\|\theta, n)$
Hypergéométrique	**HypGeo** $(x\|N, M, n)$
Binomiale-négative	**NegBin** $(x\|\theta, r)$
Géométrique	**Geo** $(x\|\theta)$
Pascale	**Pas** $(x\|m, \theta)$
Poisson	**Pn** $(x\|\lambda)$
Binomiale-Bêta	**BinBet** $(x\|\alpha, \beta, n)$
Binomiale-Bêta négative	**NegBinBet** $(x\|\alpha, \beta, r)$
Poisson-Gamma	**PnGam** $(x\|\alpha, \beta, n)$
Poisson composée	**Pnc** $(x\|\lambda, \mu)$

Lois de probabibilté à une variable réelle

Bêta	**Bet** $(x\|\alpha,\beta)$
Gamma	**Gam** $(x\|\alpha,\beta)$
Gamma inverse	**IGam** $(x\|\alpha,\beta)$
Gamma–Gamma	**GamGam** $(x\|\alpha,\beta,n)$
Pareto	**Par** $(x\|\alpha,\beta)$
Normale (paramètre précision)	**N** $(x\|\mu,\lambda)$
Normale (paramètre variance)	**N** $(x\|\mu,\sigma^2)$
du khi-deux	$\mathbf{Chi}^2$ $(x\|\nu)$
du khi-deux décentrée	$\mathbf{Chi}^2$ $(x\|\nu,\lambda)$
Logistic	**Lo** $(x\|\alpha,\beta)$
Student (t)	**St** $(x\|\mu,\lambda,\alpha)$
Fisher-Snedecor	**FS** $(x\|\alpha,\beta)$
Uniforme	**Uni** $(x\|\theta_1,\theta_2)$
Exponentielle	**Ex** $(x\|\lambda)$
du khi inverse	**IChi** $(x\|\nu)$
Racine carrée Gamma inverse	$\mathbf{IGam}^{-1/2}$ $(x\|\alpha,\beta)$
Pareto inverse	**IPar** $(x\|\alpha,\beta)$
Cauchy	**Cau** $(x\|\lambda)$
Rayleight	**Ray** $(x\|\theta)$
Log-Normale	**LogN** $(x\|\mu,\Lambda)$
Normale généralisée	**Ngen** $(x\|\alpha,\beta)$
Weibull	**Wei** $(x\|\alpha)$
Exponentielle double	**Exd** $(x\|\lambda)$
Exponentielle tronquée	**Ext** $(x\|\lambda)$
Triangulaire	**Tri** $(x\|\theta)$
du khi	**IChi** $(x\|\nu)$
Exponentielle généralisé à un paramètres	**Exf** $(x\|f,g,h,\phi,\theta)$
Exponentielle généralisé à K paramètres	**Exfk** $(x\|f,g,\boldsymbol{h},\boldsymbol{\phi},\boldsymbol{\theta})$

Lois de probabibilté à deux variables réelles

Normale-Gamma	$\mathbf{NGam}\left(x, y \mid \mu, \lambda, \alpha, \beta\right)$
Pareto bivariable	$\mathbf{Par}_2\left(x, y \mid \alpha, \beta_0, \beta_1\right)$

Lois de probabibilté à n variables discrètes

Multinomiale	$\mathbf{Mu}_k\left(\boldsymbol{x} \mid \boldsymbol{\theta}, n\right)$
Dirichlet	$\mathbf{Di}_k\left(\boldsymbol{x} \mid \boldsymbol{\theta}\right)$
Multinomiale–Dirichlet	$\mathbf{MuDi}_k\left(\boldsymbol{x} \mid \boldsymbol{\theta}, n\right)$

Lois de probabibilté à n variables réelles

Exponentielle généralisée	$\mathbf{Exfc}_n\left(\boldsymbol{x} \mid f, g, \boldsymbol{h}, \boldsymbol{\phi}, \boldsymbol{\theta}\right)$
Normale (avec matrice de précision)	$\mathbf{N}_k\left(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\Lambda}\right)$
Normale (avec matrice de covariance)	$\mathbf{N}_k\left(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}\right)$
Student	$\mathbf{St}_k\left(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha\right)$
Wishart	$\mathbf{Wi}_k\left(\boldsymbol{X} \mid \alpha, \boldsymbol{\Lambda}\right)$

Lois de probabibilté à $n+1$ variables réelles

Normal-Gamma	$\mathbf{NGam}_k\left(\boldsymbol{x}, y \mid \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta\right)$
Normale-Wishart	$\mathbf{NWi}_k\left(\boldsymbol{x}, \boldsymbol{Y} \mid \boldsymbol{\mu}, \lambda, \alpha, \boldsymbol{B}\right)$

Lien entre les différentes lois

$\mathbf{Bin}(x\|\theta,1)$ $\mathbf{NegBin}(x\|\theta,1)$ $\mathbf{BinBet}(x\|1,1,n)$	$\mathbf{Ber}(x\|\theta)$ $\mathbf{Geo}(x\|\theta)=\mathbf{Pas}(x\|\theta)$ $\mathbf{Unid}(x\|n)=\frac{1}{n+1},\ x=0,1,\cdots,n$
$\mathbf{Bet}(x\|1,1)$ $\mathbf{Gam}(x\|0,\beta)$ $\mathbf{Gam}(x\|\alpha,1)$ $\mathbf{Gam}(x\|\frac{\nu}{2},1/2)$ $\mathbf{IGam}(x\|\frac{\nu}{2},1/2)$ $\mathbf{St}(x\|\mu,\lambda,1)$	$\mathbf{Uni}(x\|0,1)$ $\mathbf{Ex}(x\|\beta)$ $\mathbf{Erl}(x\|\alpha)$ $\mathbf{Chi}^2(x\|\nu)$ $\mathbf{IChi}^2(x\|\nu)$ $\mathbf{Cau}(x\|\mu,\lambda)$
$\mathbf{Mu}_1(x\|\theta,n)$ $\mathbf{Di}_1(x\|\alpha_1,\alpha_2)$ $\mathbf{Wi}_1(x\|\alpha,\beta)$ $\mathbf{St}_1(x\|\mu,\lambda,\alpha)$ $\mathbf{N}_1(x\|\mu,\lambda)$	$\mathbf{Bin}(x\|\theta,n)$ $\mathbf{Bet}(x\|\alpha_1,\alpha_2)$ $\mathbf{Gam}(x\|\alpha,\beta)$ $\mathbf{St}(x\|\mu,\lambda,\alpha)$ $\mathbf{N}(x\|\mu,\lambda)$
$\mathbf{BinBet}(x\|\alpha,\beta,n)$ $\mathbf{NegBinBet}(x\|\alpha,\beta,r)$ $\mathbf{PnGam}(x\|\alpha,\beta,n)$	$\int_0^1 \mathbf{Bin}(x\|\theta,n)\,\mathbf{Beta}(\theta\|\alpha,\beta)\,\mathrm{d}\theta$ $\int_0^1 \mathbf{NegBin}(x\|\theta,r)\,\mathbf{Beta}(\theta\|\alpha,\beta)\,\mathrm{d}\theta$ $\int_0^\infty \mathbf{Pn}(x\|n\lambda)\,\mathbf{Gam}(\lambda\|\alpha,\beta)\,\mathrm{d}\lambda$
$\mathbf{Par}(x\|\alpha,\beta)$ $\mathbf{St}(x\|\mu,\lambda,\alpha)$ $\mathbf{Chi}^2_{nc}(x\|\mu,\lambda)$	$\int_0^\infty \mathbf{Ex}(x-\beta\|\lambda)\,\mathbf{Gam}(\lambda\|\alpha,\beta)\,\mathrm{d}\lambda$ $\int_0^\infty \mathbf{N}(x\|\mu,\lambda y)\,\mathbf{Gam}(y\|\alpha/2,\beta/2)\,\mathrm{d}y$ $\sum_{i=1}^\infty \mathbf{Pn}(i\|\lambda/2)\,\mathbf{Chi}^2(x\|\nu+2i)$
$\lim_{\alpha\mapsto\infty} \mathbf{St}(x\|\mu,\lambda,\alpha)$ $\mathbf{St}(x\|0,1,\alpha)$	$\mathbf{N}(x\|\mu,\lambda)$ loi de Student standard
si $x\sim\mathbf{Gam}(x\|\alpha,\beta)$ si $x_i\sim\mathbf{Gam}(x_i\|\alpha_i,\beta)$ si $x\sim\mathbf{N}(x\|0,1)$ et $y\sim=\mathbf{Chi}^2(x\|\nu)$ si $x\sim\mathbf{Chi}^2(x\|\nu_1)$ et $y\sim\mathbf{Chi}^2(y\|\nu_2)$	alors $y=\frac{1}{x}\sim\mathbf{IGam}(y\|\alpha,\beta)$ alors $y=\sum_{i=1}^n x_i\sim\mathbf{Gam}\,(y\|\sum_{i=1}^n\alpha_i,\beta)$ alors $z=\frac{x}{\sqrt{x/\nu}}\sim\mathbf{St}(z\|0,1,\nu)$ alors $z=\frac{x/\nu_1}{y/\nu_2}\sim\mathbf{FS}(z\|\nu_1,\nu_2)$

Famille des lois position-échelle $p(x\|\mu,\beta) = \frac{1}{\beta} f(t)$ avec $t = \frac{x-\mu}{\beta}$		Var $[x] = \beta^2 v$	H $[x] = \log\beta + h$
Famille	$f(t)$	v	h
Normale	$(2\pi)^{-\frac{1}{2}} \exp\left[-\frac{t^2}{2}\right]$	1	$\frac{1}{2}\log(2\pi e)$
Gumbel	$\exp[-t]\exp[-\exp[-t]]$	$\pi^2/6$	$1+\gamma$
Laplace	$\frac{1}{2}\exp[-\|t\|]$	2	$1+\log 2$
Logistic	$\frac{\exp[-t]}{(1+\exp[-t])^2}$	$\pi^2/3$	2
Exponentielle	$\exp[-t], \quad x > 0$	1	1
Uniforme	$1, \quad \mu - \frac{\beta}{2} < x < \mu + \frac{\beta}{2}$	1/12	0

Famille des lois forme-échelle $p(x\|\alpha,\beta) = \frac{1}{\beta} f(t;\alpha)$ avec $t = \frac{x}{\beta}$		Var $[x] = \beta^2 v(\alpha)$	H $[x] = \log\beta + h(\alpha)$
Famille	$f(t)$	v	h
Normale généralisée $\alpha = 2$: **Rayleigh**, $\alpha = 3$: **Maxwell-Boltzmann**, $\alpha = \nu$: **khi**	$\frac{2}{\Gamma(\frac{\alpha}{2})} t^{\alpha-1}\exp[-t^2]$	$\frac{\alpha - 2\Gamma^2(\frac{\alpha+1}{2})}{\Gamma^2(\frac{\alpha}{2})}$	$\log[\Gamma(\frac{\alpha}{2})/2] + \frac{1-\alpha}{2}\psi(\frac{\alpha}{2}) + \frac{\alpha}{2}$
Normale généralisée inverse $\alpha = \nu, \beta = \sqrt{2}$: **khi inverse**	$\frac{2}{\Gamma(\frac{\alpha}{2})} t^{-\alpha-1}\exp[-t^{-2}]$	$\frac{1}{\alpha-2} - \alpha\frac{\Gamma^2(\alpha-1)}{\Gamma^2(\frac{\alpha}{2})}$	$\log[\Gamma(\frac{\alpha}{2})/2] + \frac{-\alpha}{2}\psi(\frac{\alpha}{2}) + \frac{\alpha}{2}$
Gamma $\alpha = 1$: **Exponentielle**, $\alpha = \frac{\nu}{2}, \beta = 2$: **khi-deux**, $\alpha = \nu$: **Erlang**	$\frac{1}{\Gamma(\alpha)} t^{\alpha-1}\exp[-t]$	α	$\log[\Gamma(\alpha)] + (1-\alpha)\psi(\alpha) + \alpha$
Gamma inverse $\alpha = \frac{\nu}{2}, \beta = \frac{1}{2}$: **khi-deux inverse**	$\frac{1}{\Gamma(\alpha)} t^{-\alpha+1}\exp[-t^{-1}]$	$\frac{1}{(\alpha-1)^2(\alpha-2)}, \ \alpha > 2$	$\log[\Gamma(\alpha)] - (1+\alpha)\psi(\alpha) + \alpha$
Pareto	$\alpha t^{-\alpha-1}, \ x > \beta$	$\frac{1}{(\alpha-1)^2(\alpha-2)}, \ \alpha > 2$	$\frac{1}{\alpha} - \log\alpha + 1$
Weibull	$\alpha t^{\alpha-1}\exp[-t]^{\alpha}$	$\Gamma(1+\frac{2}{\alpha})\Gamma^2(1+\frac{1}{\alpha})$	$\frac{\gamma(\alpha-1)}{\alpha} - \log\alpha + 1$

10.2.2 Notations détaillées

Lois de probabibilté à une variable discrète

Bernoulli	$\mathbf{Ber}\,(x\|\theta) = \theta^x\,(1-\theta)^{1-x}$, $0 < \theta < 1$, $x = \{0, 1\}$
Binomiale	$\mathbf{Bin}\,(x\|\theta, n) = c\,\theta^x\,(1-\theta)^{n-x}$, $c = \binom{n}{x}$, $0 < \theta < 1,\ n = 1, 2, \cdots$, $x = 0, 1, \cdots, n$
Hypergéométrique	$\mathbf{HypGeo}\,(x\|N, M, n) = c\binom{N}{x}\binom{M}{n-x}$, $c = \binom{N+M}{n}^{-1}$, $N, M = 1, 2, \cdots,\ n = 1, \cdots, N+M$, $x = a, a+1, \cdots, b,$ avec $a = \max\{0, n-M\},\ b = \min\{N, n\}$
Binomiale-négative	$\mathbf{NegBin}\,(x\|\theta, r) = c\binom{r+x-1}{r-1}(1-\theta)^x$, $c = \theta^r$, $0 < \theta < 1,\ r = 1, 2, \cdots$, $x = 0, 1, 2, \cdots$
Poisson	$\mathbf{Pn}\,(x\|\lambda) = \frac{\lambda^x}{x!}$, $c = \exp[-\lambda]$, $\lambda > 0$, $x = 0, 1, 2, \cdots$
Binomiale-Bêta	$\mathbf{BinBet}\,(x\|\alpha, \beta, n) = c\binom{n}{x}\Gamma(\alpha+x)\Gamma(\beta+n-x)$, $c = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+n)}$, $\alpha, \beta > 0,\ n = 1, 2, \cdots$, $x = 0, \cdots, n$

Lois de probabibilté à une variable discrète (suite)

Binomiale-Bêta négative	$\mathbf{NegBinBet}\,(x\|\alpha,\beta,r) = c\begin{pmatrix} r+x-1 \\ r-1\end{pmatrix}\dfrac{\Gamma(\beta+x)}{\Gamma(\alpha+\beta+x+\alpha)}$, $c = \dfrac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\beta)}$, $\alpha,\beta > 0,\ r = 1,2,\cdots,$ $x = 0,1,2,\cdots$
Poisson-Gamma	$\mathbf{PnGam}\,(x\|\alpha,\beta,n) = c\,\dfrac{\Gamma(\alpha+x)}{x\,!}\dfrac{n^x}{(\beta+n)^{\alpha+x}}$, $c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}$, $\alpha,\beta > 0,\ n = 0,1,2,\cdots,$ $x = 0,1,2,\cdots$
Poisson composée	$\mathbf{Pnc}\,(x\|\lambda,\mu) = \exp\left[-\lambda\right]\sum_{n=0}^{\infty}\dfrac{(n\mu)^x\exp\left[-n\mu\right]}{x!}\dfrac{\lambda^N}{n!}$, $\lambda,\mu > 0, \quad x = 0,1,2\cdots$
Géométrique	$\mathbf{Geo}\,(x\|\theta) = c\,(1-\theta)^{x-1}$, $c = \theta$, $\theta > 0, \quad x = 0,1,2\cdots$
Pascale	$\mathbf{Pas}\,(x\|m,\theta) = C_{x-1}^{m-1}\theta^m(1-\theta)^{x-m}$, $m > 0, 0 < \theta < 1 \quad x = 0,1,2\cdots$

Lois de probabibilté à une variable réelle

Bêta	$\mathbf{Bet}\,(x\vert\alpha,\beta) = c\,x^{\alpha-1}(1-x)^{\beta-1}$, $c = \dfrac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}$, $\alpha,\beta > 0$, $0 < x < 1$
Gamma	$\mathbf{Gam}\,(x\vert\alpha,\beta) = c\,x^{\alpha-1}\exp\left[-\beta x\right]$, $c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}$, $\alpha,\beta > 0$, $x > 0$
Gamma inverse	$\mathbf{IGam}\,(x\vert\alpha,\beta) = c\,x^{-(\alpha+1)}\exp\left[-\dfrac{\beta}{x}\right]$, $c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}$, $\alpha,\beta > 0$, $x > 0$
Gamma–Gamma	$\mathbf{GamGam}\,(x\vert\alpha,\beta,n) = c\,\dfrac{x^{n-1}}{(\beta+x)^{\alpha+n}}$, $c = \dfrac{\beta^\alpha}{\Gamma(\alpha)}\dfrac{\Gamma(\alpha+n)}{\Gamma(n)}$, $\alpha,\beta > 0,\ n = 0,1,2,\cdots$, $x = 0,1,2,\cdots$
Pareto	$\mathbf{Par}\,(x\vert\alpha,\beta) = c\,x^{-(\alpha+1)}$, $c = \alpha\beta^\alpha$, $\alpha,\beta > 0$, $x \geq \beta$
Normale	$\mathbf{N}\,(x\vert\mu,\lambda) = c\,\exp\left[-\dfrac{1}{2}\lambda(x-\mu)^2\right]$, $c = \sqrt{\dfrac{\lambda}{2\pi}}$, $\mu \in \mathbb{R},\ \lambda > 0$, $x \in \mathbb{R}$
Normale	$\mathbf{N}\,(x\vert\mu,\sigma) = c\,\exp\left[-\dfrac{1}{2\sigma^2}(x-\mu)^2\right]$, $c = \dfrac{1}{\sqrt{2\pi\sigma^2}}$, $\mu \in \mathbb{R},\ \sigma > 0$, $x \in \mathbb{R}$
du khi-deux	$\mathbf{Chi}^2\,(x\vert n) = c\,x^{n/2-1}\exp\left[-\dfrac{x}{2}\right]$, $c = \dfrac{\frac{1}{2}^{n/2}}{\Gamma(n/2)}$, $n > 0,\quad x > 0$
du khi-deux décentrée	$\mathbf{Chi}^2\,(x\vert\nu,\lambda) = \sum_{i=0}^{\infty}\mathbf{Pn}\left(x\vert\dfrac{\lambda}{2}\right)\,\chi^2\,(x\vert\nu+2i)$, $\nu,\lambda > 0$, $x > 0$

Lois de probababilté à une variable réelle (suite)

Logistic	$\mathbf{Lo}\,(x\mid\alpha,\beta) = c\,\dfrac{\exp\left[-\beta^{-1}(x-\alpha)\right]}{\left(1+\exp\left[\beta^{-1}(x-\alpha)\right]\right)^2}$, $c = \beta^{-1}$, $\alpha \in \mathbb{R},\ \beta > 0$, $x \in \mathbb{R}$
Student (t)	$\mathbf{St}\,(x\mid\mu,\lambda,\alpha) = c\left[1+\dfrac{\lambda}{\alpha}(x-\mu)^2\right]^{-(\alpha+1)/2}$, $c = \dfrac{\Gamma(\frac{\alpha+1}{2})}{\Gamma(\alpha/2)\Gamma(1/2)}\left(\dfrac{\lambda}{\alpha}\right)^{1/2}$ $\mu \in \mathbb{R},\ \lambda,\alpha > 0$, $x \in \mathbb{R}$
Fisher-Snedecor	$\mathbf{FS}\,(x\mid\alpha,\beta) = c\,\dfrac{x^{\alpha/2-1}}{(\beta+\alpha x)^{(\alpha+\beta)/2}}$, $c = \dfrac{\Gamma((\alpha+\beta)/2)}{\Gamma(\alpha/2)\Gamma(\beta/2)}\alpha^{\alpha/2}\beta^{\beta/2}$, $\alpha,\beta > 0$, $x > 0$
Uniforme	$\mathbf{Uni}\,(x\mid\theta_1,\theta_2) = c$ $c = \dfrac{1}{\theta_2-\theta_1}$, $\theta_2 > \theta_1$, $\theta_1 < x < \theta_2$
Exponentielle	$\mathbf{Ex}\,(x\mid\lambda) = c\,\exp\left[-\lambda x\right]$, $c = \lambda$, $\lambda > 0, \quad x > 0$
du khi inverse	$\mathbf{IChi}\,(x\mid\nu) = c\,x^{-(\nu/2+1)}\exp\left[-\dfrac{1}{2x^2}\right]$, $c = \dfrac{\frac{1}{2}^{\nu/2}}{\Gamma(\nu/2}$, $\nu > 0, \quad x > 0$
Racine carrée Gamma inverse	$\mathbf{IGam}^{-1/2}\,(x\mid\alpha,\beta) = c\,x^{-(2\alpha+1)}\exp\left[-\dfrac{\beta}{x^2}\right]$, $c = \dfrac{2\beta^\alpha}{\Gamma(\alpha)}$, $\alpha,\beta > 0$, $x > 0$
Pareto inverse	$\mathbf{IPar}\,(x\mid\alpha,\beta) = c\,x^{\alpha-1}$, $c = \alpha\beta^\alpha$, $\alpha,\beta > 0$, $0 < x < \beta^{-1}$

Lois de probabibilté à une variable réelle (suite)

Cauchy	$\mathbf{Cau}\,(x\|\lambda) = \dfrac{1/(\pi\lambda)}{1+(x/\lambda)^2}$, $\lambda \in \mathbb{R}, \quad x \in \mathbb{R}$
Rayleight	$\mathbf{Ray}\,(x\|\theta) = c\,x\,\exp\left[-\dfrac{x^2}{\theta}\right]$, $c = \dfrac{2}{\theta}$, $\theta > 0, \quad x > 0$
Log-Normale	$\mathbf{LogN}\,(x\|\mu, \Lambda) = c\,\exp\left[-\dfrac{(\ln x - \mu)^2}{2\Lambda^2}\right]$, $c = \dfrac{1}{\Lambda\sqrt{2\pi x}}$,
Normale généralisée	$\mathbf{Ngen}\,(x\|\alpha, \beta) = c\,x^{\alpha-1}\exp\left[-\beta x^2\right]$, $c = \dfrac{2\beta^{\alpha/2}}{\Gamma(\alpha/2)}$,
Weibull	$\mathbf{Wei}\,(x\|\alpha) = c\,x^{\alpha-1}\exp\left[-x^\beta/\alpha\right]$, $c = \dfrac{\beta}{\alpha}$, $\alpha, \beta > 0, \quad x > 0$
Exponentielle double	$\mathbf{Exd}\,(x\|\lambda) = c\,\exp\left[-\|\lambda\|x\right]$, $c = \dfrac{\lambda}{2}$, $\lambda > 0, \quad x \in \mathbb{R}$
Exponentielle tronquée	$\mathbf{Ext}\,(x\|\lambda) = \exp\left[-(x-\lambda)\right]$, $\lambda > 0, \quad x > \lambda$
du khi	$\mathbf{Chi}\,(x\|n) = c\,x^{n/2-1}\exp\left[-\dfrac{x}{2}\right]$, $c = \dfrac{\frac{1}{2}^{n/2}}{\Gamma(n/2)}$, $n > 0, \quad x > 0$

Lois de probabibilté à une variable réelle (suite)

Exponentielle généralisé à un paramètres	$\mathbf{Exf}\,(x\vert f,g,h,\phi,\theta) = f(x)\,g(\theta)\exp\left[\phi(\theta)h(x)\right],$
Exponentielle généralisé à K paramètres	$\mathbf{Exfk}\,(x\vert f,g,\boldsymbol{h},\boldsymbol{\phi},\boldsymbol{\theta}) = f(x)\,g(\boldsymbol{\theta})\exp\left[\sum_{k=1}^{K}\boldsymbol{\phi}_k(\boldsymbol{\theta})\boldsymbol{h}_k(x)\right],$

Lois de probabibilté à deux variables réelles

Normale-Gamma	$\mathbf{NGam}\,(x,y\vert\mu,\lambda,\alpha,\beta) = \mathbf{N}(x\vert\mu,\lambda y)\,\mathbf{Gam}(y\vert\alpha,\beta),$ $\mu\in\mathbb{R},\lambda,\alpha,\beta>0,$ $x\in\mathbb{R},y>0$
Pareto bivariable	$\mathbf{Par}_2\,(x,y\vert\alpha,\beta_0,\beta_1) = (y-x)^{(\alpha+2)},$ $c=\alpha(\alpha+1)(\beta_1-\beta_0)^{\alpha},$ $(\beta_0,\beta_1)\in\mathbb{R}^2,\ \beta_0<\beta_1,\alpha>0,$ $(x,y)\in\mathbb{R}^2,\ x<\beta_0,y>\beta_1$

Lois de probabibilté à n variables discrètes

Multinomiale	$\mathbf{Mu}_k\left(\boldsymbol{x}\vert\boldsymbol{\theta},n\right) = \frac{n!}{\prod_{l=1}^{k+1} x_l!}\prod_{l=1}^{k+1}\theta_l^{x_l}$, $x_{k+1} = n - \sum_{l=1}^{k} x_l, \quad \theta_{k+1} = 1 - \sum_{l=1}^{k}\theta_l$, $0 < \theta_l < 1, \quad \sum \theta_l < 1, \quad n = 1, 2, \cdots$, $x_l = 0, 1, 2, \cdots, \quad \sum x_l \leq n$
Dirichlet	$\mathbf{Di}_k\left(\boldsymbol{x}\vert\boldsymbol{\alpha}\right) = c\prod_{l=1}^{k+1} x_l^{\alpha_l - 1}$, $c = \frac{\Gamma\left(\sum_{l=1}^{k+1}\alpha_l\right)}{\prod_{l=1}^{k+1}\Gamma(\alpha_l)}$, $\alpha_l > 0, l = 1, \cdots k+1$ $0 < x_l < 1, l = 1, \cdots k+1 \quad x_{l+1} = 1 - \sum_{l=1}^{k} x_l$
Multinomiale–Dirichlet	$\mathbf{MuDi}_k\left(\boldsymbol{x}\vert\boldsymbol{\alpha},n\right) = c\prod_{l=1}^{k+1}\frac{\alpha_l^{[x_l]}}{x_l!}$, $c = \frac{n!}{\sum_{l=1}^{k+1}\alpha_l^{[n]}}$, $\alpha^{[s]} = \prod_{l=1}^{s}(\alpha + l - 1), \quad x_{k+1} = n - \sum_{l=1}^{k} x_l$, $\alpha_l > 0, n = 1, 2, \cdots$, $x_l = 0, 1, 2, \cdots, \quad \sum_{l=1}^{k} x_l < n$

Lois de probabibilté à n variables réelles

Exponentielle	$\mathbf{Exfc}\,(x \mid a, b, \boldsymbol{\theta})$
Exponentielle généralisée	$\mathbf{Exfc}\,(\boldsymbol{x} \mid f, g, \boldsymbol{h}, \boldsymbol{\phi}, \boldsymbol{\theta})$
Normale	$\mathbf{N}_k\,(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\Lambda}) = c\,\exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^t\boldsymbol{\Lambda}(\boldsymbol{x}-\boldsymbol{\mu})\right]$, $c = \lvert\boldsymbol{\Lambda}\rvert^{1/2}\,(2\pi)^{-\frac{k}{2}}$, $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0$, $\boldsymbol{x} \in \mathbb{R}^k$
Normale	$\mathbf{N}_k\,(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\Sigma}) = c\,\exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^t\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right]$, $c = \lvert\boldsymbol{\Sigma}\rvert^{-1/2}\,(2\pi)^{-\frac{k}{2}}$, $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Sigma} > 0$, $\boldsymbol{x} \in \mathbb{R}^k$
Student	$\mathbf{St}_k\,(\boldsymbol{x} \mid \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha) = c\,\left[1 + \frac{1}{\alpha}(\boldsymbol{x}-\boldsymbol{\mu})^t\boldsymbol{\Lambda}(\boldsymbol{x}-\boldsymbol{\mu})\right]^{-(\alpha+k)/2}$, $c = \frac{\Gamma\left((\alpha+k)/2\right)}{\Gamma(\alpha/2)(\alpha\pi)^{k/2}}\left(\frac{\Lambda}{\alpha}\right)^{1/2}$, $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha > 0$, $\boldsymbol{x} \in \mathbb{R}^k$
Wishart	$\mathbf{Wi}_k\,(\boldsymbol{X} \mid \alpha, \boldsymbol{\Lambda}) = c\,\lvert\boldsymbol{X}\rvert^{\alpha-(k+1)/2}\exp\left[-\mathrm{tr}(\boldsymbol{\Lambda}\boldsymbol{X})\right]$, $c = \frac{\lvert\boldsymbol{\Lambda}\rvert^{\alpha}}{\Gamma_k(\alpha)}$, $\boldsymbol{\Lambda}$ une matrice de dimensions $k \times k$, $\boldsymbol{X}$ une matrice symétrique d.p. de dimensions $k \times k$, $X_{i,j} = X_{j,i}, \quad i, j = 1, \cdots, k$, $2\alpha > k - 1$

Lois de probabibilté à $n+1$ variables réelles

Normal-Gamma	$\mathbf{NGam}_k\,(\boldsymbol{x}, y \mid \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta) = \mathbf{N}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, y\boldsymbol{\Lambda})\,\mathbf{Gam}(y \mid \alpha, \beta)$, $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha, \beta > 0$, $\boldsymbol{x} \in \mathbb{R}^k, y > 0$
Normale-Wishart	$\mathbf{NWi}_k\,(\boldsymbol{x}, \boldsymbol{Y} \mid \boldsymbol{\mu}, \lambda, \alpha, \boldsymbol{B}) = \mathbf{N}_k(\boldsymbol{x} \mid \boldsymbol{\mu}, \lambda\boldsymbol{Y})\,\mathbf{Wi}_k(\boldsymbol{Y} \mid \alpha, \boldsymbol{B})$, $\boldsymbol{\mu} \in \mathbb{R}^k, \lambda > 0, 2\alpha > k - 1, \boldsymbol{B} > 0$, $\boldsymbol{x} \in \mathbb{R}^k, Y_{i,j} = Y_{j,i}, i, j = 1, \cdots, k$

10.2.3 Définitions

Bernoulli

$\mathbf{Ber}\,(x|\theta) = \theta^x\,(1-\theta)^{1-x}$,
$0 < \theta < 1$,
$x = \{0, 1\}$,

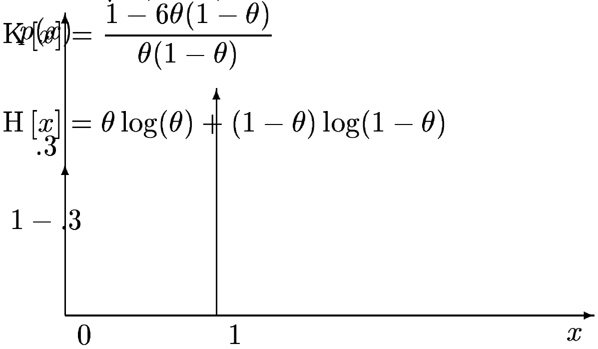
$$\begin{cases} \Pr\{X = 1\} = \theta, \\ \Pr\{X = 0\} = 1 - \theta \end{cases}$$

$\mathrm{E}\,[x] = \theta$,
$\mathrm{Var}\,[x] = \theta(1-\theta)$,
$\mathrm{M}\,[x] = \theta$

$$\mathrm{S}\,[x] = \frac{1-2\theta}{\sqrt{\theta(1-\theta)}}$$

$$\mathrm{K}\,[x] = \frac{1-6\theta(1-\theta)}{\theta(1-\theta)}$$

$$\mathrm{H}\,[x] = \theta\log(\theta) + (1-\theta)\log(1-\theta)$$



Binomiale

$\mathbf{Bin}\,(x|\theta, n) = \binom{n}{x} \theta^x \, (1-\theta)^{n-x}$,
$0 < \theta < 1$, $n = 1, 2, \cdots$,
$x = 0, 1, \cdots, n$,

$$\begin{cases} \Pr\{X = 0\} = (1-\theta)^n, \\ \Pr\{X = x\} = \binom{n}{x} \theta^x \, (1-\theta)^{n-x} \\ \Pr\{X = n\} = \theta^n \end{cases}$$

$\mathrm{E}\,[x] = n\theta$,
$\mathrm{Var}\,[x] = n\theta(1-\theta)$,
$\mathrm{M}\,[x] = \mathrm{Floor}\,[(n+1)\theta]$,

$$\mathrm{S}\,[x] = \frac{1-2\theta}{\sqrt{n\theta(1-\theta)}}$$
$$\mathrm{K}\,[x] = \frac{1-6\theta(1-\theta)}{n\theta(1-\theta)}$$

Cumulants : $K_1 = n\theta, \quad K_{r+1} = \theta(\theta-1)\dfrac{\partial K_r}{\partial \theta}, r \geq 1$

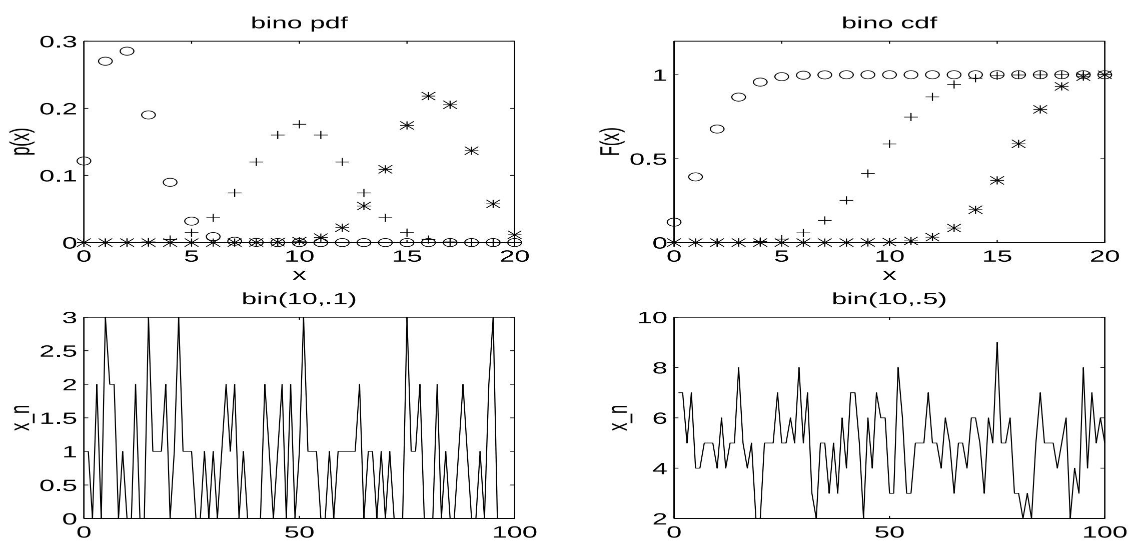
Fonction caractéristique : $F(t) = (1 - \theta + \theta e^{-it})^n$

$\mathrm{H}\,[x] = ?$

Autres notations : $p = \theta, q = 1 - \theta$

$n = 1$: **Bernouilli**, $\mathbf{Bin}\,(x|\theta, 1) = \mathbf{Ber}\,(x|\theta)$,

si $\{x_1, \cdots, x_k\} \sim \mathbf{Bin}\,(x|\theta, n_i)\,, i = 1, \cdots k$,
alors $s = \sum x_i \sim \mathbf{Bin}\left(s|\theta, \sum n_i\right)$



Hypergéométrique

$$\mathbf{HypGeo}\,(x|N,M,n) = \binom{N+M}{n}^{-1}\binom{N}{x}\binom{M}{n-x},$$

$N, M = 1, 2, \cdots,\ n = 1, \cdots, N+M,$
$x = a, a+1, \cdots, b,$ avec $a = \max\{0, n-M\}, b = \min\{N, n\},$

$$\begin{cases} \Pr\{X=a\} = ?, \\ \Pr\{X=x\} = ? \\ \Pr\{X=b\} = ? \end{cases}$$

$$\mathrm{E}[x] = \frac{Nn}{N+M},$$
$$\mathrm{Var}[x] = \frac{nNM}{(N+M)^2}\frac{N+M-n}{N+M-1},$$
$$\mathrm{M}[x] = \mathrm{Floor}\left[\frac{(n+1)(N+1)}{N+M+2}\right]$$

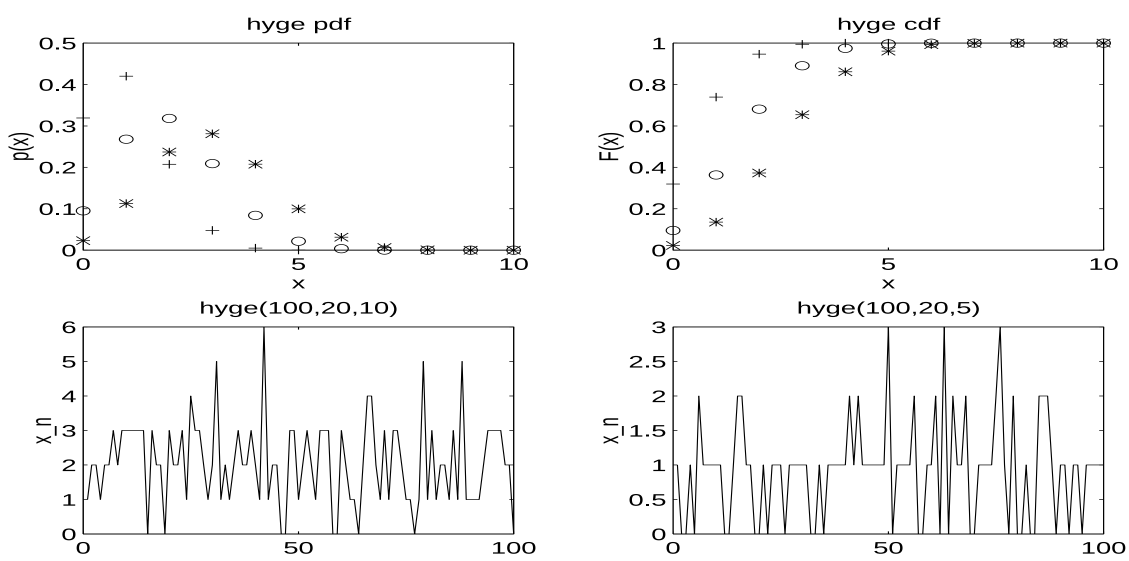
$\mathrm{S}[x] = ?$
$\mathrm{K}[x] = ?$

$\mathrm{H}[x] = ?$

Autres notations : $p = \dfrac{N}{N+M}, q = 1-p = \dfrac{M}{N+M},$

$$\mathrm{E}[x] = np,$$
$$\mathrm{Var}[x] = npq\frac{N+M-n}{N+M-1},$$

$$\mathrm{S}[x] = \frac{q-p}{\sqrt{npq}}\left(\frac{N+M-1}{N+M-n}\right)^{1/2}\frac{N+M-2n}{N+M-2}$$



Binomiale-négative

$\mathbf{NegBin}\,(x|\theta,r)=\theta^r\binom{r+x-1}{r}(1-\theta)^x,$
$0<\theta<1,\; r=1,2,\cdots,$
$x=0,1,2,\cdots,$

$$\begin{cases}\Pr\{X=0\}=\theta^r,\\ \Pr\{X=1\}=r\,\theta^r\\ \Pr\{X=n\}=\theta^r\binom{r+n-1}{r-1}(1-\theta)^n\end{cases}$$

$\mathrm{E}\,[x]=r\dfrac{1-\theta}{\theta},$
$\mathrm{Var}\,[x]=r\dfrac{1-\theta}{\theta^2},$
$$\mathrm{M}\,[x]=\begin{cases}0,1 & \text{si}\quad r(1-\theta)=0\\ 0 & \text{si}\quad r(1-\theta)<0\\ \frac{r(1-\theta)}{\theta} & \text{si}\quad r(1-\theta)>1\end{cases}$$

$\mathrm{S}\,[x]=?$
$\mathrm{K}\,[x]=?$

$\mathrm{H}\,[x]=?$

Autres notations : $p=\theta, Q=\dfrac{1}{p}, P=\dfrac{1-p}{p}$

$\mathrm{E}\,[x]=rP,$
$\mathrm{Var}\,[x]=rPQ$

$\mathrm{S}\,[x]=\dfrac{Q+P}{\sqrt{rPQ}}$
$\mathrm{K}\,[x]=\dfrac{1+6PQ}{rPQ}$
Cumulants : $K_1=rP,\quad K_{n+1}=PQ\dfrac{\partial K_n}{\partial Q}, n\geq 1$
Fonction caractéristique : $F(t)=(Q-Pe^{it})^-r$

$r=1:$ **Géométrique ou Pascal**
$\mathbf{NegBin}\,(x|\theta,1)=\mathbf{Geo}\,(x|\theta),$

si $\{x_1,\cdots,x_k\}\sim\mathbf{NegBin}\,(x|\theta,r_i)\,, i=1,\cdots k,$
alors $s=\sum x_i\sim\mathbf{NegBin}\left(s|\theta,\sum r_i\right)$

Poisson

$\mathbf{Pn}\,(x|\lambda) = \exp\left[-\lambda\right] \dfrac{\lambda^x}{x!},$
$\lambda > 0,$
$x = 0, 1, 2, \cdots,$

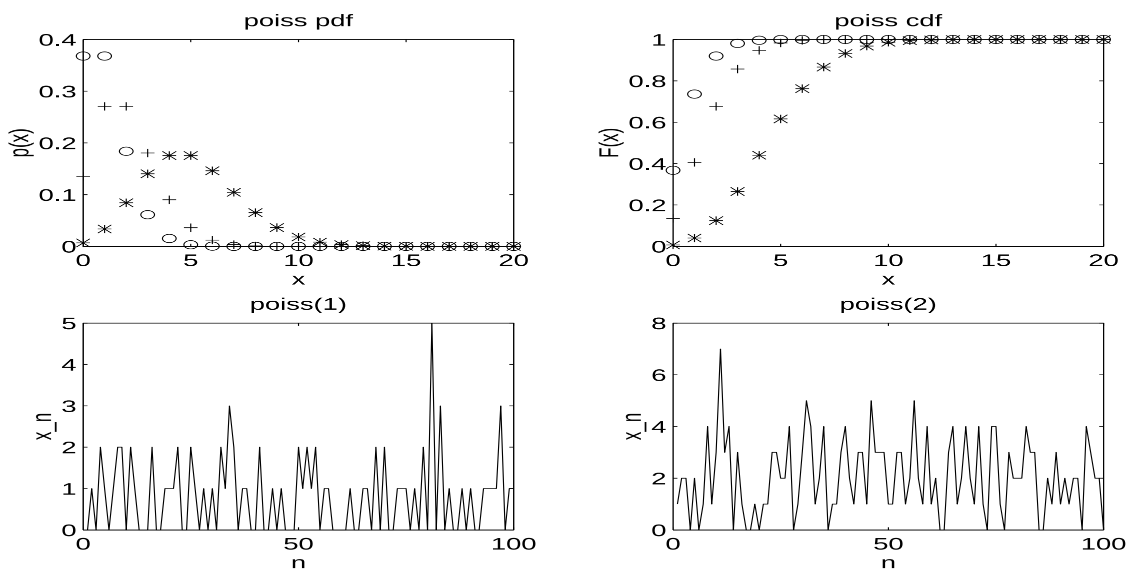
$$\begin{cases} \Pr\{X = 0\} = \exp\left[-\lambda\right], \\ \Pr\{X = 1\} = \lambda \exp\left[-\lambda\right] \\ \Pr\{X = n\} = \exp\left[-\lambda\right] \frac{\lambda^n}{n!} \end{cases}$$

$\mathrm{E}\,[x] = \lambda,$
$\mathrm{Var}\,[x] = \lambda,$
$\mathrm{M}\,[x] = \lambda$
$\mathrm{S}\,[x] = \lambda^{-1/2}$
$\mathrm{K}\,[x] = \lambda^{-1}$

Fonction caractéristique : $F(t) = e^{\lambda}(e^{it} - 1)$

$\mathrm{H}\,[x] = ?$

si $\{x_1, \cdots, x_k\} \sim \mathbf{Pn}\,(x|\lambda_i)\,, i = 1, \cdots k,$
alors $s = \sum x_i \sim \mathbf{Pn}\left(s|\sum \lambda_i\right)$



Binomiale-Bêta

$$\mathbf{BinBet}\,(x|\alpha,\beta,n) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+n)}\binom{n}{x}\Gamma(\alpha+x)\Gamma(\beta+n-x),$$

$\alpha,\beta > 0,\ n = 1,2,\cdots,$
$x = 0,\cdots,n,$

$$\begin{cases} \Pr\{X=0\} = \frac{\Gamma(\alpha+\beta)\Gamma(\beta+n)}{\Gamma(\beta)\Gamma(\alpha+\beta+n)}, \\ \Pr\{X=1\} = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+n)} n\Gamma(\alpha+1)\Gamma(\beta+n-1) \\ \Pr\{X=n\} = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+n)}{\Gamma(\alpha)\Gamma(\alpha+\beta+n)} \end{cases}$$

$\mathrm{E}\,[x] = n\dfrac{\alpha}{\alpha+\beta},$
$\mathrm{Var}\,[x] = \dfrac{n\alpha\beta}{(\alpha+\beta)^2}\dfrac{\alpha+\beta+n}{\alpha+\beta+1},$
$\mathrm{M}\,[x] = \dfrac{(n+1)(\alpha+1)}{\alpha+\beta-2}$
$\mathrm{S}\,[x] = ?$
$\mathrm{K}\,[x] = ?$

$\mathrm{H}\,[x] = ?$

$$\mathbf{BinBet}\,(x|\alpha,\beta,n) = \int_0^1 \mathbf{Bin}\,(x|\theta,n)\,\mathbf{Bet}\,(\theta|\alpha,\beta)\;\mathrm{d}\theta$$

$\alpha = \beta = 1:$ loi uniforme discrète

Binomiale-Bêta négative

$$\mathbf{NegBinBet}\,(x|\alpha,\beta,r) = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\beta)}\binom{r+x-1}{r-1}\frac{\Gamma(\beta+x)}{\Gamma(\alpha+\beta+x+1)},$$

$\alpha, \beta > 0,\ r = 1, 2, \cdots,$
$x = 0, 1, 2, \cdots,$

$$\begin{cases} \Pr\{X=0\} = ?, \\ \Pr\{X=1\} = ? \\ \Pr\{X=n\} = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\beta)}\binom{r+n-1}{r-1}\frac{\Gamma(\beta+n)}{\Gamma(\alpha+\beta+n+\alpha)} \end{cases}$$

$\mathrm{E}[x] = \frac{r\beta}{\alpha-1}, \alpha > 1,$
$\mathrm{Var}[x] = \frac{r\beta}{\alpha-1}\left[\frac{\alpha+\beta+r-1}{\alpha-2} + \frac{r\beta}{(\alpha-1)(\alpha-2)}\right], \alpha > 2,$
$\mathrm{M}[x] = ?$

$\mathrm{S}[x] = ?$
$\mathrm{K}[x] = ?$

$\mathrm{H}[x] = ?$

$$\mathbf{NegBinBet}\,(x|\alpha,\beta,r) = \int_0^1 \mathbf{NegBin}\,(x|\theta,r)\,\mathbf{Bet}\,(\theta|\alpha,\beta)\ \mathrm{d}\theta$$

Poisson-Gamma

$$\mathbf{PnGam}\left(x|\alpha,\beta,n\right)=\frac{\beta^\alpha}{\Gamma(\alpha)}\frac{\Gamma(\alpha+x)}{x\,!}\frac{n^x}{(\beta+n)^{\alpha+x}},$$

$\alpha,\beta>0,\ n=0,1,2,\cdots,$
$x=0,1,2,\cdots,$

$$\begin{cases}\Pr\left\{X=0\right\}=\frac{\beta^\alpha}{(\beta+n)^\alpha},\\ \Pr\left\{X=1\right\}=\frac{\beta^\alpha\Gamma(x+1)}{\Gamma(x)}\frac{n}{(\beta+n)^\alpha},\\ \Pr\left\{X=n\right\}=\frac{\beta^\alpha}{\Gamma(\alpha)}\frac{\Gamma(\alpha+n)}{n\,!}\frac{n^n}{(\beta+n)^{\alpha+n}}\end{cases}$$

$$\mathrm{E}\left[x\right]=\frac{n\alpha}{\beta},$$

$$\mathrm{Var}\left[x\right]=\frac{n\alpha(\beta+n)}{\beta^2},$$

$$\mathrm{M}\left[x\right]=\begin{cases}0,1 & \text{si}\quad \alpha n=\beta+n\\ 0 & \text{si}\quad \alpha n<\beta+n\\ \frac{n(\alpha-1)}{\beta} & \text{si}\quad \alpha n>\beta+n\end{cases}$$

$\mathrm{S}\left[x\right]=?$
$\mathrm{K}\left[x\right]=?$

$\mathrm{H}\left[x\right]=?$

$$\mathbf{PnGam}\left(x|\alpha,\beta,n\right)=\int_0^\infty \mathbf{Pn}\left(x|n\lambda\right)\mathbf{Gam}\left(\lambda|\alpha,\beta\right)\,\mathrm{d}\lambda$$

$$\alpha=\frac{\nu}{2},\beta=\frac{1}{2}:\quad \mathbf{IGam}\left(x|\frac{\nu}{2},\frac{1}{2}\right)=\mathbf{Chi}^{-2}\left(x|\nu\right),$$

Poisson composée

$$\mathbf{Pnc}\left(x|\lambda,\mu\right)=\exp\left[-\lambda\right]\sum_{n=0}^{\infty}\frac{(n\mu)^{x}\exp\left[-n\mu\right]}{x!}\frac{\lambda^{N}}{n!},$$

$\lambda,\mu>0,\quad x=0,1,2\cdots,$

$$\begin{cases}\Pr\left\{X=0\right\}=?,\\ \Pr\left\{X=1\right\}=?\\ \Pr\left\{X=n\right\}=\exp\left[-\lambda\right]\sum_{n=0}^{\infty}\frac{(n\mu)^{n}\exp[-n\mu]}{n!}\frac{\lambda^{N}}{n!},\end{cases}$$

$\mathrm{E}\left[x\right]=,$
$\mathrm{Var}\left[x\right]=,$
$\mathrm{M}\left[x\right]=$
$\mathrm{S}\left[x\right]=?$
$\mathrm{K}\left[x\right]=?$

$\mathrm{H}\left[x\right]=?$

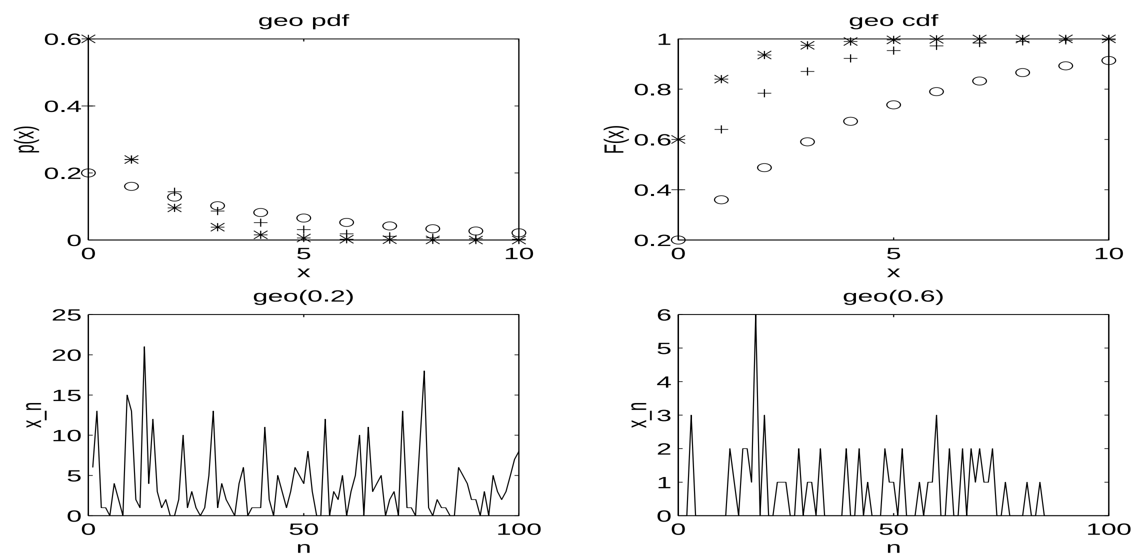
Géométrique

$\mathbf{Geo}(x|\theta) = \theta(1-\theta)^x,$
$\theta > 0, \quad x = 0, 1, 2\cdots,$

$$\begin{cases} \Pr\{X=0\} = \theta, \\ \Pr\{X=1\} = \theta(1-\theta) \\ \Pr\{X=n\} = \theta(1-\theta)^n \end{cases}$$

$\mathrm{E}[x] = \dfrac{1}{\theta},$
$\mathrm{Var}[x] = \dfrac{1-\theta}{\theta^2},$
$\mathrm{M}[x] =$
$\mathrm{S}[x] = \dfrac{2-\theta}{\sqrt{1-\theta}}$
$\mathrm{K}[x] = \dfrac{\theta^2-6\theta+6}{1-\theta}$

$\mathrm{H}[x] = \ln\theta + \dfrac{1-\theta}{\theta}\ln(1-\theta)$



Pascale

$\mathbf{Pas}\,(x|m,\theta) = \binom{m-1}{x-1}\,\theta^m(1-\theta)^{x-m},$
$m > 0, 0 < \theta < 1 \quad x \geq m,$

$$\begin{cases} \Pr\{X=1\} = (m-1)\theta^m(1-\theta)^m \\ \Pr\{X=n\} = \binom{m-1}{n-1}\,\theta^m(1-\theta)^{n-m} \end{cases} \qquad \begin{cases} \binom{n}{p} = \frac{n!}{p!(n-p)!} \\ \binom{n}{0} = \binom{n}{n} = 1 \\ \binom{n}{1} = \binom{n}{n-1} = n \end{cases}$$

$\mathrm{E}\,[x] = \dfrac{m}{\theta},$
$\mathrm{Var}\,[x] = \dfrac{m(1-\theta)}{\theta^2},$
$\mathrm{M}\,[x] = ?$
$\mathrm{S}\,[x] = ?$
$\mathrm{K}\,[x] = ?$
$\mathrm{S}\,[x] = \dfrac{2-\theta}{\sqrt{m(1-\theta)}}$
$\mathrm{K}\,[x] = \dfrac{\theta^2 - 6\theta + 6}{m(1-\theta)}$

$\mathrm{H}\,[x] = ?$

Lois de probabibilté à une variable réelle

Bêta

$$\mathbf{Bet}\,(x|\alpha,\beta) = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)}\, x^{\alpha-1}(1-x)^{\beta-1} = \frac{1}{B(\alpha,\beta)}\, x^{\alpha-1}(1-x)^{\beta-1},$$

$\alpha,\beta > 0,$
$0 < x < 1,$

$$B(\alpha,\beta) = \int_0^\infty x^{\alpha-1}(1-x)^{\beta-1}$$

$$F(x|\alpha,\beta) = \frac{1}{B(\alpha,\beta)}\int_0^x x^{\alpha-1}(1-x)^{\beta-1}$$

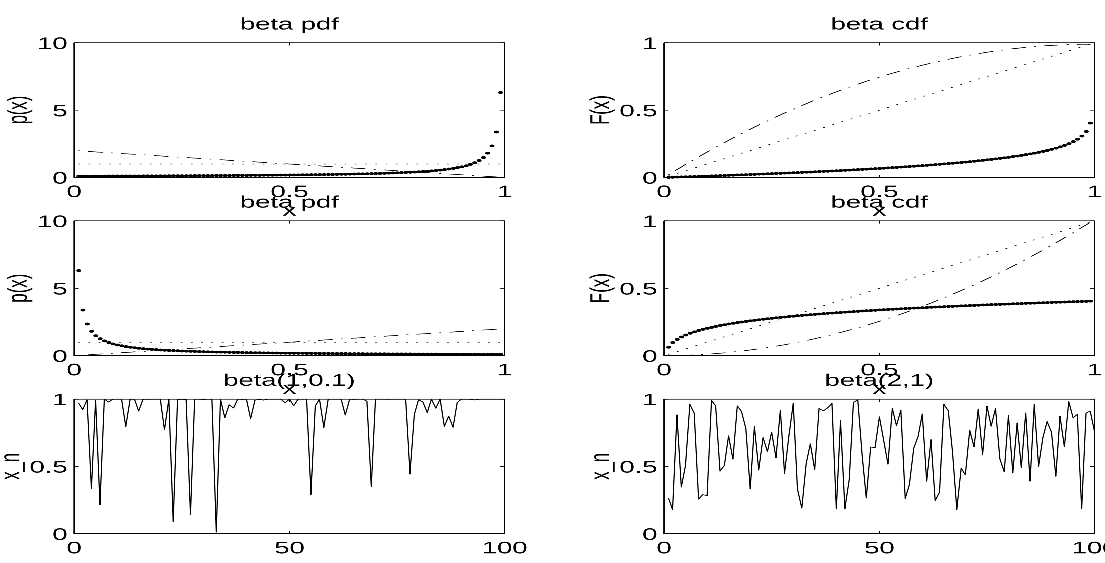
$$\mathrm{E}\,[x] = \frac{\alpha}{\alpha+\beta},$$
$$\mathrm{Var}\,[x] = \frac{\alpha\beta}{(\alpha+\beta)^2(\alpha+\beta+1)},$$
$$\mathrm{M}\,[x] = \frac{\alpha-1}{\alpha+\beta-2}\ \text{si}\quad \alpha,\beta>1$$

$$\mathrm{S}\,[x] = \frac{2(\beta-\alpha)\sqrt{\alpha+\beta+1}}{(\alpha+\beta+2)\sqrt{\alpha\beta}}$$
$$\mathrm{K}\,[x] = \frac{3(\alpha+\beta+1)[2(\alpha+\beta)^2+\alpha\beta(\alpha+\beta+6)]}{\alpha\beta(\alpha+\beta+2)(\alpha+\beta+3)}$$

$$\mathrm{H}\,[x] = ?$$

$$\mathrm{H}\,[x] = \log[B(\alpha,\beta)] \quad -(\alpha-1)[\psi(\alpha)-\psi(\alpha+\beta)] \quad -(\beta-1)[\psi(\beta)-\psi(\alpha+\beta)]$$

si $\quad x \sim \mathbf{Bet}\,(x|\alpha,\beta)\,,\quad$ alors $\quad y = 1-x \sim \mathbf{Bet}\,(y|\beta,\alpha)\,,$



Gamma

$$\mathbf{Gam}\,(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\, x^{\alpha-1} \exp\left[-\beta x\right],$$

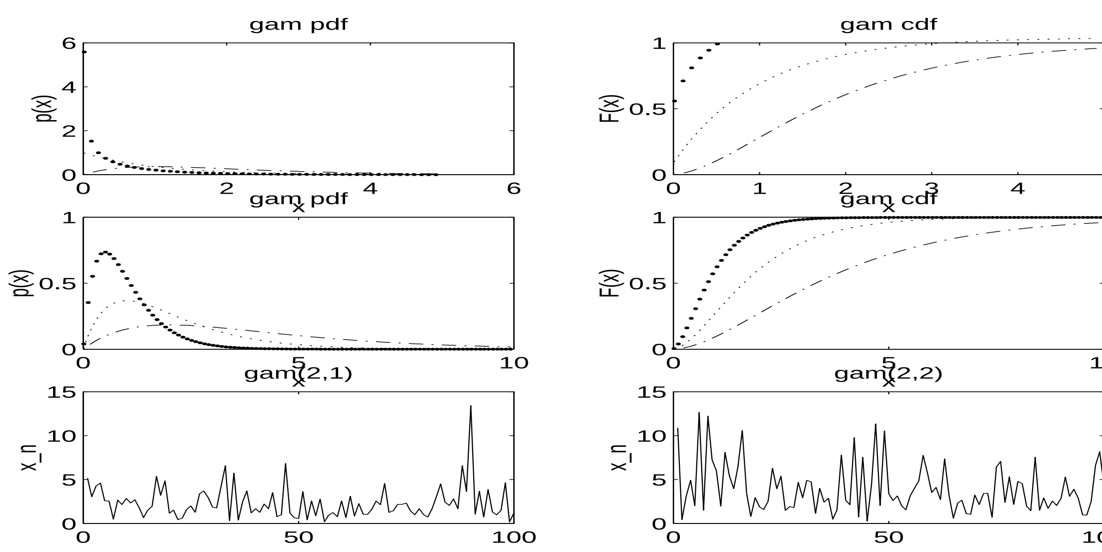
$\alpha, \beta > 0,$
$x > 0,$

$$\mathrm{E}\,[x] = \frac{\alpha}{\beta},$$
$$\mathrm{Var}\,[x] = \frac{\alpha}{\beta^2},$$
$$\mathrm{M}\,[x] = \frac{\alpha-1}{\beta}, \text{si} \quad \alpha > 1$$
$$\mathrm{S}\,[x] = \frac{2}{\sqrt{\alpha}}$$
$$\mathrm{K}\,[x] = \frac{6}{\alpha}$$

$$\mathrm{H}\,[x] = -\log\beta + \log[\Gamma(\alpha)] + (1-\alpha)\psi(\alpha) + \alpha$$

$\alpha = 1 :$ **Exponentielle**, $\mathbf{Gam}\,(x|1,\beta) = \mathbf{Ex}\,(x|\beta)\,,$
$\beta = 1 :$ **Erlang**, $\mathbf{Gam}\,(x|\alpha,1) = \mathbf{Erlang}\,(x|\alpha)\,,$
$\alpha = \frac{\nu}{2}, \beta = \frac{1}{2} :$ $\mathbf{Gam}\left(x|\frac{\nu}{2},\frac{1}{2}\right) = \mathbf{Chi}^2\,(x|\nu)\,,$

si $\{x_1,\cdots,x_k\} \sim \mathbf{Gam}\,(x|\alpha_i,\beta)\,, i = 1,\cdots k,$
alors $s = \sum x_i \sim \mathbf{Gam}\left(s|\sum \alpha_i,\beta\right)$



Gamma inverse

$$\mathbf{IGam}\,(x|\alpha,\beta) = \frac{\beta^\alpha}{\Gamma(\alpha)}\,x^{-(\alpha+1)}\exp\left[-\frac{\beta}{x}\right],$$

$\alpha, \beta > 0,$

$x > 0,$

$$\mathrm{E}\,[x] = \frac{\beta}{\alpha-1}, \text{si} \quad \alpha > 1,$$

$$\mathrm{Var}\,[x] = \frac{\beta^2}{(\alpha-1)(\alpha-2)}, \text{si} \quad \alpha > 2,$$

$$\mathrm{M}\,[x] = \frac{\beta}{\alpha+1}, \text{si} \quad \alpha < 1$$

$\mathrm{S}\,[x] = ?$

$\mathrm{K}\,[x] = ?$

$$\mathrm{H}\,[x] = \log\beta + \log[\Gamma(\alpha)] + (1-\alpha)\psi(\alpha) + \alpha$$

$$\alpha = \frac{\nu}{2}, \beta = \frac{1}{2}: \quad \mathbf{IGam}\left(x|\frac{\nu}{2},\frac{1}{2}\right) = \mathbf{Chi}^{-2}\,(x|\nu)\,,$$

du khi-deux

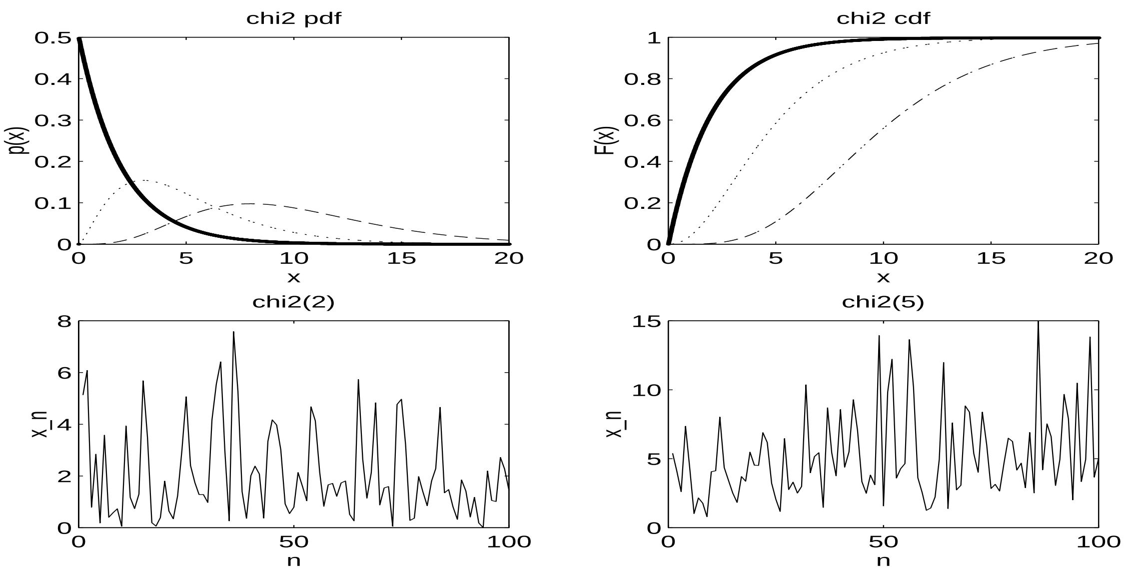
$$\mathbf{Chi}^2(x|n) = \frac{(\frac{1}{2})^{n/2}}{\Gamma(1/2)}\, x^{n/2-1} \exp\left[-\frac{x}{2}\right],$$

$n > 0, \quad x > 0,$

$\mathrm{E}[x] = n,$
$\mathrm{Var}[x] = 2n,$
$\mathrm{M}[x] = n - 2 \quad \mathrm{si}\, n > 2$

$\mathrm{S}[x] = 2\sqrt{2/n}$
$\mathrm{K}[x] = \dfrac{12}{n}$

$\mathrm{H}[x] = ?$



du khi-deux décentrée

$$\mathbf{Chi}^2(x|\nu,\lambda) = \sum_{i=0}^{\infty} \mathbf{Pn}\left(i|\frac{\lambda}{2}\right)\ \chi^2\left(x|\nu+2i\right),$$

$\nu, \lambda > 0,$
$x > 0,$

$\mathrm{E}\left[x\right] = \nu + \lambda,$
$\mathrm{Var}\left[x\right] = 2(\nu + 2\lambda),$
$\mathrm{M}\left[x\right]: \quad \mathbf{Chi}^2\left(\mathrm{M}\left[x\right]|\nu,\lambda\right) = \mathbf{Chi}^2\left(\mathrm{M}\left[x\right]|\nu-2,\lambda\right)$
$\mathrm{S}\left[x\right] = ?$
$\mathrm{K}\left[x\right] = ?$

$\mathrm{H}\left[x\right] = ?$

$$\mathbf{Chi}^2\left(x|\nu,\lambda\right) = \sum_{i=0}^{\infty} \mathbf{Pn}\left(i|\lambda/2\right) \mathbf{Chi}^2\left(x|\nu+2i\right)$$

si $\quad x \sim \mathbf{Gamm}(x|\alpha,\beta), \text{alors} \quad y = 2\beta x \sim \mathbf{Chi}^2(y|\alpha)$
si $\quad \{x_1,\cdots,x_k\} \sim \mathbf{N}\left(x|\mu_i,1\right), i = 1,\cdots k,$
alors $\quad s = \sum x_i^2 \sim \mathbf{Chi}^2\left(s|k, \sum \mu_i^2\right)$
si $\quad \{x_1,\cdots,x_k\} \sim \mathbf{Chi}^2\left(x|\nu_i,\lambda_i\right), i = 1,\cdots k,$
alors $\quad z = \sum x_i \sim \mathbf{Chi}^2\left(z|\sum \nu_i, \sum \lambda_i\right)$

du khi inverse

$$\mathbf{IChi}\,(x|\nu) = \frac{\frac{1}{2}^{\nu/2}}{\Gamma(\nu/2} \, x^{-(\nu/2+1)} \exp\left[-\frac{1}{2x^2}\right],$$

$\nu > 0, \quad x > 0,$

$\mathrm{E}\,[x] =,$
$\mathrm{Var}\,[x] =,$
$\mathrm{M}\,[x] =$
$\mathrm{S}\,[x] =?$
$\mathrm{K}\,[x] =?$

$\mathrm{H}\,[x] =?$

Racine carrée Gamma inverse

$$\mathbf{IGam}^{-\frac{1}{2}}(x|\alpha,\beta) = \frac{2\beta^{\alpha}}{\Gamma(\alpha)}\,x^{-(2\alpha+1)}\exp\left[-\frac{\beta}{x^2}\right],$$

$\alpha, \beta > 0,$
$x > 0,$

$\mathrm{E}[x] = ?,$
$\mathrm{Var}[x] = ?,$
$\mathrm{M}[x] = ?$
$\mathrm{S}[x] = ?$
$\mathrm{K}[x] = ?$

$\mathrm{H}[x] = ?$

Pareto

$\mathbf{Par}(x|\alpha,\beta) = \alpha\beta^\alpha x^{-(\alpha+1)},$
$\alpha,\beta > 0,$
$x \geq \beta,$

$\mathrm{E}[x] = \dfrac{\alpha\beta}{\alpha-1}, \text{si} \quad \alpha > 1,$
$\mathrm{Var}[x] = \dfrac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}, \text{si} \quad \alpha > 2,$
$\mathrm{M}[x] = \beta$
$\mathrm{S}[x] = ?$
$\mathrm{K}[x] = ?$

$\mathrm{H}[x] = \dfrac{1}{\alpha} - \log\alpha + 1$

$$\mathbf{Par}(x|\alpha,\beta) = \int_0^\infty \mathbf{Ex}(x-\beta|\lambda)\,\mathbf{Gam}(\lambda|\alpha,\beta)\;\mathrm{d}\lambda$$

Pareto inverse

$\mathbf{IPar}\,(x|\alpha,\beta) = \alpha\beta^{\alpha}x^{\alpha-1}$,
$\alpha, \beta > 0$,
$0 < x < \beta^{-1}$,

$\mathrm{E}\,[x] =$,
$\mathrm{Var}\,[x] =$,
$\mathrm{M}\,[x] =$
$\mathrm{S}\,[x] = ?$
$\mathrm{K}\,[x] = ?$

$\mathrm{H}\,[x] = ?$

Normale (paramètres : μ, λ)

$$\mathbf{N}(x|\mu,\lambda) = \sqrt{\frac{\lambda}{2\pi}} \exp\left[-\frac{1}{2}\lambda(x-\mu)^2\right],$$

$\mu \in \mathbb{R}$, $\lambda > 0$,
$x \in \mathbb{R}$,

$\mathrm{E}[x] = \mu$,
$\mathrm{Var}[x] = \lambda^{-1}$,
$\mathrm{M}[x] = \mu$
$\mathrm{S}[x] = 0$
$\mathrm{K}[x] = 0$

Cumulants : $K_1 = \mu, \quad K_2 = \frac{1}{\lambda}, \quad K_n = 0,\ n > 2$

Fonction caractéristique : $F(t) = \exp\left[i\mu t - \frac{t^2}{2\lambda}\right]$

$\mathrm{H}[x] = \frac{1}{2} + \ln(2\pi/\lambda)$

si $\{x_1, \cdots, x_k\} \sim \mathbf{N}(x|\mu_i, \lambda_i),\ i = 1, \cdots k,$

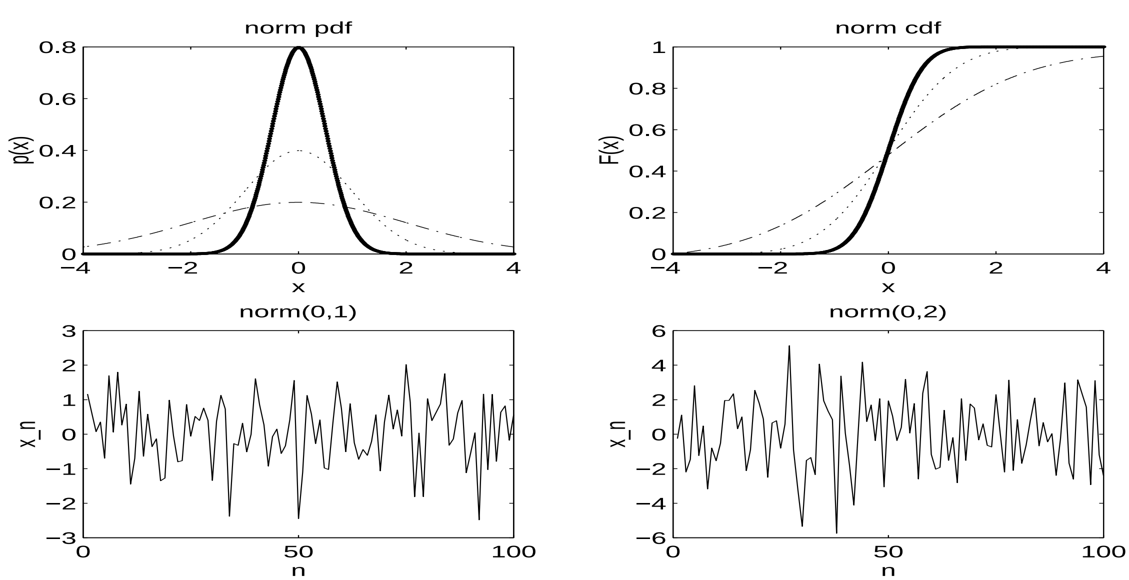
alors $y = a + \sum_{i=1}^{k} b_i\, x_i \sim \mathbf{N}\left(y|a + \sum_{i=1}^{k} b_i\, \mu_i, \left(\sum_{i=1}^{k} b_i^2/\lambda_i\right)^{-1}\right)$

si $\{x_1, \cdots, x_k\} \sim \mathbf{N}(x|0, 1),\ i = 1, \cdots k,$

alors $s = \sum_{i=1}^{k} x_i^2 \sim \mathbf{Chi}^2(s|k)$

si $\{x_1, \cdots, x_k\} \sim \mathbf{N}(x|\mu_i, 1),\ i = 1, \cdots k,$

alors $s = \sum_{i=1}^{k} x_i^2 \sim \mathbf{Chi}^2\left(s|k, \sum_{i=1}^{k} \mu_i^2\right)$



Normale (paramètres : μ, σ^2)

$$\mathbf{N}\left(x|\mu,\sigma^2\right) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right],$$

$\mu \in \mathbb{R}$, $\sigma > 0$,
$x \in \mathbb{R}$,

$\mathrm{E}[x] = \mu$,
$\mathrm{Var}[x] = \sigma^2$,
$\mathrm{M}[x] = \mu$
$\mathrm{S}[x] = 0$
$\mathrm{K}[x] = 0$

Cumulants : $K_1 = \mu$, $K_2 = \sigma$, $K_n = 0$, $n > 2$

Fonction caractéristique : $F(t) = \exp\left[i\mu t - \frac{\sigma t^2}{2}\right]$

$$\mathrm{H}[x] = \frac{1}{2} + \ln(2\pi\sigma^2)$$

si $\{x_1, \cdots, x_k\} \sim \mathbf{N}\left(x|\mu_i, \sigma_i^2\right), i = 1, \cdots k,$

alors $y = a + \sum_{i=1}^{k} b_i\, x_i \sim \mathbf{N}\left(y|a + \sum_{i=1}^{k} b_i\, \mu_i, \sum_{i=1}^{k} b_i^2 \sigma_i^2\right)$

si $\{x_1, \cdots, x_k\} \sim \mathbf{N}(x|0,1), i = 1, \cdots k,$

alors $s = \sum_{i=1}^{k} x_i^2 \sim \mathbf{Chi}^2(s|k)$

si $\{x_1, \cdots, x_k\} \sim \mathbf{N}(x|\mu_i,1), i = 1, \cdots k,$

alors $s = \sum_{i=1}^{k} x_i^2 \sim \mathbf{Chi}^2\left(s|k, \sum_{i=1}^{k} \mu_i^2\right)$

Exponentielle

$\mathbf{Ex}\,(x|\lambda) = \lambda \exp\left[-\lambda x\right],$

$\lambda > 0, \quad x > 0,$

$\mathrm{E}\left[x\right] = \lambda,$

$\mathrm{Var}\left[x\right] = \lambda,$

$\mathrm{M}\left[x\right] = \lambda$

$\mathrm{S}\left[x\right] = 2$

$\mathrm{K}\left[x\right] = 6$

$\mathrm{H}\left[x\right] = 1 + \ln \lambda$

Autre notation

$\mathbf{Ex}\,(x|\alpha, \beta) = \frac{1}{\beta} \exp\left[-(x-\alpha)/\beta\right],$

$x > \alpha, \quad \alpha \in \mathbb{R}, \quad \beta > 0,$

$\mathrm{E}\left[x\right] = \alpha + \beta,$

$\mathrm{Var}\left[x\right] = \beta^2,$

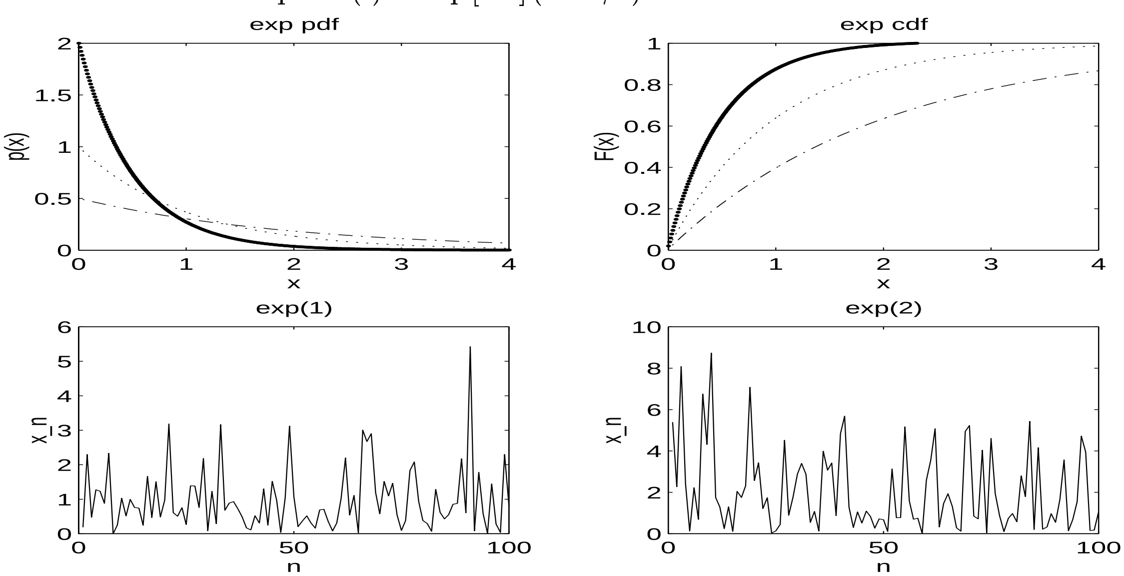
$\mathrm{M}\left[x\right] = \alpha$

$\mathrm{S}\left[x\right] = 2$

$\mathrm{K}\left[x\right] = 6$

Cumulants : $K_1 = \alpha + \beta, \quad K_n = \Gamma(n)\beta^n,\ n > 1$

Fonction caractéristique : $F(t) = \exp\left[i\alpha t\right]\left(1 - i\beta t\right)^{-1}$



Gamma–Gamma

$$\mathbf{GamGam}\,(x|\alpha,\beta,n) = \frac{\beta^\alpha}{\Gamma(\alpha)}\frac{\Gamma(\alpha+n)}{\Gamma(n)}\frac{x^{n-1}}{(\beta+x)^{\alpha+n}},$$

$\alpha,\beta > 0,\ n = 0,1,2,\cdots,$
$x = 0,1,2,\cdots,$

$\mathrm{E}\,[x] = n\dfrac{\beta}{\alpha-1}$, si $\alpha > 1$,
$\mathrm{Var}\,[x] = \dfrac{\beta^2(n^2+n(\alpha-1))}{(\alpha-1)^2(\alpha-2)}$, si $\alpha > 2$,
$\mathrm{M}\,[x] = ?$
$\mathrm{S}\,[x] = ?$
$\mathrm{K}\,[x] = ?$

$\mathrm{H}\,[x] = ?$

$$\mathbf{GamGam}\,(x|\alpha,\beta,n) = \int_0^\infty \mathbf{Gam}\,(x|n,\lambda)\,\mathbf{Gam}\,(\lambda|\alpha,\beta)\ \mathrm{d}\lambda$$

Student

$$\mathbf{St}\,(x|\mu,\lambda,\alpha) = \frac{\Gamma(\frac{\alpha+1}{2})}{\Gamma(\frac{\alpha}{2})\Gamma(\frac{1}{2})}\left(\frac{\lambda}{\alpha}\right)^{\frac{1}{2}}\left[1+\frac{\lambda}{\alpha}(x-\mu)^2\right]^{-(\alpha+1)/2},$$

$\mu \in \mathbb{R},\ \lambda,\alpha > 0,$
$x \in \mathbb{R},$

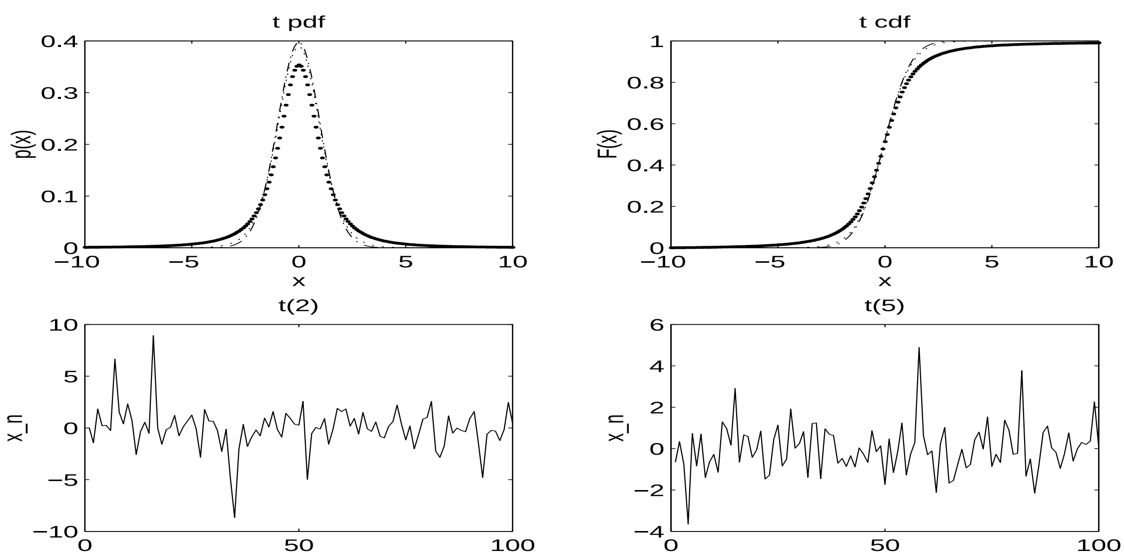
$\mathrm{E}\,[x] = \mu,$ si $\alpha > 1,$
$\mathrm{Var}\,[x] = \frac{1}{\lambda}\frac{\alpha}{\alpha-2},$ si $\alpha > 2,$
$\mathrm{M}\,[x] = \mu$

$\mathrm{S}\,[x] = ?$
$\mathrm{K}\,[x] = ?$

$$\mathrm{H}\,[x] = \log[\alpha^{\frac{1}{2}}B(\frac{1}{2},\frac{\alpha}{2})] + \frac{1+\alpha}{2}[\psi(\frac{\alpha}{2}+\frac{1}{2})-\psi(\frac{\alpha}{2})]$$

$$\mathbf{St}\,(x|\mu,\lambda,\alpha) = \int_0^\infty \mathbf{N}\,(x|\mu,\lambda y)\,\mathbf{Gam}\left(y|\frac{\alpha}{2},\frac{\alpha}{2}\right)\,\mathrm{d}y$$

$\alpha = 1:\quad \mathbf{St}\,(x|\mu,\lambda,1) = \mathbf{Cauchy}\,(x|\mu,\lambda)$
$\alpha \mapsto \infty:\quad \lim_{\alpha\mapsto\infty}\mathbf{St}\,(x|\mu,\lambda,\alpha) = \mathbf{N}\,(x|\mu,\lambda)$
si $\quad x \sim \mathbf{St}(x|\mu,\lambda,\nu)$
alors $\quad y = \lambda^{\frac{1}{2}}(x-\mu) \sim \mathbf{St}(x|0,1,\nu)$
si $\quad x \sim \mathbf{N}(x|0,1)$ et $y \sim \mathbf{Chi}^2(y|\nu)$
alors $z = \frac{x}{(y/\nu)^{\frac{1}{2}}} \sim \mathbf{St}(x|0,1,\nu)$



Fisher-Snedecor

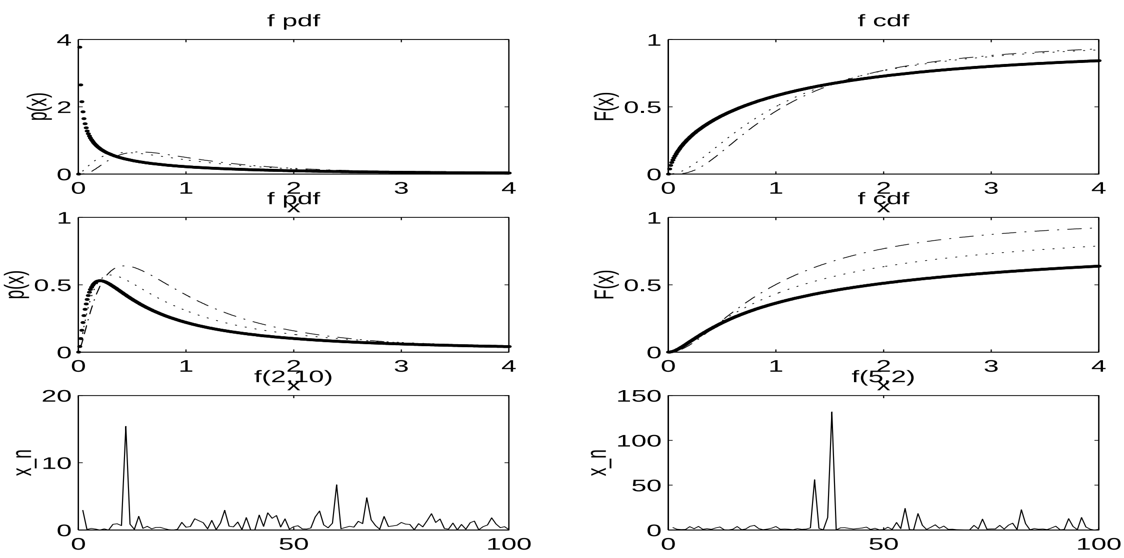
$$\mathbf{FS}\left(x|\alpha,\beta\right)=\frac{\Gamma((\alpha+\beta)/2)}{\Gamma(\alpha/2)\Gamma(\beta/2)}\alpha^{\alpha/2}\beta^{\beta/2}\frac{x^{\alpha/2-1}}{(\beta+\alpha x)^{(\alpha+\beta)/2}}=\frac{\alpha^{\alpha/2}\beta^{\beta/2}}{B(\alpha/2,\beta/2)}x^{\alpha/2-1}(\beta+\alpha x)^{-(\alpha+\beta)/2},$$

$\alpha,\beta>0,$
$x>0,$

$\mathrm{E}\left[x\right]=\frac{\beta}{\beta-2},\mathrm{si}\quad\beta>2,$
$\mathrm{Var}\left[x\right]=\frac{2\beta^2}{\alpha(\beta-4)}\frac{\alpha+\beta-2}{(\beta-2)^2},\mathrm{si}\quad\beta>4,$
$\mathrm{M}\left[x\right]=\frac{\beta}{\beta+2}\frac{\alpha-2}{\alpha},\mathrm{si}\quad\beta>2$
$\mathrm{S}\left[x\right]=?$
$\mathrm{K}\left[x\right]=?$
Les points d'inflexion: $\frac{\beta\sqrt{(\alpha-2)(\alpha+\beta)}}{\alpha(\beta+2)}$

$$\mathrm{H}\left[x\right]=\log\left[\frac{\alpha}{\beta}B\left(\frac{\alpha}{2},\frac{\beta}{2}\right)\right]+(1-\frac{\alpha}{2})\psi(\frac{\alpha}{2})-(1-\frac{\beta}{2})\psi(\frac{\beta}{2})+(\frac{\alpha+\beta}{2})\psi(\frac{\alpha+\beta}{2})$$

si $\quad x\sim\mathbf{Chi}^2(x|\nu_1)$ et $y\sim\mathbf{Chi}^2(y|\nu_2)$
alors $\quad z=\frac{x/\nu_1}{(y/\nu_2)}\sim\mathbf{FS}\left(z|\nu_1,\nu_2\right)$
si $\quad x\sim\mathbf{Bet}(x|\alpha,\beta)$
alors $\quad y=\frac{\beta x}{\alpha(1-x)}\sim\mathbf{FS}\left(y|2\alpha,2\alpha\right)$



Logistic

$\mathbf{Lo}\,(x|\alpha,\beta) = \beta^{-1}\,\dfrac{\exp\left[-\beta^{-1}(x-\alpha)\right]}{\left(1+\exp\left[\beta^{-1}(x-\alpha)\right]\right)^2},$

$= \dfrac{1}{4\beta}\,\mathbf{sech}^2\left\{\dfrac{1}{2}\dfrac{x-\alpha}{\beta}\right\},$

$\alpha \in \mathbb{R},\ \beta > 0,$

$x \in \mathbb{R},$

$\mathrm{E}\,[x] = \alpha,$

$\mathrm{Var}\,[x] = \beta^2\pi^2/3,$

$\mathrm{M}\,[x] = \alpha$

$\mathrm{S}\,[x] = ?$

$\mathrm{K}\,[x] = ?$

$\mathrm{H}\,[x] = ?$

Uniforme

$\mathbf{Uni}\,(x|\theta_1,\theta_2) = \dfrac{1}{\theta_2-\theta_1},$

$\theta_2 > \theta_1,$

$\theta_1 < x < \theta_2,$

$\mathrm{E}\,[x] = \dfrac{\theta_2+\theta_1}{2},$

$\mathrm{Var}\,[x] = \dfrac{(\theta_2-\theta_1)^2}{12},$

$\mathrm{M}\,[x] = ?$

$\mathrm{S}\,[x] = 0$

$\mathrm{K}\,[x] = -1.2$

$\mathrm{H}\,[x] = \ln(\theta_2-\theta_1)$

Lois de probabibilté à une variable réelle (suite)

Cauchy

$\mathbf{Cau}\,(x|\lambda) = \dfrac{1/(\pi\lambda)}{1+(x/\lambda)^2}$,
$\lambda \in \mathbb{R}, \quad x \in \mathbb{R}$,

$\mathrm{E}\,[x]$ n'existe pas,
$\mathrm{Var}\,[x] =$ n'existe pas
$\mathrm{M}\,[x] = \dfrac{1}{\pi\lambda}$
$\mathrm{S}\,[x] =$ n'existe pas,
$\mathrm{K}\,[x] =$ n'existe pas,
Fonction caractéristique : $F(t) = e^{-\lambda|t|}$

$\mathrm{H}\,[x] = \ln(4\pi\lambda)$

Autre notation :
$\mathbf{Cau}\,(x|\lambda,\alpha) = \frac{1/(\pi\lambda)}{1+(\frac{x-\alpha}{\lambda})^2}$,
$\lambda \in \mathbb{R}, \quad x \in \mathbb{R}$,

$\mathrm{E}\,[x]$ n'existe pas,
$\mathrm{Var}\,[x] =$ n'existe pas
$\mathrm{M}\,[x] = \dfrac{1}{\pi\lambda}$
$\mathrm{S}\,[x] =$ n'existe pas,
$\mathrm{K}\,[x] =$ n'existe pas,
Fonction caractéristique : $F(t) = e^{-\lambda|t|+i\alpha t}$

$\mathrm{H}\,[x] = \ln(4\pi\lambda)$

RayleightRay $(x\|\theta)$

Log-Normale

$$\mathbf{LogN}\left(x|\mu,\lambda\right) = \frac{1}{\sqrt{2\pi}\lambda x}\ \exp\left[-\frac{(\ln x-\mu)^2}{2\lambda^2}\right],$$

$\mathrm{E}\left[x\right] =,$
$\mathrm{Var}\left[x\right] = \exp\left[2\mu - 2\lambda^2\right] - \exp\left[2\mu - \lambda^2\right],$
$\mathrm{M}\left[x\right] =?$

$\mathrm{H}\left[x\right] = \mu + \log(2\pi e\lambda^2)$

Normale généralisée

$$\mathbf{Ngen}\,(x|\alpha,\beta) = \frac{2\beta^{\alpha/2}}{\Gamma(\alpha/2)}\,x^{\alpha-1}\exp\left[-\beta x^2\right],$$

$\mathrm{E}\,[x] =,$
$\mathrm{Var}\,[x] =,$
$\mathrm{M}\,[x] =?$

$\mathrm{H}\,[x] =?$

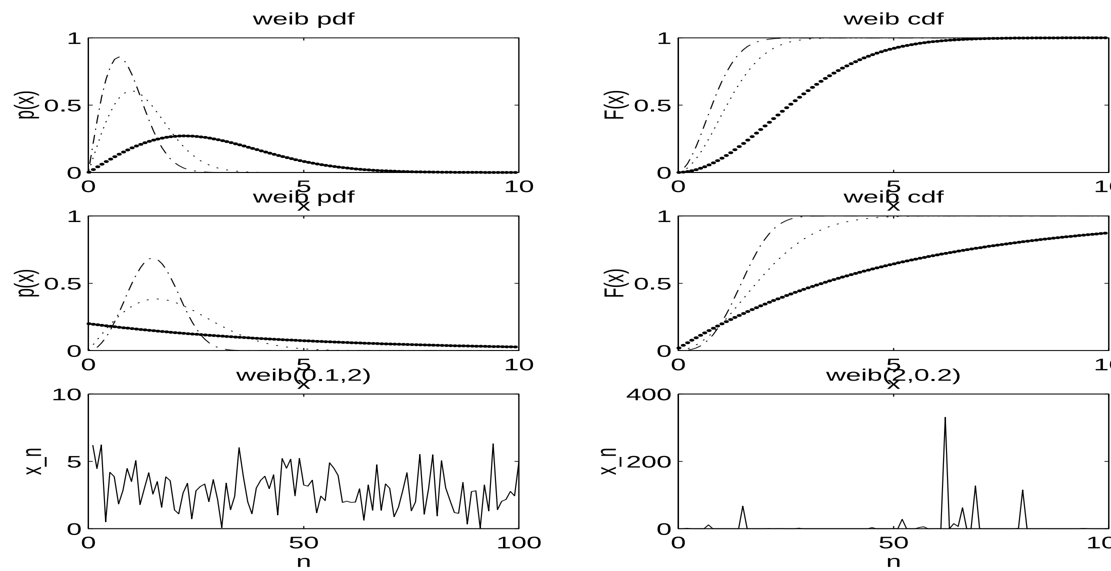
Weibull

$$\mathbf{Wei}\,(x|\alpha,\beta) = \frac{\beta}{\alpha}\,x^{\alpha-1}\exp\left[-x^{\beta}/\alpha\right],$$
$\alpha,\beta > 0, \quad x > 0,$

$\mathrm{E}\,[x] = \alpha^{1/\beta}\Gamma(1+1/\beta)$
$\mathrm{Var}\,[x] = \alpha^{2/\beta}[\Gamma(1+2/\beta) - \Gamma^2(1+1/\beta)]$
$\mathrm{M}\,[x] =$
$\mathrm{S}\,[x] = ?$
$\mathrm{K}\,[x] = ?$

Fonction de répartition : $F(x) = 1 - \exp\left[-x^{\beta}/\alpha\right], \quad F^{-1}(p) = (\alpha \ln \frac{1}{1-p})^{1/\beta}$

$\mathrm{H}\,[x] = ?$



Exponentielle double

$\mathbf{Exd}(x|\lambda) = \frac{\lambda}{2} \exp[-|\lambda|x],$

$\lambda > 0, \quad x \in \mathbb{R},$

$\mathrm{E}[x] = 0,$

$\mathrm{Var}[x] = \frac{2}{\lambda^2},$

$\mathrm{M}[x] = 0$

$\mathrm{S}[x] = 0$

$\mathrm{K}[x] = ?$

$\mathrm{H}[x] = ?$

Exponentielle tronquée

$\mathbf{Ext}\,(x|\lambda) = \exp\left[-(x-\lambda)\right],$

$\lambda > 0, \quad x > \lambda,$

$\mathrm{E}\,[x] =,$

$\mathrm{Var}\,[x] =,$

$\mathrm{M}\,[x] =$

$\mathrm{H}\,[x] =?$

Triangulaire

$\mathbf{Tri}\,(x|\theta) =,$

$\theta > 0, \quad ,$

$\mathrm{E}\,[x] =,$

$\mathrm{Var}\,[x] =,$

$\mathrm{M}\,[x] =$

$\mathrm{H}\,[x] =?$

du khi

$$\mathbf{Chi}(x|n) = \frac{\frac{1}{2}^{n/2}}{\Gamma(n/2)}\, x^{n/2-1} \exp\left[-\frac{x}{2}\right],$$

$n > 0, \quad x > 0,$

$\mathrm{E}[x] = n,$

$\mathrm{Var}[x] = 2n,$

$\mathrm{M}[x] =?$

$\mathrm{S}[x] = 2\sqrt{\frac{2}{n}}$

$\mathrm{K}[x] = \frac{12}{n}$

$\mathrm{H}[x] =?$

Famille exponentielle généralisée à un paramètres θ

$$\mathbf{Exf}\,(x|f,g,h,\phi,\theta) = f(x)\,g(\theta)\exp\left[\phi(\theta)h(x)\right],$$

$f(x)$	$g(\theta)$	$h(x)$	$\phi(\theta)$	$p(x\|\theta)$
1	$1-\theta$	x	$\log\frac{\theta}{1-\theta}$	$\mathbf{Ber}(x\|\theta)$
$\frac{1}{x!}$	$\exp\left[-\theta\right]$	x	$\log(\theta)$	$\mathbf{Pn}(x\|\theta)$
1	θ	x	$-\theta$	$\mathbf{Ex}(x\|\theta)$
$\frac{1}{\sqrt{2\pi}}$	$\sqrt{\theta}$	x^2	$-\frac{1}{2}\theta$	$\mathbf{N}(x\|0,\theta)$
1	$\frac{1}{\theta}$	0	θ	$\mathbf{Uni}(x\|0,\theta)$
$\exp\left[-x\right]$	$\exp\left[\theta\right]$	0	θ	$\mathbf{Ext}(x\|\theta) = \exp\left[-(x-\theta)\right]$

Famille exponentielle à k paramètres

$$\mathbf{Exfk}\,(x|f, g, \boldsymbol{h}, \boldsymbol{\phi}, \boldsymbol{\theta}) = f(x)\, g(\boldsymbol{\theta}) \exp\left[\sum_{k=1}^{K} \boldsymbol{\phi}_k(\boldsymbol{\theta}) \boldsymbol{h}_k(x)\right],$$

$f(x)$	$\boldsymbol{\theta}$	$g(\boldsymbol{\theta})$	$\boldsymbol{h}(x)$	$\boldsymbol{\phi}(\boldsymbol{\theta})$	$p(x\|\boldsymbol{\theta})$
$\frac{1}{\sqrt{2\pi}}$	(μ, λ)	$\sqrt{\lambda} \exp\left[-\frac{1}{2}\lambda\mu^2\right]$	$\{x, x^2\}$	$\{\lambda\mu, -\frac{1}{2}\lambda\}$	$\mathbf{N}(x\|\mu, \lambda)$
$\frac{1}{\sqrt{2\pi}}$	(μ, σ^2)	$\frac{1}{\sigma} \exp\left[-\frac{1}{2}\frac{\mu^2}{\sigma^2}\right]$	$\{x, x^2\}$	$\left\{\frac{\mu}{\sigma^2}, -\frac{1}{2\sigma^2}\right\}$	$\mathbf{N}(x\|\mu, \sigma^2)$
1	$(\theta - 2, \theta - 1)$	$\frac{1}{\theta_2 - \theta_1}$	$\{0, 0\}$	$\{0, 0\}$	$\mathbf{Uni}(x\|\theta_1, \theta_2)$
1	$\{\alpha_1, \cdots, \alpha_k\}$	$\frac{\Gamma(\alpha_1)\cdots\Gamma(\alpha_k)}{\Gamma(\alpha_1+\cdots\alpha_k)}$	$\{\log x_1, \cdots, \log x_k\}$	$\{\alpha_1, \cdots, \alpha_k\}$	$\mathbf{Di}_k(\boldsymbol{x}\|\alpha_1, \alpha_k)$

Lois de probabibilté à deux variables réelles

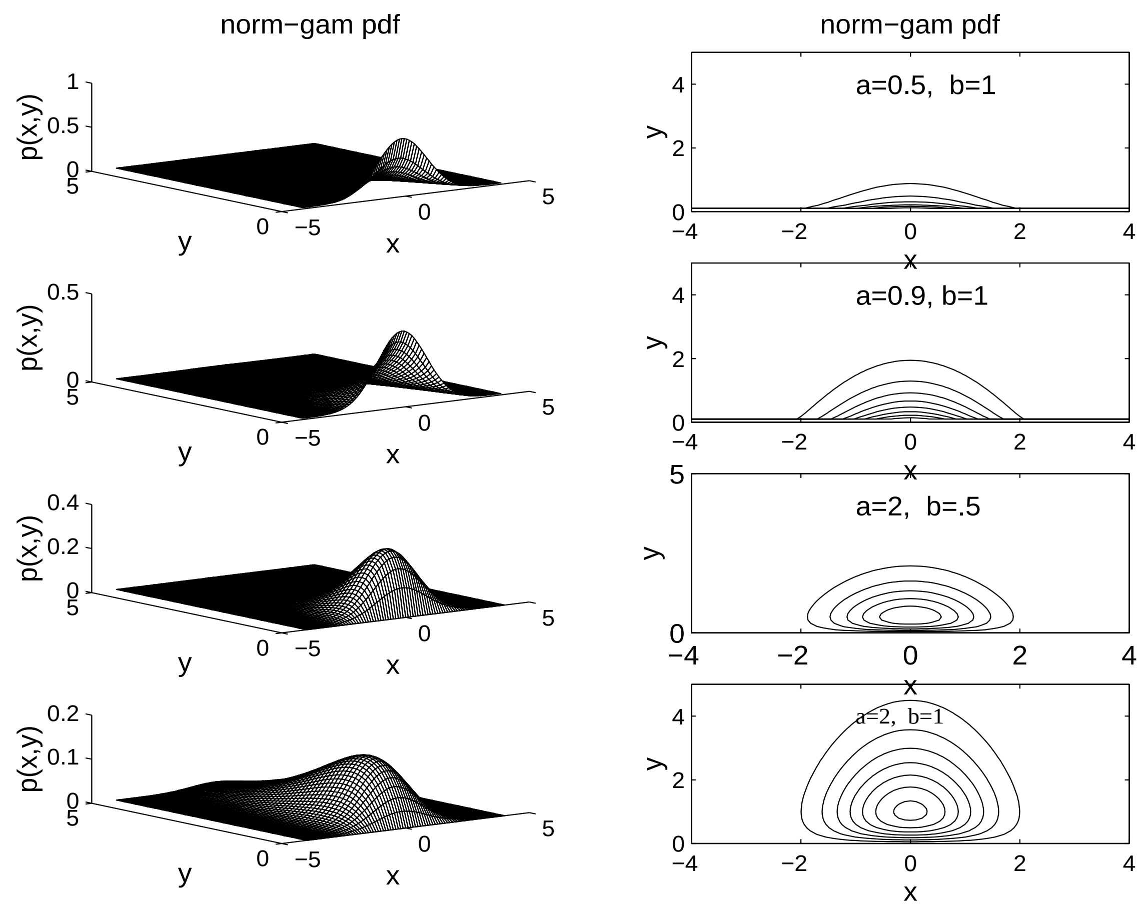
Normale-Gamma

$\mathbf{NGam}\,(x, y|\mu, \lambda, \alpha, \beta) = \mathbf{N}(x|\mu, \lambda y)\, \mathbf{Gam}(y|\alpha, \beta),$
$\mu \in \mathbb{R}, \lambda, \alpha, \beta > 0,$
$x \in \mathbb{R}, y > 0,$

$\mathrm{E}\,[x] = \mu$
$\mathrm{E}\,[y] = \dfrac{\alpha}{\beta}$

$$p(x) = \int_{-\infty}^{+\infty} p(x, y)\,\mathrm{d}x = \int_{-\infty}^{+\infty} \mathbf{NGam}\,(x, y|\mu, \lambda, \alpha, \beta)\ \mathrm{d}x = \mathbf{St}\left(x|\mu, \lambda\frac{\alpha}{\beta}, 2\alpha\right),$$
$$p(y) = \int_{0}^{\infty} p(x, y)\,\mathrm{d}y = \int_{0}^{\infty} \mathbf{NGam}\,(x, y|\mu, \lambda, \alpha, \beta)\ \mathrm{d}y = \mathbf{Gam}(y|\alpha, \beta),$$
$$p(x|y) = \frac{p(x, y)}{p(y)} = \mathbf{N}(x|\mu, \lambda y),$$

$\mathrm{E}\,[x] = \mu, \quad \text{si } 2\alpha > 1, \quad \mathrm{Var}\,[x] = \dfrac{2\beta}{2\alpha - 2}\dfrac{1}{\lambda} \quad \text{si } \alpha > 1$
$\mathrm{E}\,[y] = \dfrac{\alpha}{\beta}, \quad \mathrm{Var}\,[y] = \dfrac{\alpha}{\beta^2}$
$\mathrm{E}\,[xy] = \mu, \quad \mathrm{Var}\,[x|y] = \lambda y$



Pareto bivariable

$\mathbf{Par}_2\,(x,y|\alpha,\beta_0,\beta_1) = \alpha(\alpha+1)(\beta_1-\beta_0)^{\alpha}(y-x)^{(\alpha+2)}$,
$(\beta_0,\beta_1) \in \mathbb{R}^2$, $\beta_0 < \beta_1, \alpha > 0$,
$(x,y) \in \mathbb{R}^2$, $x < \beta_0, y > \beta_1$,

$\mathrm{E}\,[x] = \dfrac{\alpha\beta_0-\beta_1}{\alpha-1}$, si $\quad \alpha > 1$,
$\mathrm{E}\,[y] = \dfrac{\alpha\beta_1-\beta_0}{\alpha-1}$, si $\quad \alpha > 1$,
$\mathrm{Var}\,[x] = \mathrm{Var}\,[y] = \dfrac{\alpha(\beta_1-\beta_0)^2}{(\alpha-1)^2(\alpha-2)}$, si $\quad \alpha > 2$,
$\mathrm{Corr}\,[x,y] = -\dfrac{1}{\alpha}$,

$t_1 = \beta_1 - x$, $t_2 = \beta_0 - y \sim \mathbf{Par}(t|\beta_1-\beta_0,\alpha)$

$\mathrm{H}\,[x] = ?$

Lois de probabibilté à n variables discrètes

Multinomiale

$$\mathbf{Mu}_k\left(\boldsymbol{x}|\boldsymbol{\theta},n\right)=\frac{n!}{\prod_{l=1}^{k+1}x_l!}\prod_{l=1}^{k+1}\theta_l^{x_l},$$

$$\boldsymbol{x}=\{x_1,\cdots,x_k\},\quad \boldsymbol{\theta}=\{\theta_1,\cdots,\theta_k\}$$

$$x_{k+1}=n-\sum_{l=1}^{k}x_l,\quad \theta_{k+1}=1-\sum_{l=1}^{k}\theta_l,$$

$$0<\theta_l<1,\quad \sum\theta_l<1,\quad n=1,2,\cdots,$$

$$x_l=0,1,2,\cdots,\quad \sum x_l\leq n,$$

$$\mathrm{E}\left[x_i\right]=n\theta_i,$$
$$\mathrm{Var}\left[x_j\right]=n\theta_i(1-\theta_i),$$
$$\mathrm{Cov}\left[x_i,x_j\right]=-n\theta_i\theta_j,$$

$$\mathrm{H}\left[x\right]=?$$

$$n\theta_i<\mathrm{M}\left[x_i\right]<(n+k-1)\theta_i,\quad i=1,\cdots,k$$

$$k=1:\qquad \mathbf{Mu}_1(x|\theta,n)=\mathbf{Bin}(x|\theta,n)$$

si $\quad p(x_1,\cdots,x_k)=\mathbf{Mu}_k\left(\boldsymbol{x}|\boldsymbol{\theta},n\right),\quad$ alors
$p(x_1,\cdots,x_m)=\mathbf{Mu}_m\left(x_1,\cdots,x_m|\theta_1,\cdots,\theta_m,n\right),\quad m\leq k,$ et
$p(x_1,\cdots,x_m|x_{m+1},\cdots,x_k)=\mathbf{Mu}_m\left(x_1,\cdots,x_m|\theta'_1,\cdots,\theta'_m,n'\right),$
avec $\theta'_i=\dfrac{\theta}{\sum_{j=1}^{m}\theta_j}\quad$ et $\quad n'=n-\displaystyle\sum_{j=m+1}^{k}\theta_j$

si on définit
$$y_1=x_1+\cdots+x_{i_1},$$
$$y_2=x_{i_1+1}+\cdots+x_{2_1},$$
$$\vdots$$
$$y_t=x_{i_{t-1}+1}+\cdots+x_{i_t},\quad 1\leq i_1\leq\cdots\leq i_t\leq k,$$
alors $\quad p(y_1,\cdots,y_t)=\mathbf{Mu}_t\left(y_1,\cdots,y_t|\theta'_1,\cdots,\theta'_t,n\right),$
avec
$$\theta'_1=\theta_1+\cdots+\theta_{i_1},$$
$$\theta'_2=\theta_{i_1+1}+\cdots+\theta_{2_1},$$
$$\vdots$$
$$\theta'_t=\theta_{i_{t-1}+1}+\cdots+\theta_{i_t}.$$

si $\quad \{\boldsymbol{x}_1,\cdots,\boldsymbol{x}_m\}\sim\mathbf{Mu}_k\left(\boldsymbol{x}|\boldsymbol{\theta},n_i\right),i=1,\cdots k,$
alors $\quad \boldsymbol{z}=\displaystyle\sum_{i=1}^{m}\boldsymbol{x}_i\sim\mathbf{Mu}_k\left(\boldsymbol{z}|\boldsymbol{\theta},\sum n_i\right)$

si $\quad \{x_1,\cdots,x_k\}\sim\mathbf{Pn}\left(x_i|\lambda_i\right),i=1,\cdots k,$
alors $\quad \left(x_1,\cdots,x_k|\sum x_i=n\right)\sim\mathbf{Mu}_k\left(\boldsymbol{x}|\boldsymbol{\theta},n\right),$ avec $\theta_i=\dfrac{\lambda_i}{\sum\lambda_j}$

Dirichlet

$$\mathbf{Di}_k\left(\boldsymbol{x}|\boldsymbol{\alpha}\right)=\frac{\Gamma\left(\sum_{l=1}^{k+1}\alpha_l\right)}{\prod_{l=1}^{k+1}\Gamma(\alpha_l)}\prod_{l=1}^{k+1}x_l^{\alpha_l-1},$$
$$\boldsymbol{x}=\{x_1,\cdots,x_k\},\quad \boldsymbol{\alpha}=\{\alpha_1,\cdots,\alpha_k,\alpha_{k+1}\}$$
$$\alpha_l>0, l=1,\cdots k+1$$
$$0<x_l<1, l=1,\cdots k+1 \quad x_{l+1}=1-\sum_{l=1}^{k}x_l,$$

$$\mathrm{E}\left[x_i\right]=\frac{\alpha_i}{\sum_{l=1}^{k+1}\alpha_l},$$
$$\mathrm{Var}\left[x_j\right]=\frac{\mathrm{E}\left[x_i\right]\left(1-\mathrm{E}\left[x_i\right]\right)}{1+\sum_{l=1}^{k+1}\alpha_l},$$
$$\mathrm{Cov}\left[x_i,x_j\right]=\frac{\mathrm{E}\left[x_i\right]\mathrm{E}\left[x_i\right]}{1+\sum_{l=1}^{k+1}\alpha_l},$$
$$\mathrm{M}\left[x_i\right]=\frac{\alpha_i-1}{\sum_{l=1}^{k+1}\alpha_l-k-1},\ i=1,\cdots,k$$

$$k=1: \qquad \mathbf{Di}_1(x|\alpha_1,\alpha_2)=\mathbf{Bet}(x|\alpha_1,\alpha_2)$$

si $\quad \boldsymbol{x}=\{x_1,\cdots,x_k\}\sim\mathbf{Di}_k\left(\boldsymbol{x}|\boldsymbol{\alpha}\right)$ alors

$$p(x_1,\cdots,x_m)=\mathbf{Di}_m\left(x_1,\cdots,x_m|\alpha_1,\cdots,\alpha_m,\sum_{j=m+1}^{k+1}\alpha_j\right),\quad m\leq k,\quad \text{et}$$
$$p(x_1',\cdots,x_m'|x_{m+1},\cdots,x_k)=\mathbf{Di}_m\left(x_1',\cdots,x_m'|\alpha_1,\cdots,\alpha_m,\alpha_{k+1}\right),$$
avec $x_i'=\dfrac{x_i}{1-\sum_{j=m+1}^{k}x_j}$
$$p(x_i'|x_{m+1},\cdots,x_k)=\mathbf{Bet}\left(x_i'|\alpha_i,\sum_{j=1}^{m}\alpha_j+\alpha_{k+1}-\alpha_i\right),\quad i=1,\cdots,m$$

si on définit
$$\begin{aligned} y_1&=x_1+\cdots+x_{i_1},\\ y_2&=x_{i_1+1}+\cdots+x_{2_1},\\ &\vdots\\ y_t&=x_{i_{t-1}+1}+\cdots+x_{i_t},\quad 1\leq i_1\leq\cdots\leq i_t\leq k,\end{aligned}$$
alors $p(y_1,\cdots,y_t)=\mathbf{Mu}_t\left(y_1,\cdots,y_t|\alpha_1',\cdots,\alpha_t'\right),$

avec
$$\begin{aligned} \alpha_1'&=\alpha_1+\cdots+\alpha_{i_1},\\ \alpha_2'&=\alpha_{i_1+1}+\cdots+\alpha_{2_1},\\ &\vdots\\ \alpha_t'&=\alpha_{i_{t-1}+1}+\cdots+\alpha_{i_t},\\ \alpha_{t+1}'&=\alpha_{k+1}.\end{aligned}$$

Multinomiale–Dirichlet

$$\mathbf{MuDi}_k\left(\boldsymbol{x}|\boldsymbol{\alpha},n\right) = \frac{n!}{\sum_{l=1}^{k+1}\alpha_l^{[n]}}\prod_{l=1}^{k+1}\frac{\alpha_l^{[x_l]}}{x_l!},$$

$$\boldsymbol{x} = \{x_1,\cdots,x_k\},\quad \boldsymbol{\alpha} = \{\alpha_1,\cdots,\alpha_k,\alpha_{k+1}\}$$

$$\alpha^{[s]} = \prod_{l=1}^{s}(\alpha+l-1),\quad x_{k+1} = n - \sum_{l=1}^{k}x_l,$$

$$\alpha_l > 0, n = 1,2,\cdots,$$

$$x_l = 0,1,2,\cdots,\quad \sum_{l=1}^{k}x_l < n,$$

$$\mathrm{E}\left[x_i\right] = np_i,\quad p_i = \frac{\alpha_i}{\sum_{l=1}^{k+1}\alpha_l},$$

$$\mathrm{Var}\left[x_j\right] = \frac{n+\sum_{l=1}^{k+1}\alpha_l}{1+\sum_{l=1}^{k+1}\alpha_l}np_i(1-p_i),$$

$$\mathrm{Cov}\left[x_i,x_j\right] = -\frac{n+\sum_{l=1}^{k+1}\alpha_l}{1+\sum_{l=1}^{k+1}\alpha_l}np_ip_j,$$

si $\quad\boldsymbol{x} = \{x_1,\cdots,x_k\} \sim \mathbf{MuDi}_k\left(\boldsymbol{x}|\boldsymbol{\alpha},n\right)$ alors

$$p(x_1,\cdots,x_m) = \mathbf{MuDi}_m\left(x_1,\cdots,x_m|\alpha_1,\cdots,\alpha_m,\sum_{j=1}^{k+1}\alpha_j - \sum_{j=1}^{m}\alpha_j, n\right),\quad m \leq k,\quad \text{et}$$

$$p(x_i) = \mathbf{BinBet}\left(x_i|\alpha_i,\sum_{j=1}^{k+1}\alpha_j - \alpha_i, n\right),\quad i = 1,\cdots,m \leq k$$

$$p(x_1,\cdots,x_m|x_{m+1},\cdots,x_k) = \mathbf{MuDi}_m\left(x_1,\cdots,x_m|\alpha_1,\cdots,\alpha_m,\sum_{j=1}^{k+1}\alpha_j - \sum_{j=s+1}^{k}\alpha_j, n - \sum_{j=1}^{s}x_j\right),$$

$$p(x_i|x_{m+1},\cdots,x_k) = \mathbf{BinBet}\left(x_i|\alpha_i,\sum_{j=1}^{m}\alpha_j + \alpha_{k+1} - \alpha_i\right),\quad i = 1,\cdots,m$$

$$p(x_{s+1},\cdots,x_k|x_1,\cdots,x_s) = \mathbf{MuDi}_{k-s}(x_{s+1},\cdots,x_k|\alpha_{s+1},\cdots,\alpha_k,\sum_{j=1}^{k+1}\alpha_j - \sum_{j=s+1}^{k}\alpha_j, n - \sum_{j=1}^{s}x_j)$$

si on définit
$$y_1 = x_1 + \cdots + x_{i_1},$$
$$y_2 = x_{i_1+1} + \cdots + x_{2_1},$$
$$\vdots$$
$$y_t = x_{i_{t-1}+1} + \cdots + x_{i_t},\quad 1 \leq i_1 \leq \cdots \leq i_t \leq k,$$

alors $\quad p(y_1,\cdots,y_t) = \mathbf{Mu}_t\left(y_1,\cdots,y_t|\alpha'_1,\cdots,\alpha'_t,n\right),$

avec
$$\alpha'_1 = \alpha_1 + \cdots + \alpha_{i_1},$$
$$\alpha'_2 = \alpha_{i_1+1} + \cdots + \alpha_{2_1},$$
$$\vdots$$
$$\alpha'_t = \alpha_{i_{t-1}+1} + \cdots + \alpha_{i_t},\quad 1 \leq i_1 \leq \cdots \leq i_t \leq k,$$

Lois de probabibilté à n variables réelles

Exponentielle

$\mathbf{Exfc}_n\,(\boldsymbol{x}|a,b,\boldsymbol{\phi}) = a(\boldsymbol{x})\exp\left[\boldsymbol{\phi}^t\boldsymbol{x} - b(\boldsymbol{\phi})\right]$,

$\boldsymbol{x} = \{x_1,\cdots,x_k\},\ x_i = h_i(x)$,

$\boldsymbol{\phi} = \{\phi_1,\cdots,\phi_k\},\ \phi_i = c_i\phi_i(\boldsymbol{\theta}), i = 1,\cdots,k$

$\mathrm{E}\left[\boldsymbol{x}\right] = \nabla b(\boldsymbol{\phi}), \quad \mathrm{E}\left[x_i\right] = \dfrac{\partial b(\boldsymbol{\phi})}{\partial \phi_i}$

$\mathrm{Var}\left[\boldsymbol{x}\right] = \nabla^2 b(\boldsymbol{\phi}), \quad \mathrm{Var}\left[x_i\right] = \dfrac{\partial^2 b(\boldsymbol{\phi})}{\partial {\phi_i}^2}$

$\mathrm{Cov}\left[x_i, x_j\right] = \dfrac{\partial^2 b(\boldsymbol{\phi})}{\partial \phi_i \partial \phi_j}$

si $\quad \boldsymbol{x}_1,\cdots,\boldsymbol{x}_n \sim \mathbf{Exfc}_n\,(\boldsymbol{x}_i|a,b,\boldsymbol{\phi})$

alors $\quad \boldsymbol{y} = \sum_{i=1}^{n} \boldsymbol{x}_i \sim \mathbf{Exfc}_n\left(\boldsymbol{y}|a^{[n]}, nb, \boldsymbol{\phi}\right)$,

avec

$a^{[n]}(\boldsymbol{x}) = a(\boldsymbol{x}) * \cdots * a(\boldsymbol{x}), \quad$ et

$nb(\boldsymbol{\phi}) = \log \int a^{[n]}(\boldsymbol{s}) \exp\left[\boldsymbol{\phi}^t\boldsymbol{s}\right]\,\mathrm{d}\boldsymbol{s}.$

Normale (paramètres : $\boldsymbol{\mu}, \boldsymbol{\Lambda}$)

$$\mathbf{N}_k\left(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}\right) = (2\pi)^{-\frac{k}{2}} \left|\boldsymbol{\Lambda}\right|^{\frac{1}{2}} \exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^t \boldsymbol{\Lambda}(\boldsymbol{x}-\boldsymbol{\mu})\right],$$

$\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0,$
$\boldsymbol{x} \in \mathbb{R}^k,$

$\mathrm{E}\left[\boldsymbol{x}\right] = \boldsymbol{\mu},$
$\mathrm{Var}\left[x_i\right] = (\boldsymbol{\Lambda}^{-1})_{ii},$
$\mathrm{Cov}\left[x_i, x_j\right] = (\boldsymbol{\Lambda}^{-1})_{ij},$

$\mathrm{M}\left[\boldsymbol{x}\right] = \boldsymbol{\mu},$

si $\quad \boldsymbol{x} \sim \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}), \quad$ alors $\quad z = (\boldsymbol{x}-\boldsymbol{\mu})^t \boldsymbol{\Lambda}(\boldsymbol{x}-\boldsymbol{\mu}) \sim \mathbf{khi}^2(z|k)$

si $\quad \boldsymbol{x} = \begin{pmatrix} \boldsymbol{x}_1 \\ \boldsymbol{x}_2 \end{pmatrix} \sim \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda})$

avec $\quad \boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \quad \boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} \\ \boldsymbol{\Lambda}_{21} & \boldsymbol{\Lambda}_{22} \end{pmatrix}$

alors
$p(\boldsymbol{x}_1) = \mathbf{N}_{k_1}(\boldsymbol{x}_1|\boldsymbol{\mu}_1, \boldsymbol{\Lambda}_{11}),$

$p(\boldsymbol{x}_2) = \mathbf{N}_{k_2}(\boldsymbol{x}_2|\boldsymbol{\mu}_2, \boldsymbol{\Lambda}_{22}),$

$p(\boldsymbol{x}_1|\boldsymbol{x}_2) = \mathbf{N}_{k_1}(\boldsymbol{x}_1|\boldsymbol{\mu}', \boldsymbol{\Lambda}_{11}) \quad$ avec $\quad \boldsymbol{\mu}' = \boldsymbol{\mu}_1 - \boldsymbol{\Lambda}_{11}^{-1}\boldsymbol{\Lambda}_{12}(\boldsymbol{x}_2 - \boldsymbol{\mu}_2),$

$p(\boldsymbol{x}_2|\boldsymbol{x}_1) = \mathbf{N}_{k_2}(\boldsymbol{x}_2|\boldsymbol{\mu}', \boldsymbol{\Lambda}_{22}) \quad$ avec $\quad \boldsymbol{\mu}' = \boldsymbol{\mu}_2 - \boldsymbol{\Lambda}_{22}^{-1}\boldsymbol{\Lambda}_{21}(\boldsymbol{x}_1 - \boldsymbol{\mu}_1),$

Normale (paramètres : $\boldsymbol{\mu}, \boldsymbol{\Sigma}$)

$$\mathbf{N}_k\left(\boldsymbol{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}\right) = (2\pi)^{-\frac{k}{2}}\,|\boldsymbol{\Sigma}|^{-\frac{1}{2}}\,\exp\left[-\frac{1}{2}(\boldsymbol{x}-\boldsymbol{\mu})^t\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu})\right],$$

$\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Sigma} > 0,$
$\boldsymbol{x} \in \mathbb{R}^k,$

$\mathrm{E}\left[\boldsymbol{x}\right] = \boldsymbol{\mu},$
$\mathrm{Var}\left[x_i\right] = \boldsymbol{\Sigma}_{ii},$
$\mathrm{Cov}\left[x_i, x_j\right] = \boldsymbol{\Sigma}_{ij},$

$\mathrm{M}\left[\boldsymbol{x}\right] = \boldsymbol{\mu},$

si $\quad \boldsymbol{x} \sim \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu},\boldsymbol{\Sigma}),\quad$ alors $\quad z = (\boldsymbol{x}-\boldsymbol{\mu})^t\boldsymbol{\Sigma}^{-1}(\boldsymbol{x}-\boldsymbol{\mu}) \sim \mathbf{Chi}_2(z|k)$

si $\quad \boldsymbol{x} = \begin{pmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\end{pmatrix} \sim \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu},\boldsymbol{\Sigma})$
avec $\quad \boldsymbol{\mu} = \begin{pmatrix}\boldsymbol{\mu}_1\\ \boldsymbol{\mu}_2\end{pmatrix},\quad \boldsymbol{\Sigma} = \begin{pmatrix}\boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12}\\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22}\end{pmatrix}$

alors
$p(\boldsymbol{x}_1) = \mathbf{N}_{k_1}(\boldsymbol{x}_1|\boldsymbol{\mu}_1,\boldsymbol{\Sigma}_{11}),$

$p(\boldsymbol{x}_2) = \mathbf{N}_{k_2}(\boldsymbol{x}_2|\boldsymbol{\mu}_2,\boldsymbol{\Sigma}_{22}),$

$p(\boldsymbol{x}_1|\boldsymbol{x}_2) = \mathbf{N}_{k_1}(\boldsymbol{x}_1|\boldsymbol{\mu}',\boldsymbol{\Sigma}_{11})\quad$ avec $\quad \boldsymbol{\mu}' = \boldsymbol{\mu}_1 - \boldsymbol{\Sigma}_{11}\boldsymbol{\Sigma}_{12}^{-1}(\boldsymbol{x}_2-\boldsymbol{\mu}_2),$

$p(\boldsymbol{x}_2|\boldsymbol{x}_1) = \mathbf{N}_{k_2}(\boldsymbol{x}_2|\boldsymbol{\mu}',\boldsymbol{\Sigma}_{22})\quad$ avec $\quad \boldsymbol{\mu}' = \boldsymbol{\mu}_2 - \boldsymbol{\Sigma}_{22}\boldsymbol{\Sigma}_{21}^{-1}(\boldsymbol{x}_1-\boldsymbol{\mu}_1),$

Student

$$\mathbf{St}_k\,(\boldsymbol{x}|\boldsymbol{\mu},\boldsymbol{\Lambda},\alpha) = \frac{\Gamma\,((\alpha+k)/2)}{\Gamma(\alpha/2)(\alpha\pi)^{k/2}}\,(\boldsymbol{\Lambda})^{\frac{1}{2}}\left[1+\frac{1}{\alpha}(\boldsymbol{x}-\boldsymbol{\mu})^t\boldsymbol{\Lambda}(\boldsymbol{x}-\boldsymbol{\mu})\right]^{-(\alpha+k)/2},$$

$\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha > 0, \alpha :$ nombre de dégrés de liberté

$\boldsymbol{x} \in \mathbb{R}^k$,

$\mathrm{E}\,[\boldsymbol{x}] = \boldsymbol{\mu}$,

$\mathrm{Var}\,[x_i] = (\boldsymbol{\Lambda}^{-1})_{ii}$,

$\mathrm{Cov}\,[x_i, x_j] = \dfrac{\alpha}{\alpha-2}(\boldsymbol{\Lambda}^{-1})_{ij}$,

$\mathrm{M}\,[x_i] =$

si $\quad \boldsymbol{x} \sim \mathbf{St}_k(\boldsymbol{x}|\boldsymbol{\mu},\boldsymbol{\Lambda},\alpha), \quad$ alors $\quad y = (\boldsymbol{x}-\boldsymbol{\mu})^t\boldsymbol{\Lambda}(\boldsymbol{x}-\boldsymbol{\mu}) \sim \mathbf{FS}_k(y|k,\alpha)$

si $\quad \boldsymbol{x} = \begin{pmatrix}\boldsymbol{x}_1\\ \boldsymbol{x}_2\end{pmatrix} \sim \mathbf{St}_k(\boldsymbol{x}|\boldsymbol{\mu},\boldsymbol{\Lambda},\alpha)$

avec $\quad \boldsymbol{\mu} = \begin{pmatrix}\boldsymbol{\mu}_1\\ \boldsymbol{\mu}_2\end{pmatrix}, \quad \boldsymbol{\Lambda} = \begin{pmatrix}\boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12}\\ \boldsymbol{\Lambda}_{21} & \boldsymbol{\Lambda}_{22}\end{pmatrix}$

alors

$p(\boldsymbol{x}_1) = \mathbf{St}_{k_1}(\boldsymbol{x}_1|\boldsymbol{\mu}_1,\boldsymbol{\Lambda}_{11},\Lambda)$,

$p(\boldsymbol{x}_2) = \mathbf{St}_{k_2}(\boldsymbol{x}_2|\boldsymbol{\mu}_2,\boldsymbol{\Lambda}_{22},\Lambda)$,

$p(\boldsymbol{x}_1|\boldsymbol{x}_2) \sim \mathbf{St}_k(\boldsymbol{x}_1|\boldsymbol{\mu}',\boldsymbol{\Lambda}',\alpha+k_2)$

avec $\quad \boldsymbol{\mu}' = \boldsymbol{\mu}_1 - \boldsymbol{\Lambda}_{11}^{-1}\boldsymbol{\Lambda}_{12}(\boldsymbol{x}_2-\boldsymbol{\mu}_2)$,

$$\boldsymbol{\Lambda}' = \frac{\alpha+k_2}{\alpha+(\boldsymbol{x}_2-\boldsymbol{\mu}_2)^t[\boldsymbol{\Lambda}_{22}-\boldsymbol{\Lambda}_{21}\boldsymbol{\Lambda}_{11}^{-1}\boldsymbol{\Lambda}_{12}]^{-1}(\boldsymbol{x}_2-\boldsymbol{\mu}_2)}\,\boldsymbol{\Lambda}_{11}$$

$p(\boldsymbol{x}_2|\boldsymbol{x}_1) \sim \mathbf{St}_k(\boldsymbol{x}_2|\boldsymbol{\mu}',\boldsymbol{\Lambda}',\alpha+k_1)$

avec $\quad \boldsymbol{\mu}' = \boldsymbol{\mu}_2 - \boldsymbol{\Lambda}_{22}^{-1}\boldsymbol{\Lambda}_{21}(\boldsymbol{x}_1-\boldsymbol{\mu}_1)$,

$$\boldsymbol{\Lambda}' = \frac{\alpha+k_1}{\alpha+(\boldsymbol{x}_1-\boldsymbol{\mu}_1)^t[\boldsymbol{\Lambda}_{11}-\boldsymbol{\Lambda}_{12}\boldsymbol{\Lambda}_{22}^{-1}\boldsymbol{\Lambda}_{21}]^{-1}(\boldsymbol{x}_1-\boldsymbol{\mu}_1)}\,\boldsymbol{\Lambda}_{22}$$

Wishart

$\mathbf{Wi}_k\left(\boldsymbol{X}|\alpha,\boldsymbol{\Lambda}\right) = c\,|\boldsymbol{X}|^{\alpha-(k+1)/2}\exp\left[-\mathrm{tr}(\boldsymbol{\Lambda}\boldsymbol{X})\right],$

$c = \dfrac{|\boldsymbol{\Lambda}|^{\alpha}}{\Gamma_k(\alpha)},\quad \text{avec}\quad \Gamma_k(\alpha) = \pi^{k(k-1)/4}\prod_{i=1}^{k}\Gamma\left(\dfrac{2\alpha+1-i}{2}\right)$

$\boldsymbol{\Lambda}$ une matrice de dimensions $k\times k$,

$\boldsymbol{X}$ une matrice symétrique d.p. de dimensions $k\times k$,

$X_{i,j} = X_{j,i},\quad i,j = 1,\cdots,k,$

$2\alpha > k-1,$

$\mathrm{E}\left[\boldsymbol{X}\right] = \alpha\boldsymbol{\Lambda}^{-1},$

$\mathrm{E}\left[\boldsymbol{X}^{-1}\right] = (\alpha-(k+1)/2)^{-1}\boldsymbol{\Lambda},$

$\mathrm{M}\left[X\right] =$

$k=1:\qquad \mathbf{Wi}_1(x|\alpha,\Lambda) = \mathbf{Gam}(x|\alpha,\Lambda)$

si $\quad \boldsymbol{X}\sim\mathbf{Wi}_k(\boldsymbol{X}|\alpha,\boldsymbol{\Lambda})$, alors

$\boldsymbol{Y} = \boldsymbol{A}\boldsymbol{X}\boldsymbol{A}^t \sim \mathbf{Wi}_m\left(\boldsymbol{Y}|\alpha,(\boldsymbol{A}\boldsymbol{X}^{-1}\boldsymbol{A}^t)^{-1}\right)$

si $\quad \{\boldsymbol{X}_1,\cdots,\boldsymbol{X}_n\},\quad \boldsymbol{X}_i\sim\mathbf{N}(\boldsymbol{X}_i|\boldsymbol{\mu},\boldsymbol{\Lambda})$, alors

$\bar{\boldsymbol{X}}_i = \dfrac{1}{n}\sum\boldsymbol{X}_i, \sim \mathbf{N}(\bar{\boldsymbol{X}}_i|\boldsymbol{\mu},n\boldsymbol{\Lambda}),$

$\boldsymbol{S} = (\boldsymbol{X}_i-\bar{\boldsymbol{X}}_i)^t(\boldsymbol{X}_i-\bar{\boldsymbol{X}}_i)\sim\mathbf{Wi}_k\left(\boldsymbol{S}|\dfrac{n-1}{2},\dfrac{1}{2}\boldsymbol{\Lambda}\right)$

si $\quad \boldsymbol{X} = \begin{pmatrix}\boldsymbol{X}_{11} & \boldsymbol{X}_{12}\\ \boldsymbol{X}_{21} & \boldsymbol{X}_{22}\end{pmatrix},\quad \boldsymbol{\Lambda}^{-1} = \begin{pmatrix}\boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12}\\ \boldsymbol{\Lambda}_{21} & \boldsymbol{\Lambda}_{22}\end{pmatrix},$

alors

$\boldsymbol{X}_{11}\sim\mathbf{Wi}_m\left(\boldsymbol{X}_{11}|\alpha,\boldsymbol{\Lambda}_{11}^{-1}\right)$

$\int |\boldsymbol{x}|^{\alpha-(k+1)/2}\exp\left[-\mathrm{tr}(\boldsymbol{\Lambda}\boldsymbol{x})\right]\,\mathrm{d}\boldsymbol{x} = c^{-1}$

Lois de probabibilté à $n+1$ variables réelles (suite)

Normal-Gamma

$\mathbf{NGam}_k\,(\boldsymbol{x}, y|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta) = \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu}, y\boldsymbol{\Lambda})\,\mathbf{Gam}(y|\alpha, \beta)$,
$\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha, \beta > 0$,
$\boldsymbol{x} \in \mathbb{R}^k, y > 0$,

$$p(\boldsymbol{x}) = \int_{-\infty}^{+\infty} p(\boldsymbol{x}, y)\,\mathrm{d}\boldsymbol{x} = \int_{-\infty}^{+\infty} \mathbf{NGam}_k\,(\boldsymbol{x}, y|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta)\ \mathrm{d}\boldsymbol{x} = \mathbf{St}\left(\boldsymbol{x}|\boldsymbol{\mu}, \frac{\alpha}{\beta}\boldsymbol{\Lambda}, 2\alpha\right),$$
$$p(y) = \int_0^{\infty} p(\boldsymbol{x}, y)\,\mathrm{d}y = \int_0^{\infty} \mathbf{NGam}_k\,(\boldsymbol{x}, y|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta)\ \mathrm{d}y = \mathbf{Gam}(y|\alpha, \beta),$$
$$p(\boldsymbol{x}|y) = \frac{p(\boldsymbol{x}, y)}{p(y)} = \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu}, y\boldsymbol{\Lambda}),$$

$$\mathrm{E}\left[\boldsymbol{x}\right] = \boldsymbol{\mu}, \quad \text{si } 2\alpha > 1, \quad \mathrm{Var}\left[\boldsymbol{x}\right] = \frac{2\beta}{2\alpha - 2}\frac{1}{\boldsymbol{\Lambda}} \quad \text{si } \alpha > 1$$
$$\mathrm{E}\left[y\right] = \frac{\alpha}{\beta}, \quad \mathrm{Var}\left[y\right] = \frac{\alpha}{\beta^2}$$
$$\mathrm{E}\left[\boldsymbol{x}\#2\right] = \boldsymbol{\mu}, \quad \mathrm{Var}\left[\boldsymbol{x}|y\right] = y\boldsymbol{\Lambda}$$

Normale-Wishart

$\mathbf{NWi}_k\,(\boldsymbol{x}, \boldsymbol{Y}|\boldsymbol{\mu}, \lambda, \alpha, \boldsymbol{B}) = \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu}, \lambda\boldsymbol{Y})\,\mathbf{Wi}_k(\boldsymbol{Y}|\alpha, \boldsymbol{B}),$
$\boldsymbol{\mu} \in \mathbb{R}^k, \lambda > 0, 2\alpha > k-1, \boldsymbol{B} > 0,$
$\boldsymbol{x} \in \mathbb{R}^k, Y_{i,j} = Y_{j,i}, i, j = 1, \cdots, k,$

$p(\boldsymbol{x}|\boldsymbol{Y}) = \mathbf{N}_k(\boldsymbol{x}|\boldsymbol{\mu}, \lambda\boldsymbol{Y}),$
$p(\boldsymbol{Y}) = \mathbf{Wi}_k(\boldsymbol{Y}|\alpha, \boldsymbol{B}),$
$p(\boldsymbol{x}) = \mathbf{St}_k(\boldsymbol{x}|\boldsymbol{\mu}, \lambda\alpha B^{-1}, 2\alpha),$

$\mathrm{E}\,[\boldsymbol{x}] = \boldsymbol{\mu}, \quad \text{si } \alpha > 1, \quad \mathrm{Var}\,[\boldsymbol{x}] = \dfrac{\alpha}{\alpha-2}\dfrac{1}{\boldsymbol{\lambda}} \quad \text{si } \alpha > 2$
$\mathrm{E}\,[Y] = \dfrac{\alpha}{B}, \quad \mathrm{Var}\,[Y] = \dfrac{\alpha}{B^2}$
$\mathrm{E}\,[\boldsymbol{x}\#2] = \boldsymbol{\mu}, \quad \mathrm{Var}\,[\boldsymbol{x}|Y] = \boldsymbol{\lambda}Y$

Index