

10.2 Lois de probabilités usuelles

10.2.1 Notations

Pour faciliter l'utilisation de ces tableaux, j'ai choisi une notation assez particulière qui est résumé dans les tableaux qui suivent.

Lois de probabilité à une variable discrète

Bernoulli	Ber ($x \theta$)
Binomiale	Bin ($x \theta, n$)
Hypergéométrique	HypGeo ($x N, M, n$)
Binomiale-négative	NegBin ($x \theta, r$)
Géométrique	Geo ($x \theta$)
Pascale	Pas ($x m, \theta$)
Poisson	Pn ($x \lambda$)
Binomiale-Bêta	BinBet ($x \alpha, \beta, n$)
Binomiale-Bêta négative	NegBinBet ($x \alpha, \beta, r$)
Poisson-Gamma	PnGam ($x \alpha, \beta, n$)
Poisson composée	Pnc ($x \lambda, \mu$)

Lois de probabilité à une variable réelle

Bêta	Bet ($x \alpha, \beta$)
Gamma	Gam ($x \alpha, \beta$)
Gamma inverse	IGam ($x \alpha, \beta$)
Gamma–Gamma	GamGam ($x \alpha, \beta, n$)
Pareto	Par ($x \alpha, \beta$)
Normale (paramètre précision)	N ($x \mu, \lambda$)
Normale (paramètre variance)	N ($x \mu, \sigma^2$)
du khi-deux	Chi ² ($x \nu$)
du khi-deux décentrée	Chi ² ($x \nu, \lambda$)
Logistic	Lo ($x \alpha, \beta$)
Student (t)	St ($x \mu, \lambda, \alpha$)
Fisher-Snedecor	FS ($x \alpha, \beta$)
Uniforme	Uni ($x \theta_1, \theta_2$)
Exponentielle	Ex ($x \lambda$)
du khi inverse	IChi ($x \nu$)
Racine carrée Gamma inverse	IGam ^{-1/2} ($x \alpha, \beta$)
Pareto inverse	IPar ($x \alpha, \beta$)
Cauchy	Cau ($x \lambda$)
Rayleigh	Ray ($x \theta$)
Log-Normale	LogN ($x \mu, \Lambda$)
Normale généralisée	Ngen ($x \alpha, \beta$)
Weibull	Wei ($x \alpha$)
Exponentielle double	Exd ($x \lambda$)
Exponentielle tronquée	Ext ($x \lambda$)
Triangulaire	Tri ($x \theta$)
du khi	IChi ($x \nu$)
Exponentielle généralisé à un paramètres	Exf ($x f, g, h, \phi, \theta$)
Exponentielle généralisé à K paramètres	Exfk ($x f, g, h, \phi, \theta$)

Lois de probabilité à deux variables réelles

Normale-Gamma	$\mathbf{NGam}(x, y \mu, \lambda, \alpha, \beta)$
Pareto bivariable	$\mathbf{Par}_2(x, y \alpha, \beta_0, \beta_1)$

Lois de probabilité à n variables discrètes

Multinomiale	$\mathbf{Mu}_k(\mathbf{x} \boldsymbol{\theta}, n)$
Dirichlet	$\mathbf{Di}_k(\mathbf{x} \boldsymbol{\theta})$
Multinomiale-Dirichlet	$\mathbf{MuDi}_k(\mathbf{x} \boldsymbol{\theta}, n)$

Lois de probabilité à n variables réelles

Exponentielle généralisée	$\mathbf{Exfc}_n(\mathbf{x} f, g, \mathbf{h}, \boldsymbol{\phi}, \boldsymbol{\theta})$
Normale (avec matrice de précision)	$\mathbf{N}_k(\mathbf{x} \boldsymbol{\mu}, \boldsymbol{\Lambda})$
Normale (avec matrice de covariance)	$\mathbf{N}_k(\mathbf{x} \boldsymbol{\mu}, \boldsymbol{\Sigma})$
Student	$\mathbf{St}_k(\mathbf{x} \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha)$
Wishart	$\mathbf{Wi}_k(\mathbf{X} \alpha, \boldsymbol{\Lambda})$

Lois de probabilité à $n + 1$ variables réelles

Normal-Gamma	$\mathbf{NGam}_k(\mathbf{x}, y \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta)$
Normale-Wishart	$\mathbf{NW}_k(\mathbf{x}, \mathbf{Y} \boldsymbol{\mu}, \lambda, \alpha, \mathbf{B})$

Lien entre les différentes lois

Bin ($x \theta, 1$) NegBin ($x \theta, 1$) BinBet ($x 1, 1, n$)	Ber ($x \theta$) Geo ($x \theta$) = Pas ($x \theta$) Unid ($x n$) = $\frac{1}{n+1}$, $x = 0, 1, \dots, n$
Bet ($x 1, 1$) Gam ($x 0, \beta$) Gam ($x \alpha, 1$) Gam ($x \frac{\nu}{2}, 1/2$) IGam ($x \frac{\nu}{2}, 1/2$) St ($x \mu, \lambda, 1$)	Uni ($x 0, 1$) Ex ($x \beta$) Erl ($x \alpha$) Chi ² ($x \nu$) IChi ² ($x \nu$) Cau ($x \mu, \lambda$)
Mu ₁ ($x \theta, n$) Di ₁ ($x \alpha_1, \alpha_2$) Wi ₁ ($x \alpha, \beta$) St ₁ ($x \mu, \lambda, \alpha$) N ₁ ($x \mu, \lambda$)	Bin ($x \theta, n$) Bet ($x \alpha_1, \alpha_2$) Gam ($x \alpha, \beta$) St ($x \mu, \lambda, \alpha$) N ($x \mu, \lambda$)
BinBet ($x \alpha, \beta, n$) NegBinBet ($x \alpha, \beta, r$) PnGam ($x \alpha, \beta, n$)	\int_0^1 Bin ($x \theta, n$) Beta ($\theta \alpha, \beta$) $d\theta$ \int_0^1 NegBin ($x \theta, r$) Beta ($\theta \alpha, \beta$) $d\theta$ \int_0^∞ Pn ($x n\lambda$) Gam ($\lambda \alpha, \beta$) $d\lambda$
Par ($x \alpha, \beta$) St ($x \mu, \lambda, \alpha$) Chi _{nc} ² ($x \mu, \lambda$)	\int_0^∞ Ex ($x - \beta \lambda$) Gam ($\lambda \alpha, \beta$) $d\lambda$ \int_0^∞ N ($x \mu, \lambda y$) Gam ($y \alpha/2, \beta/2$) dy $\sum_{i=1}^\infty$ Pn ($i \lambda/2$) Chi ² ($x \nu + 2i$)
$\lim_{\alpha \rightarrow \infty}$ St ($x \mu, \lambda, \alpha$) St ($x 0, 1, \alpha$)	N ($x \mu, \lambda$) loi de Student standard
si $x \sim$ Gam ($x \alpha, \beta$) si $x_i \sim$ Gam ($x_i \alpha_i, \beta$) si $x \sim$ N ($x 0, 1$) et $y \sim$ Chi ² ($x \nu$) si $x \sim$ Chi ² ($x \nu_1$) et $y \sim$ Chi ² ($y \nu_2$)	alors $y = \frac{x}{n} \sim$ IGam ($y \alpha, \beta$) alors $y = \sum_{i=1}^n x_i \sim$ Gam ($y \sum_{i=1}^n \alpha_i, \beta$) alors $z = \frac{x}{\sqrt{x/\nu}} \sim$ St ($z 0, 1, \nu$) alors $z = \frac{x/\nu_1}{y/\nu_2} \sim$ FS ($z \nu_1, \nu_2$)

Famille des lois position-échelle			
$p(x \mu, \beta) = \frac{1}{\beta}f(t)$ avec $t = \frac{x-\mu}{\beta}$			
Var $[x] = \beta^2v$ H $[x] = \log \beta + h$			
Famille	$f(t)$	v	h
Normale	$(2\pi)^{-\frac{1}{2}} \exp\left[-\frac{t^2}{2}\right]$	1	$\frac{1}{2} \log(2\pi e)$
Gumbel	$\exp[-t] \exp[-\exp[-t]]$	$\pi^2/6$	$1 + \gamma$
Laplace	$\frac{1}{2} \exp[- t]$	2	$1 + \log 2$
Logistic	$\frac{\exp[-t]}{(1+\exp[-t])^2}$	$\pi^2/3$	2
Exponentielle	$\exp[-t], \quad x > 0$	1	1
Uniforme	$1, \quad \mu - \frac{\beta}{2} < x < \mu + \frac{\beta}{2}$	1/12	0

Famille des lois forme-échelle			
$p(x \alpha, \beta) = \frac{1}{\beta}f(t; \alpha)$ avec $t = \frac{x}{\beta}$			
Var $[x] = \beta^2v(\alpha)$ H $[x] = \log \beta + h(\alpha)$			
Famille	$f(t)$	v	h
Normale généralisée $\alpha = 2$: Rayleigh, $\alpha = 3$: Maxwell-Boltzmann, $\alpha = \nu$: khi	$\frac{2}{\Gamma(\frac{\alpha}{2})} t^{\alpha-1} \exp[-t^2]$	$\frac{\alpha-2\Gamma^2(\frac{\alpha+1}{2})}{\Gamma^2(\frac{\alpha}{2})}$	$\log[\Gamma(\frac{\alpha}{2})/2] + \frac{1-\alpha}{2}\psi(\frac{\alpha}{2}) + \frac{\alpha}{2}$
Normale généralisée inverse $\alpha = \nu, \beta = \sqrt{2}$: khi inverse	$\frac{2}{\Gamma(\frac{\alpha}{2})} t^{-\alpha-1} \exp[-t^{-2}]$	$\frac{1}{\alpha-2} - \alpha \frac{\Gamma^2(\frac{\alpha-1}{2})}{\Gamma^2(\frac{\alpha}{2})}$	$\log[\Gamma(\frac{\alpha}{2})/2] + \frac{-\alpha}{2}\psi(\frac{\alpha}{2}) + \frac{\alpha}{2}$
Gamma $\alpha = 1$: Exponentielle, $\alpha = \frac{\nu}{2}, \beta = 2$: khi-deux, $\alpha = \nu$: Erlang	$\frac{1}{\Gamma(\alpha)} t^{\alpha-1} \exp[-t]$	α	$\log[\Gamma(\alpha)] + (1-\alpha)\psi(\alpha) + \alpha$
Gamma inverse $\alpha = \frac{\nu}{2}, \beta = \frac{1}{2}$: khi-deux inverse	$\frac{1}{\Gamma(\alpha)} t^{-\alpha+1} \exp[-t^{-1}]$	$\frac{1}{(\alpha-1)^2(\alpha-2)}, \alpha > 2$	$\log[\Gamma(\alpha)] - (1+\alpha)\psi(\alpha) + \alpha$
Pareto	$\alpha t^{-\alpha-1}, x > \beta$	$\frac{1}{(\alpha-1)^2(\alpha-2)}, \alpha > 2$	$\frac{1}{\alpha} - \log \alpha + 1$
Weibull	$\alpha t^{\alpha-1} \exp[-t]^\alpha$	$\Gamma(1 + \frac{2}{\alpha})\Gamma^2(1 + \frac{1}{\alpha})$	$\frac{\gamma(\alpha-1)}{\alpha} - \log \alpha + 1$

10.2.2 Notations détaillées

Lois de probabilité à une variable discrète

Bernoulli	$\mathbf{Ber}(x \theta) = \theta^x (1 - \theta)^{1-x},$ $0 < \theta < 1,$ $x = \{0, 1\}$
Binomiale	$\mathbf{Bin}(x \theta, n) = c \theta^x (1 - \theta)^{n-x},$ $c = \binom{n}{x},$ $0 < \theta < 1, n = 1, 2, \dots,$ $x = 0, 1, \dots, n$
Hypergéométrique	$\mathbf{HypGeo}(x N, M, n) = c \binom{N}{x} \binom{M}{n-x},$ $c = \binom{N+M}{n}^{-1},$ $N, M = 1, 2, \dots, n = 1, \dots, N+M,$ $x = a, a+1, \dots, b,$ avec $a = \max\{0, n-M\}, b = \min\{N, n\}$
Binomiale-négative	$\mathbf{NegBin}(x \theta, r) = c \binom{r+x-1}{r-1} (1 - \theta)^x,$ $c = \theta^r,$ $0 < \theta < 1, r = 1, 2, \dots,$ $x = 0, 1, 2, \dots$
Poisson	$\mathbf{Pn}(x \lambda) = \frac{\lambda^x}{x!},$ $c = \exp[-\lambda],$ $\lambda > 0,$ $x = 0, 1, 2, \dots$
Binomiale-Bêta	$\mathbf{BinBet}(x \alpha, \beta, n) = c \binom{n}{x} \Gamma(\alpha+x)\Gamma(\beta+n-x),$ $c = \frac{\Gamma(\alpha+\beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha+\beta+n)},$ $\alpha, \beta > 0, n = 1, 2, \dots,$ $x = 0, \dots, n$

Lois de probabilité à une variable discrète (suite)

Binomiale-Bêta négative	$\mathbf{NegBinBet}(x \alpha, \beta, r) = c \binom{r+x-1}{r-1} \frac{\Gamma(\beta+x)}{\Gamma(\alpha+\beta+x+\alpha)},$ $c = \frac{\Gamma(\alpha+\beta)\Gamma(\alpha+r)}{\Gamma(\alpha)\Gamma(\beta)},$ $\alpha, \beta > 0, r = 1, 2, \dots,$ $x = 0, 1, 2, \dots$
Poisson-Gamma	$\mathbf{PnGam}(x \alpha, \beta, n) = c \frac{\Gamma(\alpha+x)}{x!} \frac{n^x}{(\beta+n)^{\alpha+x}},$ $c = \frac{\beta^\alpha}{\Gamma(\alpha)},$ $\alpha, \beta > 0, n = 0, 1, 2, \dots,$ $x = 0, 1, 2, \dots$
Poisson composée	$\mathbf{Pnc}(x \lambda, \mu) = \exp[-\lambda] \sum_{n=0}^{\infty} \frac{(n\mu)^x \exp[-n\mu]}{x!} \frac{\lambda^n}{n!},$ $\lambda, \mu > 0, \quad x = 0, 1, 2, \dots$
Géométrique	$\mathbf{Geo}(x \theta) = c(1-\theta)^{x-1},$ $c = \theta,$ $\theta > 0, \quad x = 0, 1, 2, \dots$
Pascale	$\mathbf{Pas}(x m, \theta) = C_{x-1}^{m-1} \theta^m (1-\theta)^{x-m},$ $m > 0, 0 < \theta < 1 \quad x = 0, 1, 2, \dots$

Lois de probabilité à une variable réelle

Bêta	$\mathbf{Bet}(x \alpha, \beta) = c x^{\alpha-1} (1-x)^{\beta-1},$ $c = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)},$ $\alpha, \beta > 0,$ $0 < x < 1$
Gamma	$\mathbf{Gam}(x \alpha, \beta) = c x^{\alpha-1} \exp[-\beta x],$ $c = \frac{\beta^\alpha}{\Gamma(\alpha)},$ $\alpha, \beta > 0,$ $x > 0$
Gamma inverse	$\mathbf{IGam}(x \alpha, \beta) = c x^{-(\alpha+1)} \exp\left[-\frac{\beta}{x}\right],$ $c = \frac{\beta^\alpha}{\Gamma(\alpha)},$ $\alpha, \beta > 0,$ $x > 0$
Gamma-Gamma	$\mathbf{GamGam}(x \alpha, \beta, n) = c \frac{x^{n-1}}{(\beta+x)^{\alpha+n}},$ $c = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha+n)}{\Gamma(n)},$ $\alpha, \beta > 0, n = 0, 1, 2, \dots,$ $x = 0, 1, 2, \dots$
Pareto	$\mathbf{Par}(x \alpha, \beta) = c x^{-(\alpha+1)},$ $c = \alpha\beta^\alpha,$ $\alpha, \beta > 0,$ $x \geq \beta$
Normale	$\mathbf{N}(x \mu, \lambda) = c \exp\left[-\frac{1}{2}\lambda(x-\mu)^2\right],$ $c = \sqrt{\frac{\lambda}{2\pi}},$ $\mu \in \mathbb{R}, \lambda > 0,$ $x \in \mathbb{R}$
Normale	$\mathbf{N}(x \mu, \sigma) = c \exp\left[-\frac{1}{2\sigma^2}(x-\mu)^2\right],$ $c = \frac{1}{\sqrt{2\pi\sigma^2}},$ $\mu \in \mathbb{R}, \sigma > 0,$ $x \in \mathbb{R}$
du khi-deux	$\mathbf{Chi}^2(x n) = c x^{n/2-1} \exp\left[-\frac{x}{2}\right],$ $c = \frac{1}{2} \frac{1}{\Gamma(n/2)},$ $n > 0, x > 0$
du khi-deux décentrée	$\mathbf{Chi}^2(x \nu, \lambda) = \sum_{i=0}^{\infty} \mathbf{Pn}\left(x \frac{\lambda}{2}\right) \chi^2(x \nu+2i),$ $\nu, \lambda > 0,$ $x > 0$

Lois de probabilité à une variable réelle (suite)

Logistic	$\mathbf{Lo}(x \alpha, \beta) = c \frac{\exp[-\beta^{-1}(x - \alpha)]}{(1 + \exp[\beta^{-1}(x - \alpha)])^2},$ $c = \beta^{-1},$ $\alpha \in \mathbb{R}, \beta > 0,$ $x \in \mathbb{R}$
Student (t)	$\mathbf{St}(x \mu, \lambda, \alpha) = c \left[1 + \frac{\lambda}{\alpha}(x - \mu)^2\right]^{-(\alpha+1)/2},$ $c = \frac{\Gamma(\frac{\alpha+1}{2})}{\Gamma(\alpha/2)\Gamma(1/2)} \left(\frac{\lambda}{\alpha}\right)^{1/2}$ $\mu \in \mathbb{R}, \lambda, \alpha > 0,$ $x \in \mathbb{R}$
Fisher-Snedecor	$\mathbf{FS}(x \alpha, \beta) = c \frac{x^{\alpha/2-1}}{(\beta + \alpha x)^{(\alpha+\beta)/2}},$ $c = \frac{\Gamma((\alpha + \beta)/2)}{\Gamma(\alpha/2)\Gamma(\beta/2)} \alpha^{\alpha/2} \beta^{\beta/2},$ $\alpha, \beta > 0,$ $x > 0$
Uniforme	$\mathbf{Uni}(x \theta_1, \theta_2) = c$ $c = \frac{1}{\theta_2 - \theta_1},$ $\theta_2 > \theta_1,$ $\theta_1 < x < \theta_2$
Exponentielle	$\mathbf{Ex}(x \lambda) = c \exp[-\lambda x],$ $c = \lambda,$ $\lambda > 0, \quad x > 0$
du khi inverse	$\mathbf{IChi}(x \nu) = c x^{-(\nu/2+1)} \exp\left[-\frac{1}{2x^2}\right],$ $c = \frac{1^{\nu/2}}{\Gamma(\nu/2)},$ $\nu > 0, \quad x > 0$
Racine carrée Gamma inverse	$\mathbf{IGam}^{-1/2}(x \alpha, \beta) = c x^{-(2\alpha+1)} \exp\left[-\frac{\beta}{x^2}\right],$ $c = \frac{2\beta^\alpha}{\Gamma(\alpha)},$ $\alpha, \beta > 0,$ $x > 0$
Pareto inverse	$\mathbf{IPar}(x \alpha, \beta) = c x^{\alpha-1},$ $c = \alpha\beta^\alpha,$ $\alpha, \beta > 0,$ $0 < x < \beta^{-1}$

Lois de probabilité à une variable réelle (suite)

Cauchy	$\mathbf{Cau}(x \lambda) = \frac{1/(\pi\lambda)}{1 + (x/\lambda)^2},$ $\lambda \in \mathbb{R}, \quad x \in \mathbb{R}$
Rayleigh	$\mathbf{Ray}(x \theta) = c x \exp\left[-\frac{x^2}{\theta}\right],$ $c = \frac{2}{\theta},$ $\theta > 0, \quad x > 0$
Log-Normale	$\mathbf{LogN}(x \mu, \Lambda) = c \exp\left[-\frac{(\ln x - \mu)^2}{2\Lambda^2}\right],$ $c = \frac{1}{\Lambda\sqrt{2\pi x}},$
Normale généralisée	$\mathbf{Ngen}(x \alpha, \beta) = c x^{\alpha-1} \exp[-\beta x^2],$ $c = \frac{2\beta^{\alpha/2}}{\Gamma(\alpha/2)},$
Weibull	$\mathbf{Wei}(x \alpha) = c x^{\alpha-1} \exp[-x^\beta/\alpha],$ $c = \frac{\beta}{\alpha},$ $\alpha, \beta > 0, \quad x > 0$
Exponentielle double	$\mathbf{Exd}(x \lambda) = c \exp[- \lambda x],$ $c = \frac{\lambda}{2},$ $\lambda > 0, \quad x \in \mathbb{R}$
Exponentielle tronquée	$\mathbf{Ext}(x \lambda) = \exp[-(x - \lambda)],$ $\lambda > 0, \quad x > \lambda$
du khi	$\mathbf{Chi}(x n) = c x^{n/2-1} \exp\left[-\frac{x}{2}\right],$ $c = \frac{1^{n/2}}{\Gamma(n/2)},$ $n > 0, \quad x > 0$

Lois de probabilité à une variable réelle (suite)

Exponentielle généralisé à un paramètre	$\mathbf{Exp}(x f, g, h, \phi, \theta) = f(x) g(\theta) \exp [\phi(\theta) h(x)],$
Exponentielle généralisé à K paramètres	$\mathbf{Expk}(x f, g, \mathbf{h}, \phi, \boldsymbol{\theta}) = f(x) g(\boldsymbol{\theta}) \exp \left[\sum_{k=1}^K \phi_k(\boldsymbol{\theta}) h_k(x) \right],$

Lois de probabilité à deux variables réelles

Normale-Gamma	$\mathbf{NGam}(x, y \mu, \lambda, \alpha, \beta) = \mathbf{N}(x \mu, \lambda y) \mathbf{Gam}(y \alpha, \beta),$ $\mu \in \mathbf{R}, \lambda, \alpha, \beta > 0,$ $x \in \mathbf{R}, y > 0$
Pareto bivariable	$\mathbf{Par}_2(x, y \alpha, \beta_0, \beta_1) = (y - x)^{-(\alpha+2)},$ $c = \alpha(\alpha + 1)(\beta_1 - \beta_0)^\alpha,$ $(\beta_0, \beta_1) \in \mathbf{R}^2, \beta_0 < \beta_1, \alpha > 0,$ $(x, y) \in \mathbf{R}^2, x < \beta_0, y > \beta_1$

Lois de probabilité à n variables discrètes

Multinomiale	$\mathbf{Mu}_k(\mathbf{x} \boldsymbol{\theta}, n) = \frac{n!}{\prod_{l=1}^{k+1} x_l!} \prod_{l=1}^{k+1} \theta_l^{x_l},$ $x_{k+1} = n - \sum_{l=1}^k x_l, \quad \theta_{k+1} = 1 - \sum_{l=1}^k \theta_l,$ $0 < \theta_l < 1, \quad \sum_{l=1}^k \theta_l < 1, \quad n = 1, 2, \dots,$ $x_l = 0, 1, 2, \dots, \quad \sum_{l=1}^k x_l \leq n$
Dirichlet	$\mathbf{Di}_k(\mathbf{x} \boldsymbol{\alpha}) = c \prod_{l=1}^{k+1} x_l^{\alpha_l - 1},$ $c = \frac{\Gamma\left(\sum_{l=1}^{k+1} \alpha_l\right)}{\prod_{l=1}^{k+1} \Gamma(\alpha_l)},$ $\alpha_l > 0, l = 1, \dots, k + 1$ $0 < x_l < 1, l = 1, \dots, k + 1 \quad x_{l+1} = 1 - \sum_{l=1}^k x_l$
Multinomiale-Dirichlet	$\mathbf{MuDi}_k(\mathbf{x} \boldsymbol{\alpha}, n) = c \prod_{l=1}^{k+1} \frac{\alpha_l^{[x_l]}}{x_l!},$ $c = \frac{n!}{\sum_{l=1}^{k+1} \alpha_l^{[n]}},$ $\alpha^{[s]} = \prod_{l=1}^s (\alpha + l - 1), \quad x_{k+1} = n - \sum_{l=1}^k x_l,$ $\alpha_l > 0, n = 1, 2, \dots,$ $x_l = 0, 1, 2, \dots, \quad \sum_{l=1}^k x_l < n$

Lois de probabilité à n variables réelles

Exponentielle	Exfc ($x a, b, \theta$)
Exponentielle généralisée	Exfc ($x f, g, h, \phi, \theta$)
Normale	$\mathbf{N}_k(\mathbf{x} \boldsymbol{\mu}, \boldsymbol{\Lambda}) = c \exp \left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Lambda}(\mathbf{x} - \boldsymbol{\mu}) \right],$ $c = \boldsymbol{\Lambda} ^{1/2} (2\pi)^{-\frac{k}{2}},$ $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0,$ $\mathbf{x} \in \mathbb{R}^k$
Normale	$\mathbf{N}_k(\mathbf{x} \boldsymbol{\mu}, \boldsymbol{\Sigma}) = c \exp \left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) \right],$ $c = \boldsymbol{\Sigma} ^{-1/2} (2\pi)^{-\frac{k}{2}},$ $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Sigma} > 0,$ $\mathbf{x} \in \mathbb{R}^k$
Student	$\mathbf{St}_k(\mathbf{x} \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha) = c \left[1 + \frac{1}{\alpha}(\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Lambda}(\mathbf{x} - \boldsymbol{\mu}) \right]^{-(\alpha+k)/2},$ $c = \frac{\Gamma((\alpha+k)/2)}{\Gamma(\alpha/2)(\alpha\pi)^{k/2}} \left(\frac{\boldsymbol{\Lambda}}{\alpha} \right)^{1/2},$ $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha > 0,$ $\mathbf{x} \in \mathbb{R}^k$
Wishart	$\mathbf{Wi}_k(\mathbf{X} \alpha, \boldsymbol{\Lambda}) = c \mathbf{X} ^{\alpha-(k+1)/2} \exp[-\text{tr}(\boldsymbol{\Lambda}\mathbf{X})],$ $c = \frac{ \boldsymbol{\Lambda} ^\alpha}{\Gamma_k(\alpha)},$ $\boldsymbol{\Lambda} \text{ une matrice de dimensions } k \times k,$ $\mathbf{X} \text{ une matrice symétrique d.p. de dimensions } k \times k,$ $X_{i,j} = X_{j,i}, \quad i, j = 1, \dots, k,$ $2\alpha > k - 1$

Lois de probabilité à $n + 1$ variables réelles

Normal-Gamma	$\mathbf{NGam}_k(\mathbf{x}, y \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta) = \mathbf{N}_k(\mathbf{x} \boldsymbol{\mu}, y\boldsymbol{\Lambda}) \mathbf{Gam}(y \alpha, \beta),$ $\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha, \beta > 0,$ $\mathbf{x} \in \mathbb{R}^k, y > 0$
Normale-Wishart	$\mathbf{NWi}_k(\mathbf{x}, \mathbf{Y} \boldsymbol{\mu}, \lambda, \alpha, \mathbf{B}) = \mathbf{N}_k(\mathbf{x} \boldsymbol{\mu}, \lambda\mathbf{Y}) \mathbf{Wi}_k(\mathbf{Y} \alpha, \mathbf{B}),$ $\boldsymbol{\mu} \in \mathbb{R}^k, \lambda > 0, 2\alpha > k - 1, \mathbf{B} > 0,$ $\mathbf{x} \in \mathbb{R}^k, Y_{i,j} = Y_{j,i}, i, j = 1, \dots, k$

10.2.3 Définitions

Bernoulli

$$\mathbf{Ber}(x|\theta) = \theta^x (1 - \theta)^{1-x},$$

$$0 < \theta < 1,$$

$$x = \{0, 1\},$$

$$\begin{cases} \Pr\{X = 1\} = \theta, \\ \Pr\{X = 0\} = 1 - \theta \end{cases}$$

$$\mathbf{E}[x] = \theta,$$

$$\mathbf{Var}[x] = \theta(1 - \theta),$$

$$\mathbf{M}[x] = \theta$$

$$\mathbf{S}[x] = \frac{1 - 2\theta}{\sqrt{\theta(1 - \theta)}}$$

$$\mathbf{Kp}[x] = \frac{1 - 6\theta(1 - \theta)}{\theta(1 - \theta)}$$

$$\mathbf{H}[x] = \theta \log(\theta) + (1 - \theta) \log(1 - \theta)$$

$$1 - \beta$$

0 1 x

Binomiale

$$\mathbf{Bin}(x|\theta, n) = \binom{n}{x} \theta^x (1 - \theta)^{n-x},$$

$$0 < \theta < 1, n = 1, 2, \dots,$$

$$x = 0, 1, \dots, n,$$

$$\begin{cases} \Pr\{X = 0\} = (1 - \theta)^n, \\ \Pr\{X = x\} = \binom{n}{x} \theta^x (1 - \theta)^{n-x} \\ \Pr\{X = n\} = \theta^n \end{cases}$$

$$E[x] = n\theta,$$

$$\text{Var}[x] = n\theta(1 - \theta),$$

$$M[x] = \text{Floor}[(n + 1)\theta],$$

$$S[x] = \frac{1 - 2\theta}{\sqrt{n\theta(1 - \theta)}}$$

$$K[x] = \frac{1 - 6\theta(1 - \theta)}{n\theta(1 - \theta)}$$

$$\text{Cumulants} : K_1 = n\theta, \quad K_{r+1} = \theta(\theta - 1) \frac{\partial K_r}{\partial \theta}, r \geq 1$$

$$\text{Fonction caractéristique} : F(t) = (1 - \theta + \theta e^{-it})^n$$

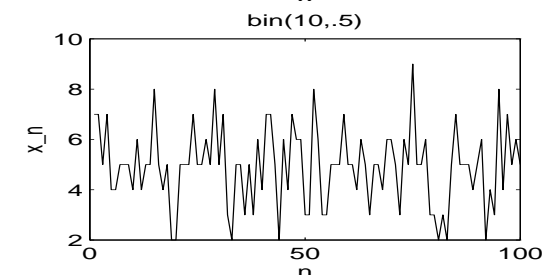
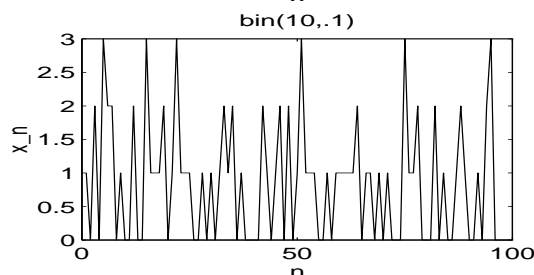
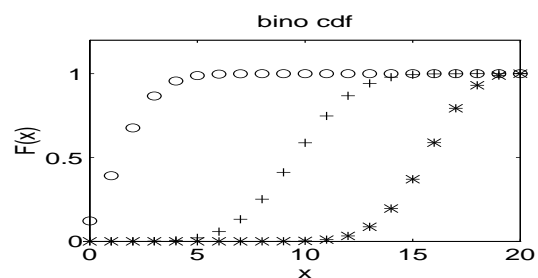
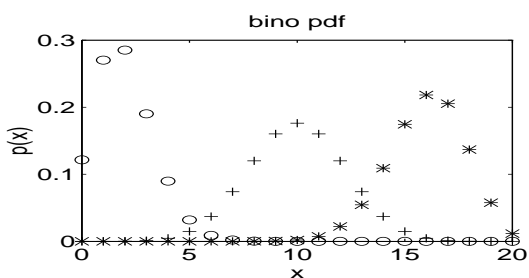
$$H[x] = ?$$

$$\text{Autres notations} : p = \theta, q = 1 - \theta$$

$$n = 1 : \quad \mathbf{Bernoulli}, \quad \mathbf{Bin}(x|\theta, 1) = \mathbf{Ber}(x|\theta),$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{Bin}(x|\theta, n_i), i = 1, \dots, k,$$

$$\text{alors } s = \sum x_i \sim \mathbf{Bin}(s|\theta, \sum n_i)$$



Hypergéométrique

$$\text{HypGeo}(x|N, M, n) = \binom{N+M}{n}^{-1} \binom{N}{x} \binom{M}{n-x},$$

$$N, M = 1, 2, \dots, n = 1, \dots, N+M,$$

$$x = a, a+1, \dots, b, \text{ avec } a = \max\{0, n-M\}, b = \min\{N, n\},$$

$$\begin{cases} \Pr\{X = a\} = ?, \\ \Pr\{X = x\} = ? \\ \Pr\{X = b\} = ? \end{cases}$$

$$\mathbb{E}[x] = \frac{Nn}{N+M},$$

$$\text{Var}[x] = \frac{nNM}{(N+M)^2} \frac{N+M-n}{N+M-1},$$

$$M[x] = \text{Floor} \left[\frac{(n+1)(N+1)}{N+M+2} \right]$$

$$S[x] = ?$$

$$K[x] = ?$$

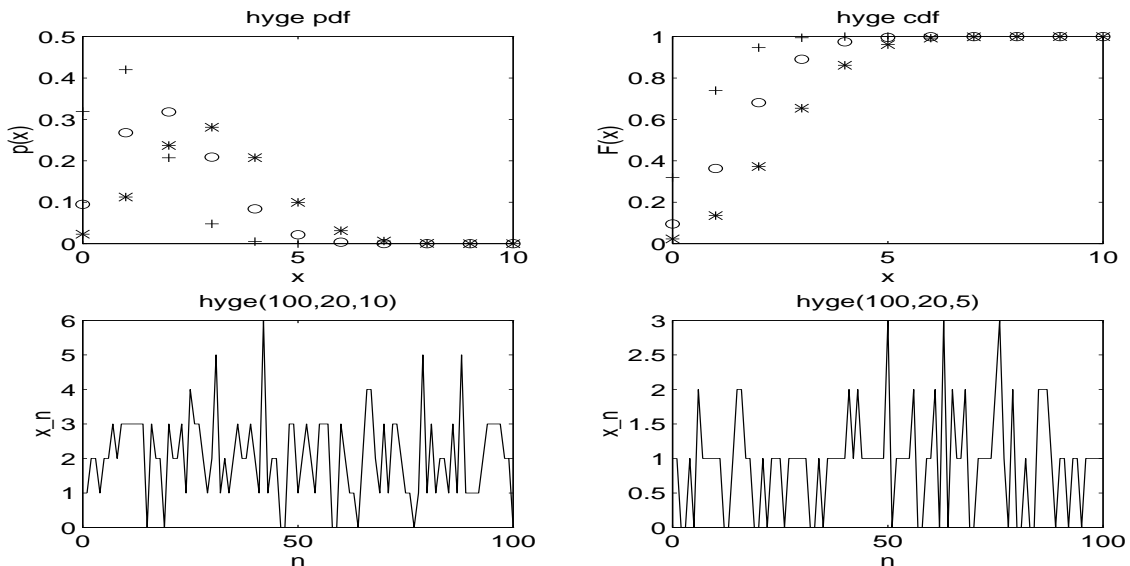
$$H[x] = ?$$

$$\text{Autres notations : } p = \frac{N}{N+M}, q = 1-p = \frac{M}{N+M},$$

$$\mathbb{E}[x] = np,$$

$$\text{Var}[x] = npq \frac{N+M-n}{N+M-1},$$

$$S[x] = \frac{q-p}{\sqrt{npq}} \left(\frac{N+M-1}{N+M-n} \right)^{1/2} \frac{N+M-2n}{N+M-2}$$



Binomiale-négative

$$\mathbf{NegBin}(x|\theta, r) = \theta^r \binom{r+x-1}{r} (1-\theta)^x,$$

$$0 < \theta < 1, r = 1, 2, \dots,$$

$$x = 0, 1, 2, \dots,$$

$$\begin{cases} \Pr\{X = 0\} = \theta^r, \\ \Pr\{X = 1\} = r\theta^r \\ \Pr\{X = n\} = \theta^r \binom{r+n-1}{r-1} (1-\theta)^n \end{cases}$$

$$E[x] = r \frac{1-\theta}{\theta},$$

$$\text{Var}[x] = r \frac{1-\theta}{\theta^2},$$

$$M[x] = \begin{cases} 0, 1 & \text{si } r(1-\theta) = 0 \\ 0 & \text{si } r(1-\theta) < 0 \\ \frac{r(1-\theta)}{\theta} & \text{si } r(1-\theta) > 1 \end{cases}$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = ?$$

$$\text{Autres notations : } p = \theta, Q = \frac{1}{p}, P = \frac{1-p}{p}$$

$$E[x] = rP,$$

$$\text{Var}[x] = rPQ$$

$$S[x] = \frac{Q+P}{\sqrt{rPQ}}$$

$$K[x] = \frac{1+6PQ}{rPQ}$$

$$\text{Cumulants : } K_1 = rP, \quad K_{n+1} = PQ \frac{\partial K_n}{\partial Q}, n \geq 1$$

$$\text{Fonction caractéristique : } F(t) = (Q - Pe^{it})^{-r}$$

$r = 1$: **Géométrique ou Pascal**

$$\mathbf{NegBin}(x|\theta, 1) = \mathbf{Geo}(x|\theta),$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{NegBin}(x|\theta, r_i), i = 1, \dots, k,$$

$$\text{alors } s = \sum x_i \sim \mathbf{NegBin}(s|\theta, \sum r_i)$$

Poisson

$$\mathbf{Pn}(x|\lambda) = \exp[-\lambda] \frac{\lambda^x}{x!},$$

$$\lambda > 0,$$

$$x = 0, 1, 2, \dots,$$

$$\begin{cases} \Pr\{X = 0\} = \exp[-\lambda], \\ \Pr\{X = 1\} = \lambda \exp[-\lambda] \\ \Pr\{X = n\} = \exp[-\lambda] \frac{\lambda^n}{n!} \end{cases}$$

$$\mathbf{E}[x] = \lambda,$$

$$\mathbf{Var}[x] = \lambda,$$

$$\mathbf{M}[x] = \lambda$$

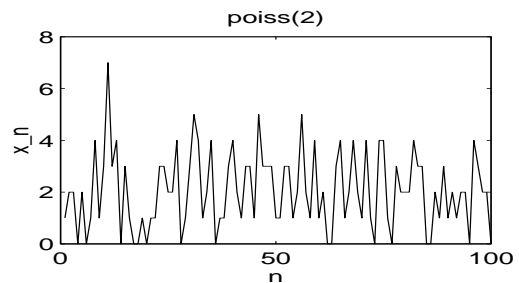
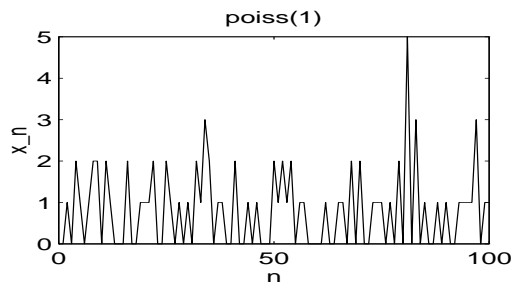
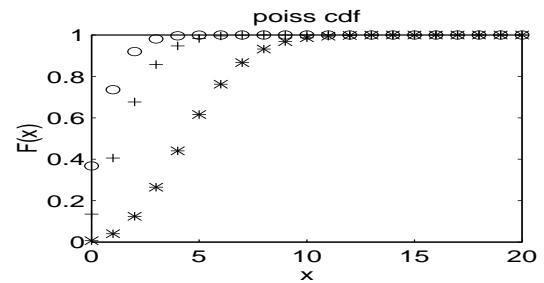
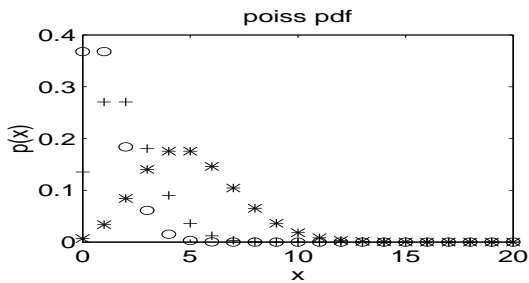
$$\mathbf{S}[x] = \lambda^{-1/2}$$

$$\mathbf{K}[x] = \lambda^{-1}$$

Fonction caractéristique : $F(t) = e^\lambda(e^{it} - 1)$

$\mathbf{H}[x] = ?$

si $\{x_1, \dots, x_k\} \sim \mathbf{Pn}(x|\lambda_i), i = 1, \dots, k,$
alors $s = \sum x_i \sim \mathbf{Pn}(s|\sum \lambda_i)$



Binomiale-Bêta

$$\mathbf{BinBet}(x|\alpha, \beta, n) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha + \beta + n)} \binom{n}{x} \Gamma(\alpha + x)\Gamma(\beta + n - x),$$

$$\alpha, \beta > 0, n = 1, 2, \dots,$$

$$x = 0, \dots, n,$$

$$\begin{cases} \Pr\{X = 0\} = \frac{\Gamma(\alpha + \beta)\Gamma(\beta + n)}{\Gamma(\beta)\Gamma(\alpha + \beta + n)}, \\ \Pr\{X = 1\} = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)\Gamma(\alpha + \beta + n)} n\Gamma(\alpha + 1)\Gamma(\beta + n - 1) \\ \Pr\{X = n\} = \frac{\Gamma(\alpha + \beta)\Gamma(\alpha + n)}{\Gamma(\alpha)\Gamma(\alpha + \beta + n)} \end{cases}$$

$$E[x] = n \frac{\alpha}{\alpha + \beta},$$

$$\text{Var}[x] = \frac{n\alpha\beta}{(\alpha + \beta)^2} \frac{\alpha + \beta + n}{\alpha + \beta + 1},$$

$$M[x] = \frac{(n + 1)(\alpha + 1)}{\alpha + \beta - 2}$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = ?$$

$$\mathbf{BinBet}(x|\alpha, \beta, n) = \int_0^1 \mathbf{Bin}(x|\theta, n) \mathbf{Bet}(\theta|\alpha, \beta) d\theta$$

$$\alpha = \beta = 1 : \text{ loi uniforme discrète}$$

Binomiale-Bêta négative

$$\text{NegBinBet}(x|\alpha, \beta, r) = \frac{\Gamma(\alpha + \beta)\Gamma(\alpha + r)}{\Gamma(\alpha)\Gamma(\beta)} \binom{r + x - 1}{r - 1} \frac{\Gamma(\beta + x)}{\Gamma(\alpha + \beta + x + 1)},$$

$$\alpha, \beta > 0, r = 1, 2, \dots,$$

$$x = 0, 1, 2, \dots,$$

$$\begin{cases} \Pr\{X = 0\} = ?, \\ \Pr\{X = 1\} = ? \\ \Pr\{X = n\} = \frac{\Gamma(\alpha + \beta)\Gamma(\alpha + r)}{\Gamma(\alpha)\Gamma(\beta)} \binom{r + n - 1}{r - 1} \frac{\Gamma(\beta + n)}{\Gamma(\alpha + \beta + n + \alpha)} \end{cases}$$

$$E[x] = \frac{r\beta}{\alpha - 1}, \alpha > 1,$$

$$\text{Var}[x] = \frac{r\beta}{\alpha - 1} \left[\frac{\alpha + \beta + r - 1}{\alpha - 2} + \frac{r\beta}{(\alpha - 1)(\alpha - 2)} \right], \alpha > 2,$$

$$M[x] = ?$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = ?$$

$$\text{NegBinBet}(x|\alpha, \beta, r) = \int_0^1 \text{NegBin}(x|\theta, r) \text{Bet}(\theta|\alpha, \beta) d\theta$$

Poisson-Gamma

$$\mathbf{PnGam}(x|\alpha, \beta, n) = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha + x)}{x!} \frac{n^x}{(\beta + n)^{\alpha+x}},$$

$$\alpha, \beta > 0, n = 0, 1, 2, \dots,$$

$$x = 0, 1, 2, \dots,$$

$$\begin{cases} \Pr\{X = 0\} = \frac{\beta^\alpha}{(\beta+n)^\alpha}, \\ \Pr\{X = 1\} = \frac{\beta^\alpha \Gamma(x+1)}{\Gamma(x)} \frac{n}{(\beta+n)^\alpha}, \\ \Pr\{X = n\} = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha+n)}{n!} \frac{n^n}{(\beta+n)^{\alpha+n}} \end{cases}$$

$$\mathbf{E}[x] = \frac{n\alpha}{\beta},$$

$$\mathbf{Var}[x] = \frac{n\alpha(\beta + n)}{\beta^2},$$

$$\mathbf{M}[x] = \begin{cases} 0, 1 & \text{si } \alpha n = \beta + n \\ 0 & \text{si } \alpha n < \beta + n \\ \frac{n(\alpha-1)}{\beta} & \text{si } \alpha n > \beta + n \end{cases}$$

$$\mathbf{S}[x] = ?$$

$$\mathbf{K}[x] = ?$$

$$\mathbf{H}[x] = ?$$

$$\mathbf{PnGam}(x|\alpha, \beta, n) = \int_0^\infty \mathbf{Pn}(x|n\lambda) \mathbf{Gam}(\lambda|\alpha, \beta) d\lambda$$

$$\alpha = \frac{\nu}{2}, \beta = \frac{1}{2} : \mathbf{IGam}\left(x \middle| \frac{\nu}{2}, \frac{1}{2}\right) = \mathbf{Chi}^{-2}(x|\nu),$$

Poisson composée

$$\mathbf{Pnc} (x|\lambda, \mu) = \exp[-\lambda] \sum_{n=0}^{\infty} \frac{(n\mu)^x \exp[-n\mu]}{x!} \frac{\lambda^N}{n!},$$

$$\lambda, \mu > 0, \quad x = 0, 1, 2 \dots,$$

$$\begin{cases} \Pr\{X = 0\} = ?, \\ \Pr\{X = 1\} = ? \\ \Pr\{X = n\} = \exp[-\lambda] \sum_{n=0}^{\infty} \frac{(n\mu)^n \exp[-n\mu]}{n!} \frac{\lambda^N}{n!}, \end{cases}$$

$$\mathbf{E}[x] = ,$$

$$\mathbf{Var}[x] = ,$$

$$\mathbf{M}[x] =$$

$$\mathbf{S}[x] = ?$$

$$\mathbf{K}[x] = ?$$

$$\mathbf{H}[x] = ?$$

Géométrie

$$\text{Geo}(x|\theta) = \theta(1-\theta)^x,$$

$$\theta > 0, \quad x = 0, 1, 2, \dots,$$

$$\begin{cases} \Pr\{X = 0\} = \theta, \\ \Pr\{X = 1\} = \theta(1-\theta) \\ \Pr\{X = n\} = \theta(1-\theta)^n \end{cases}$$

$$E[x] = \frac{1}{\theta},$$

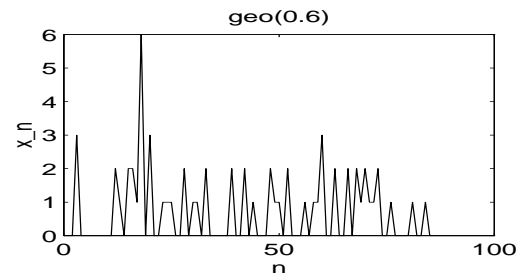
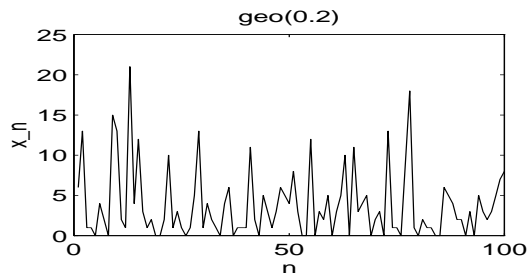
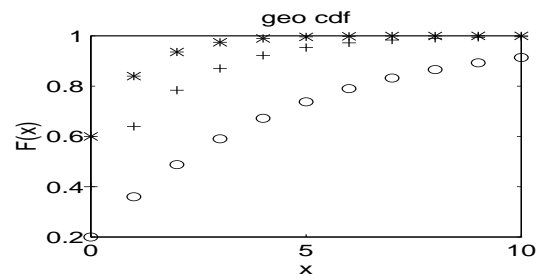
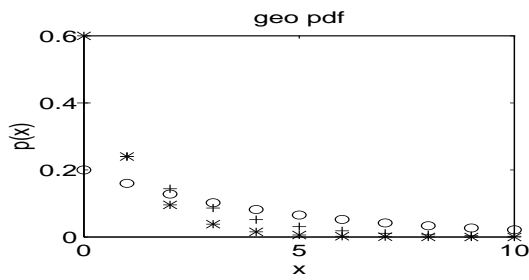
$$\text{Var}[x] = \frac{1-\theta}{\theta^2},$$

$$M[x] =$$

$$S[x] = \frac{2-\theta}{\sqrt{1-\theta}}$$

$$K[x] = \frac{\theta^2 - 6\theta + 6}{1-\theta}$$

$$H[x] = \ln \theta + \frac{1-\theta}{\theta} \ln(1-\theta)$$



Pascale

$$\mathbf{Pas}(x|m, \theta) = \binom{m-1}{x-1} \theta^m (1-\theta)^{x-m},$$

$$m > 0, 0 < \theta < 1 \quad x \geq m,$$

$$\left\{ \begin{array}{l} \Pr\{X=1\} = (m-1)\theta^m(1-\theta)^m \\ \Pr\{X=n\} = \binom{m-1}{n-1} \theta^m(1-\theta)^{n-m} \end{array} \right. \quad \left\{ \begin{array}{l} \binom{n}{p} = \frac{n!}{p!(n-p)!} \\ \binom{n}{0} = \binom{n}{n} = 1 \\ \binom{n}{1} = \binom{n}{n-1} = n \end{array} \right.$$

$$E[x] = \frac{m}{\theta},$$

$$\text{Var}[x] = \frac{m(1-\theta)}{\theta^2},$$

$$M[x] = ?$$

$$S[x] = ?$$

$$K[x] = ?$$

$$S[x] = \frac{2-\theta}{\sqrt{m(1-\theta)}}$$

$$K[x] = \frac{\theta^2 - 6\theta + 6}{m(1-\theta)}$$

$$H[x] = ?$$

Lois de probabilité à une variable réelle

Bêta

$$\mathbf{Bet}(x|\alpha, \beta) = \frac{\Gamma(\alpha + \beta)}{\Gamma(\alpha)\Gamma(\beta)} x^{\alpha-1}(1-x)^{\beta-1} = \frac{1}{B(\alpha, \beta)} x^{\alpha-1}(1-x)^{\beta-1},$$

$$\alpha, \beta > 0,$$

$$0 < x < 1,$$

$$B(\alpha, \beta) = \int_0^1 x^{\alpha-1}(1-x)^{\beta-1} dx$$

$$F(x|\alpha, \beta) = \frac{1}{B(\alpha, \beta)} \int_0^x t^{\alpha-1}(1-t)^{\beta-1} dt$$

$$E[x] = \frac{\alpha}{\alpha + \beta},$$

$$\text{Var}[x] = \frac{\alpha\beta}{(\alpha + \beta)^2(\alpha + \beta + 1)},$$

$$M[x] = \frac{\alpha - 1}{\alpha + \beta - 2} \text{ si } \alpha, \beta > 1$$

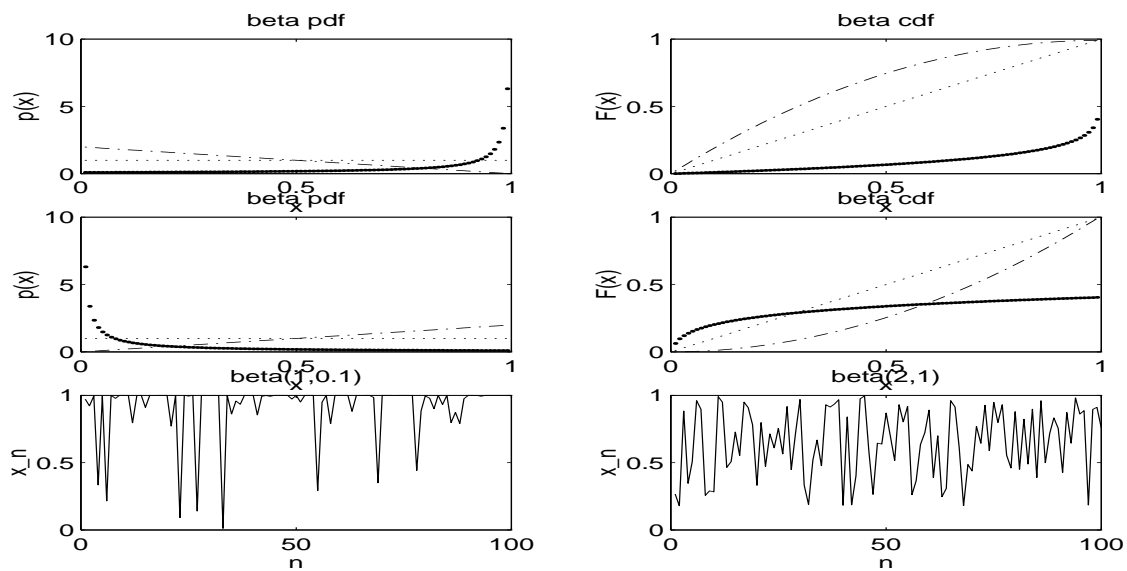
$$S[x] = \frac{2(\beta - \alpha)\sqrt{\alpha + \beta + 1}}{(\alpha + \beta + 2)\sqrt{\alpha\beta}}$$

$$K[x] = \frac{3(\alpha + \beta + 1)[2(\alpha + \beta)^2 + \alpha\beta(\alpha + \beta + 6)]}{\alpha\beta(\alpha + \beta + 2)(\alpha + \beta + 3)}$$

$$H[x] = ?$$

$$H[x] = \log[B(\alpha, \beta)] - (\alpha - 1)[\psi(\alpha) - \psi(\alpha + \beta)] - (\beta - 1)[\psi(\beta) - \psi(\alpha + \beta)]$$

si $x \sim \mathbf{Bet}(x|\alpha, \beta)$, alors $y = 1 - x \sim \mathbf{Bet}(y|\beta, \alpha)$,



Gamma

$$\mathbf{Gam}(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{\alpha-1} \exp[-\beta x],$$

$$\alpha, \beta > 0,$$

$$x > 0,$$

$$E[x] = \frac{\alpha}{\beta},$$

$$\text{Var}[x] = \frac{\alpha}{\beta^2},$$

$$M[x] = \frac{\alpha - 1}{\beta}, \text{ si } \alpha > 1$$

$$S[x] = \frac{2}{\sqrt{\alpha}}$$

$$K[x] = \frac{6}{\alpha}$$

$$H[x] = -\log \beta + \log[\Gamma(\alpha)] + (1 - \alpha)\psi(\alpha) + \alpha$$

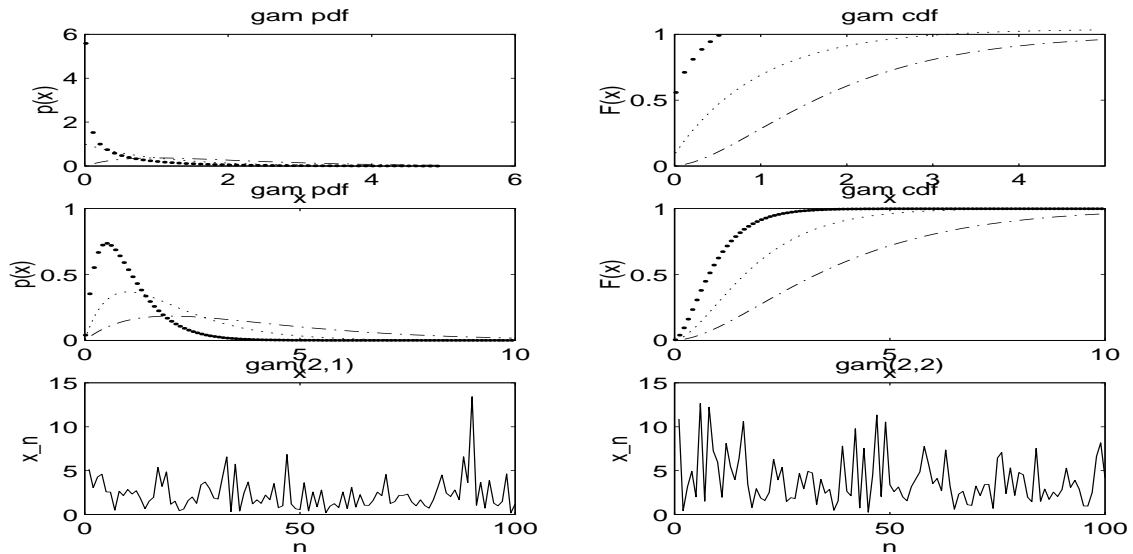
$$\alpha = 1 : \text{ Exponentielle, } \mathbf{Gam}(x|1, \beta) = \mathbf{Ex}(x|\beta),$$

$$\beta = 1 : \text{ Erlang, } \mathbf{Gam}(x|\alpha, 1) = \mathbf{Erlang}(x|\alpha),$$

$$\alpha = \frac{\nu}{2}, \beta = \frac{1}{2} : \mathbf{Gam}\left(x|\frac{\nu}{2}, \frac{1}{2}\right) = \mathbf{Chi}^2(x|\nu),$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{Gam}(x|\alpha_i, \beta), i = 1, \dots, k,$$

$$\text{alors } s = \sum x_i \sim \mathbf{Gam}\left(s|\sum \alpha_i, \beta\right)$$



Gamma inverse

$$\mathbf{IGam}(x|\alpha, \beta) = \frac{\beta^\alpha}{\Gamma(\alpha)} x^{-(\alpha+1)} \exp\left[-\frac{\beta}{x}\right],$$

$$\alpha, \beta > 0,$$

$$x > 0,$$

$$E[x] = \frac{\beta}{\alpha - 1}, \text{ si } \alpha > 1,$$

$$\text{Var}[x] = \frac{\beta^2}{(\alpha - 1)(\alpha - 2)}, \text{ si } \alpha > 2,$$

$$M[x] = \frac{\beta}{\alpha + 1}, \text{ si } \alpha < 1$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = \log \beta + \log[\Gamma(\alpha)] + (1 - \alpha)\psi(\alpha) + \alpha$$

$$\alpha = \frac{\nu}{2}, \beta = \frac{1}{2} : \quad \mathbf{IGam}\left(x \mid \frac{\nu}{2}, \frac{1}{2}\right) = \mathbf{Chi}^{-2}(x|\nu),$$

du khi-deux

$$\text{Chi}^2(x|n) = \frac{(\frac{1}{2})^{n/2}}{\Gamma(1/2)} x^{n/2-1} \exp\left[-\frac{x}{2}\right],$$

$$n > 0, \quad x > 0,$$

$$E[x] = n,$$

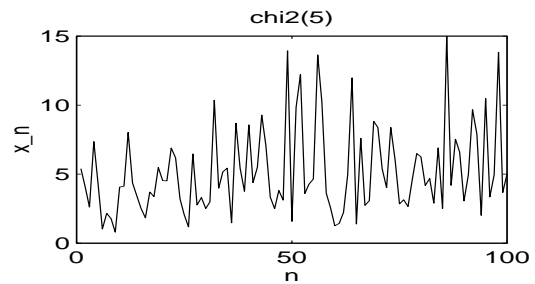
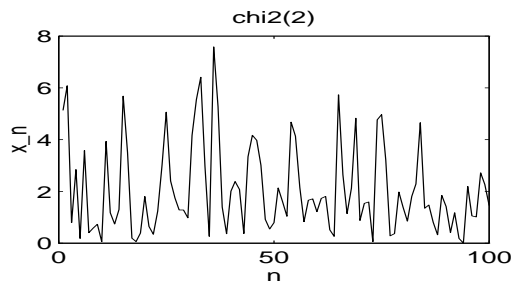
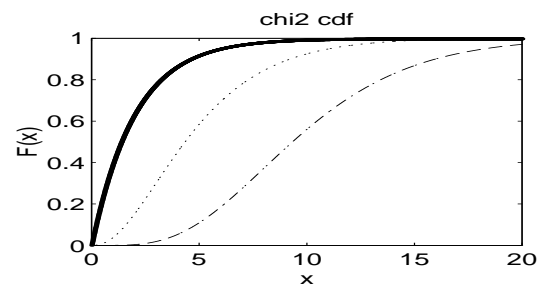
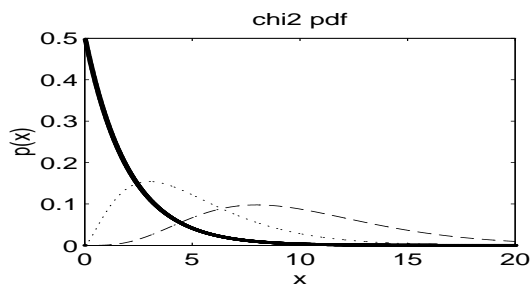
$$\text{Var}[x] = 2n,$$

$$M[x] = n - 2 \quad \text{si } n > 2$$

$$S[x] = 2\sqrt{2/n}$$

$$K[x] = \frac{12}{n}$$

$$H[x] = ?$$



du khi-deux décentrée

$$\mathbf{Chi}^2(x|\nu, \lambda) = \sum_{i=0}^{\infty} \mathbf{Pn} \left(i | \frac{\lambda}{2} \right) \chi^2(x|\nu + 2i),$$

$$\nu, \lambda > 0,$$

$$x > 0,$$

$$\mathbf{E}[x] = \nu + \lambda,$$

$$\mathbf{Var}[x] = 2(\nu + 2\lambda),$$

$$\mathbf{M}[x] : \mathbf{Chi}^2(\mathbf{M}[x]|\nu, \lambda) = \mathbf{Chi}^2(\mathbf{M}[x]|\nu - 2, \lambda)$$

$$\mathbf{S}[x] = ?$$

$$\mathbf{K}[x] = ?$$

$$\mathbf{H}[x] = ?$$

$$\mathbf{Chi}^2(x|\nu, \lambda) = \sum_{i=0}^{\infty} \mathbf{Pn}(i|\lambda/2) \mathbf{Chi}^2(x|\nu + 2i)$$

si $x \sim \mathbf{Gamm}(x|\alpha, \beta)$, alors $y = 2\beta x \sim \mathbf{Chi}^2(y|\alpha)$

si $\{x_1, \dots, x_k\} \sim \mathbf{N}(x|\mu_i, 1), i = 1, \dots, k$,

alors $s = \sum x_i^2 \sim \mathbf{Chi}^2\left(s|k, \sum \mu_i^2\right)$

si $\{x_1, \dots, x_k\} \sim \mathbf{Chi}^2(x|\nu_i, \lambda_i), i = 1, \dots, k$,

alors $z = \sum x_i \sim \mathbf{Chi}^2\left(z|\sum \nu_i, \sum \lambda_i\right)$

du khi inverse

$$\mathbf{IChi}(x|\nu) = \frac{1^{\nu/2}}{\Gamma(\nu/2)} x^{-(\nu/2+1)} \exp\left[-\frac{1}{2x^2}\right],$$
$$\nu > 0, \quad x > 0,$$

$$\mathbf{E}[x] =,$$

$$\mathbf{Var}[x] =,$$

$$\mathbf{M}[x] =$$

$$\mathbf{S}[x] =?$$

$$\mathbf{K}[x] =?$$

$$\mathbf{H}[x] =?$$

Racine carrée Gamma inverse

$$\mathbf{IGam}^{-\frac{1}{2}}(x|\alpha, \beta) = \frac{2\beta^\alpha}{\Gamma(\alpha)} x^{-(2\alpha+1)} \exp\left[-\frac{\beta}{x^2}\right],$$

$$\alpha, \beta > 0,$$

$$x > 0,$$

$$E[x] = ?,$$

$$\text{Var}[x] = ?,$$

$$M[x] = ?$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = ?$$

Pareto

$$\mathbf{Par}(x|\alpha, \beta) = \alpha\beta^\alpha x^{-(\alpha+1)},$$
$$\alpha, \beta > 0,$$
$$x \geq \beta,$$

$$\mathbf{E}[x] = \frac{\alpha\beta}{\alpha-1}, \text{ si } \alpha > 1,$$

$$\mathbf{Var}[x] = \frac{\alpha\beta^2}{(\alpha-1)^2(\alpha-2)}, \text{ si } \alpha > 2,$$

$$\mathbf{M}[x] = \beta$$

$$\mathbf{S}[x] = ?$$

$$\mathbf{K}[x] = ?$$

$$\mathbf{H}[x] = \frac{1}{\alpha} - \log \alpha + 1$$

$$\mathbf{Par}(x|\alpha, \beta) = \int_0^\infty \mathbf{Ex}(x - \beta|\lambda) \mathbf{Gam}(\lambda|\alpha, \beta) d\lambda$$

Pareto inverse

$$\mathbf{IPar}(x|\alpha, \beta) = \alpha\beta^\alpha x^{\alpha-1},$$

$$\alpha, \beta > 0,$$

$$0 < x < \beta^{-1},$$

$$E[x] =,$$

$$\text{Var}[x] =,$$

$$M[x] =$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = ?$$

Normale (paramètres : μ, λ)

$$\mathbf{N}(x|\mu, \lambda) = \sqrt{\frac{\lambda}{2\pi}} \exp\left[-\frac{1}{2}\lambda(x - \mu)^2\right],$$

$$\mu \in \mathbb{R}, \lambda > 0,$$

$$x \in \mathbb{R},$$

$$\mathbf{E}[x] = \mu,$$

$$\mathbf{Var}[x] = \lambda^{-1},$$

$$\mathbf{M}[x] = \mu$$

$$\mathbf{S}[x] = 0$$

$$\mathbf{K}[x] = 0$$

$$\text{Cumulants: } K_1 = \mu, \quad K_2 = \frac{1}{\lambda}, \quad K_n = 0, \quad n > 2$$

$$\text{Fonction caractéristique: } F(t) = \exp\left[i\mu t - \frac{t^2}{2\lambda}\right]$$

$$\mathbf{H}[x] = \frac{1}{2} + \ln(2\pi/\lambda)$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{N}(x|\mu_i, \lambda_i), \quad i = 1, \dots, k,$$

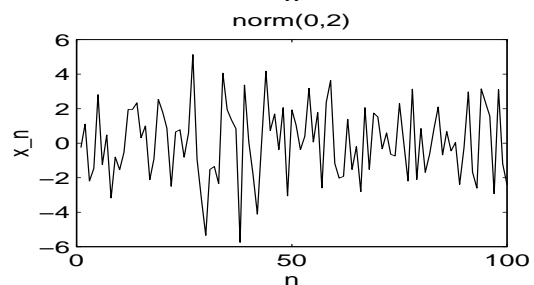
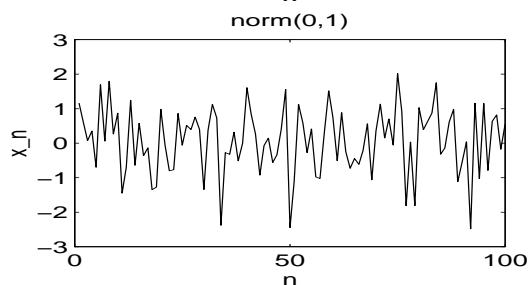
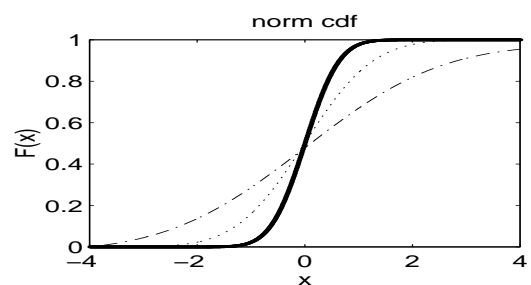
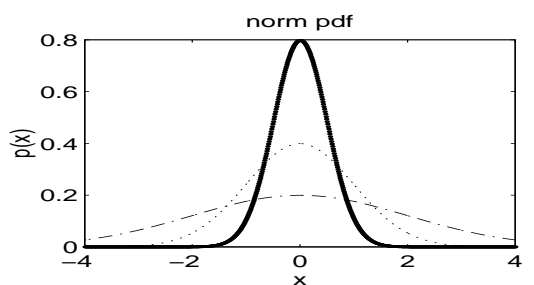
$$\text{alors } y = a + \sum_{i=1}^k b_i x_i \sim \mathbf{N}\left(y|a + \sum_{i=1}^k b_i \mu_i, \left(\sum_{i=1}^k b_i^2 / \lambda_i\right)^{-1}\right)$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{N}(x|0, 1), \quad i = 1, \dots, k,$$

$$\text{alors } s = \sum_{i=1}^k x_i^2 \sim \mathbf{Chi}^2(s|k)$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{N}(x|\mu_i, 1), \quad i = 1, \dots, k,$$

$$\text{alors } s = \sum_{i=1}^k x_i^2 \sim \mathbf{Chi}^2\left(s|k, \sum_{i=1}^k \mu_i^2\right)$$



Normale (paramètres : μ, σ^2)

$$\mathbf{N}(x|\mu, \sigma^2) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[-\frac{1}{2\sigma^2}(x - \mu)^2\right],$$

$$\mu \in \mathbb{R}, \sigma > 0,$$

$$x \in \mathbb{R},$$

$$\mathbf{E}[x] = \mu,$$

$$\mathbf{Var}[x] = \sigma^2,$$

$$\mathbf{M}[x] = \mu$$

$$\mathbf{S}[x] = 0$$

$$\mathbf{K}[x] = 0$$

$$\text{Cumulants : } K_1 = \mu, \quad K_2 = \sigma, \quad K_n = 0, \quad n > 2$$

$$\text{Fonction caractéristique : } F(t) = \exp\left[i\mu t - \frac{\sigma t^2}{2}\right]$$

$$\mathbf{H}[x] = \frac{1}{2} + \ln(2\pi\sigma^2)$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{N}(x|\mu_i, \sigma_i^2), \quad i = 1, \dots, k,$$

$$\text{alors } y = a + \sum_{i=1}^k b_i x_i \sim \mathbf{N}\left(y|a + \sum_{i=1}^k b_i \mu_i, \sum_{i=1}^k b_i^2 \sigma_i^2\right)$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{N}(x|0, 1), \quad i = 1, \dots, k,$$

$$\text{alors } s = \sum_{i=1}^k x_i^2 \sim \mathbf{Chi}^2(s|k)$$

$$\text{si } \{x_1, \dots, x_k\} \sim \mathbf{N}(x|\mu_i, 1), \quad i = 1, \dots, k,$$

$$\text{alors } s = \sum_{i=1}^k x_i^2 \sim \mathbf{Chi}^2\left(s|k, \sum_{i=1}^k \mu_i^2\right)$$

Exponentielle

$$\mathbf{E}x(x|\lambda) = \lambda \exp[-\lambda x],$$

$$\lambda > 0, \quad x > 0,$$

$$\mathbf{E}[x] = \lambda,$$

$$\mathbf{Var}[x] = \lambda,$$

$$\mathbf{M}[x] = \lambda$$

$$\mathbf{S}[x] = 2$$

$$\mathbf{K}[x] = 6$$

$$\mathbf{H}[x] = 1 + \ln \lambda$$

Autre notation

$$\mathbf{E}x(x|\alpha, \beta) = \frac{1}{\beta} \exp[-(x - \alpha)/\beta],$$

$$x > \alpha, \quad \alpha \in \mathbf{R}, \quad \beta > 0,$$

$$\mathbf{E}[x] = \alpha + \beta,$$

$$\mathbf{Var}[x] = \beta^2,$$

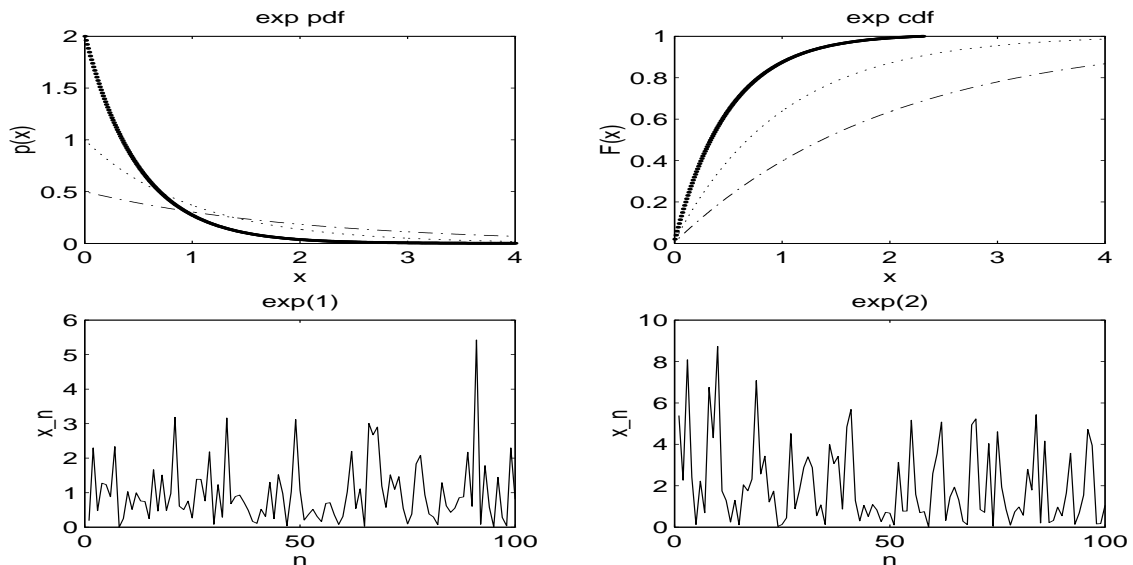
$$\mathbf{M}[x] = \alpha$$

$$\mathbf{S}[x] = 2$$

$$\mathbf{K}[x] = 6$$

$$\text{Cumulants: } K_1 = \alpha + \beta, \quad K_n = \Gamma(n)\beta^n, \quad n > 1$$

$$\text{Fonction caractéristique: } F(t) = \exp[i\alpha t] (1 - i\beta t)^{-1}$$



Gamma–Gamma

$$\mathbf{GamGam}(x|\alpha, \beta, n) = \frac{\beta^\alpha}{\Gamma(\alpha)} \frac{\Gamma(\alpha + n)}{\Gamma(n)} \frac{x^{n-1}}{(\beta + x)^{\alpha+n}},$$

$$\alpha, \beta > 0, n = 0, 1, 2, \dots,$$

$$x = 0, 1, 2, \dots,$$

$$E[x] = n \frac{\beta}{\alpha - 1}, \text{ si } \alpha > 1,$$

$$\text{Var}[x] = \frac{\beta^2(n^2 + n(\alpha - 1))}{(\alpha - 1)^2(\alpha - 2)}, \text{ si } \alpha > 2,$$

$$M[x] = ?$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = ?$$

$$\mathbf{GamGam}(x|\alpha, \beta, n) = \int_0^\infty \mathbf{Gam}(x|n, \lambda) \mathbf{Gam}(\lambda|\alpha, \beta) d\lambda$$

Student

$$\mathbf{St}(x|\mu, \lambda, \alpha) = \frac{\Gamma(\frac{\alpha+1}{2})}{\Gamma(\frac{\alpha}{2})\Gamma(\frac{1}{2})} \left(\frac{\lambda}{\alpha}\right)^{\frac{1}{2}} \left[1 + \frac{\lambda}{\alpha}(x - \mu)^2\right]^{-(\alpha+1)/2},$$

$$\mu \in \mathbb{R}, \lambda, \alpha > 0,$$

$$x \in \mathbb{R},$$

$$E[x] = \mu, \text{ si } \alpha > 1,$$

$$\text{Var}[x] = \frac{1}{\lambda} \frac{\alpha}{\alpha - 2}, \text{ si } \alpha > 2,$$

$$M[x] = \mu$$

$$S[x] = ?$$

$$K[x] = ?$$

$$H[x] = \log[\alpha^{\frac{1}{2}} B(\frac{1}{2}, \frac{\alpha}{2})] + \frac{1 + \alpha}{2} [\psi(\frac{\alpha}{2} + \frac{1}{2}) - \psi(\frac{\alpha}{2})]$$

$$\mathbf{St}(x|\mu, \lambda, \alpha) = \int_0^{\infty} \mathbf{N}(x|\mu, \lambda y) \mathbf{Gam}\left(y|\frac{\alpha}{2}, \frac{\alpha}{2}\right) dy$$

$$\alpha = 1 : \mathbf{St}(x|\mu, \lambda, 1) = \mathbf{Cauchy}(x|\mu, \lambda)$$

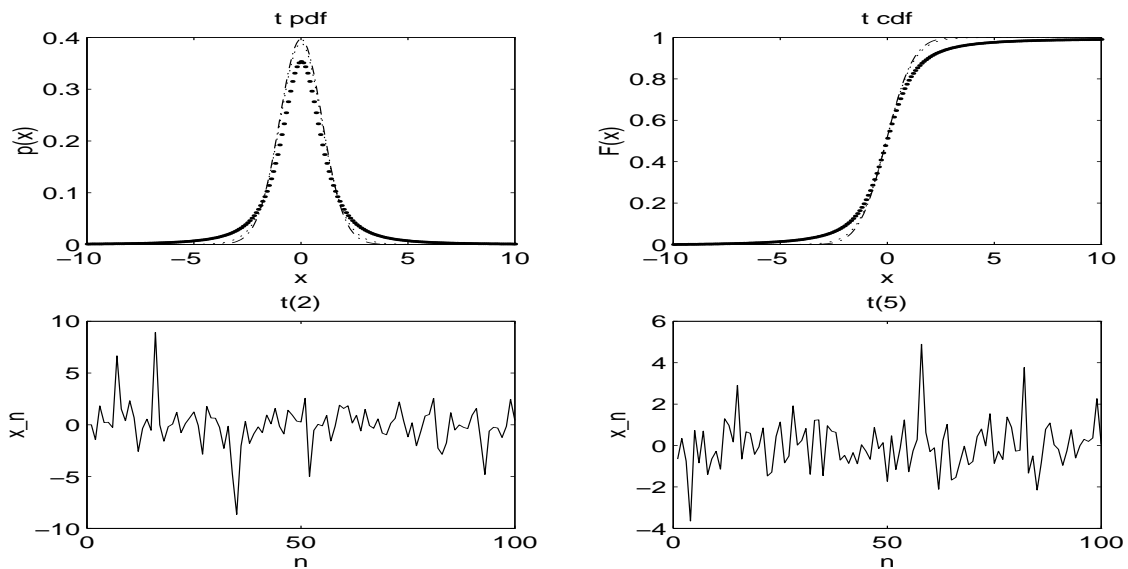
$$\alpha \mapsto \infty : \lim_{\alpha \rightarrow \infty} \mathbf{St}(x|\mu, \lambda, \alpha) = \mathbf{N}(x|\mu, \lambda)$$

$$\text{si } x \sim \mathbf{St}(x|\mu, \lambda, \nu)$$

$$\text{alors } y = \lambda^{\frac{1}{2}}(x - \mu) \sim \mathbf{St}(x|0, 1, \nu)$$

$$\text{si } x \sim \mathbf{N}(x|0, 1) \text{ et } y \sim \mathbf{Chi}^2(y|\nu)$$

$$\text{alors } z = \frac{x}{(y/\nu)^{\frac{1}{2}}} \sim \mathbf{St}(x|0, 1, \nu)$$



Fisher-Snedecor

$$\mathbf{FS}(x|\alpha, \beta) = \frac{\Gamma((\alpha + \beta)/2)}{\Gamma(\alpha/2)\Gamma(\beta/2)} \alpha^{\alpha/2} \beta^{\beta/2} \frac{x^{\alpha/2-1}}{(\beta + \alpha x)^{(\alpha+\beta)/2}} = \frac{\alpha^{\alpha/2} \beta^{\beta/2}}{B(\alpha/2, \beta/2)} x^{\alpha/2-1} (\beta + \alpha x)^{-(\alpha+\beta)/2},$$

$$\alpha, \beta > 0,$$

$$x > 0,$$

$$E[x] = \frac{\beta}{\beta - 2}, \text{ si } \beta > 2,$$

$$\text{Var}[x] = \frac{2\beta^2}{\alpha(\beta - 4)} \frac{\alpha + \beta - 2}{(\beta - 2)^2}, \text{ si } \beta > 4,$$

$$M[x] = \frac{\beta}{\beta + 2} \frac{\alpha - 2}{\alpha}, \text{ si } \beta > 2$$

$$S[x] = ?$$

$$K[x] = ?$$

$$\text{Les points d'inflexion: } \frac{\beta\sqrt{(\alpha - 2)(\alpha + \beta)}}{\alpha(\beta + 2)}$$

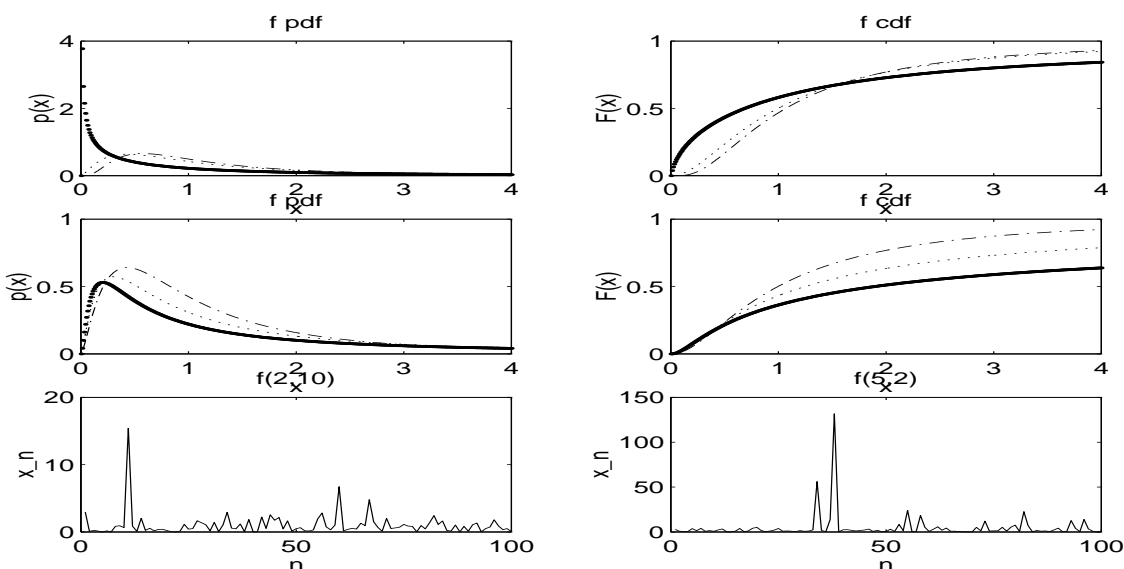
$$H[x] = \log \left[\frac{\alpha}{\beta} B \left(\frac{\alpha}{2}, \frac{\beta}{2} \right) \right] + \left(1 - \frac{\alpha}{2} \right) \psi \left(\frac{\alpha}{2} \right) - \left(1 - \frac{\beta}{2} \right) \psi \left(\frac{\beta}{2} \right) + \left(\frac{\alpha + \beta}{2} \right) \psi \left(\frac{\alpha + \beta}{2} \right)$$

$$\text{si } x \sim \mathbf{Chi}^2(x|\nu_1) \text{ et } y \sim \mathbf{Chi}^2(y|\nu_2)$$

$$\text{alors } z = \frac{x/\nu_1}{(y/\nu_2)} \sim \mathbf{FS}(z|\nu_1, \nu_2)$$

$$\text{si } x \sim \mathbf{Bet}(x|\alpha, \beta)$$

$$\text{alors } y = \frac{\beta x}{\alpha(1-x)} \sim \mathbf{FS}(y|2\alpha, 2\alpha)$$



Logistic

$$\mathbf{Lo}(x|\alpha, \beta) = \beta^{-1} \frac{\exp[-\beta^{-1}(x - \alpha)]}{(1 + \exp[\beta^{-1}(x - \alpha)])^2},$$

$$= \frac{1}{4\beta} \operatorname{sech}^2 \left\{ \frac{1}{2} \frac{x - \alpha}{\beta} \right\},$$

$$\alpha \in \mathbf{R}, \beta > 0,$$

$$x \in \mathbf{R},$$

$$\mathbf{E}[x] = \alpha,$$

$$\mathbf{Var}[x] = \beta^2 \pi^2 / 3,$$

$$\mathbf{M}[x] = \alpha$$

$$\mathbf{S}[x] = ?$$

$$\mathbf{K}[x] = ?$$

$$\mathbf{H}[x] = ?$$

Uniforme

$$\mathbf{Uni}(x|\theta_1, \theta_2) = \frac{1}{\theta_2 - \theta_1},$$

$$\theta_2 > \theta_1,$$

$$\theta_1 < x < \theta_2,$$

$$\mathbf{E}[x] = \frac{\theta_2 + \theta_1}{2},$$

$$\mathbf{Var}[x] = \frac{(\theta_2 - \theta_1)^2}{12},$$

$$\mathbf{M}[x] = ?$$

$$\mathbf{S}[x] = 0$$

$$\mathbf{K}[x] = -1.2$$

$$\mathbf{H}[x] = \ln(\theta_2 - \theta_1)$$

Lois de probabilité à une variable réelle (suite)

Cauchy

$$\mathbf{Cau}(x|\lambda) = \frac{1/(\pi\lambda)}{1 + (x/\lambda)^2},$$

$$\lambda \in \mathbf{R}, \quad x \in \mathbf{R},$$

$$E[x] \quad \text{n'existe pas,}$$

$$\text{Var}[x] = \quad \text{n'existe pas}$$

$$M[x] = \frac{1}{\pi\lambda}$$

$$S[x] = \quad \text{n'existe pas,}$$

$$K[x] = \quad \text{n'existe pas,}$$

$$\text{Fonction caractéristique : } F(t) = e^{-\lambda|t|}$$

$$H[x] = \ln(4\pi\lambda)$$

Autre notation :

$$\mathbf{Cau}(x|\lambda, \alpha) = \frac{1/(\pi\lambda)}{1 + (\frac{x-\alpha}{\lambda})^2},$$

$$\lambda \in \mathbf{R}, \quad x \in \mathbf{R},$$

$$E[x] \quad \text{n'existe pas,}$$

$$\text{Var}[x] = \quad \text{n'existe pas}$$

$$M[x] = \frac{1}{\pi\lambda}$$

$$S[x] = \quad \text{n'existe pas,}$$

$$K[x] = \quad \text{n'existe pas,}$$

$$\text{Fonction caractéristique : } F(t) = e^{-\lambda|t|+iat}$$

$$H[x] = \ln(4\pi\lambda)$$

RayleighRay ($x|\theta$)

Log-Normale

$$\mathbf{LogN}(x|\mu, \lambda) = \frac{1}{\sqrt{2\pi\lambda x}} \exp\left[-\frac{(\ln x - \mu)^2}{2\lambda^2}\right],$$

$$E[x] =,$$

$$\text{Var}[x] = \exp[2\mu - 2\lambda^2] - \exp[2\mu - \lambda^2],$$

$$M[x] = ?$$

$$H[x] = \mu + \log(2\pi e\lambda^2)$$

Normale généralisée

$$\mathbf{Ngen}(x|\alpha, \beta) = \frac{2\beta^{\alpha/2}}{\Gamma(\alpha/2)} x^{\alpha-1} \exp[-\beta x^2],$$

$$\mathbf{E}[x] =,$$

$$\mathbf{Var}[x] =,$$

$$\mathbf{M}[x] = ?$$

$$\mathbf{H}[x] = ?$$

Weibull

$$\text{Wei}(x|\alpha, \beta) = \frac{\beta}{\alpha} x^{\alpha-1} \exp[-x^\beta/\alpha],$$

$$\alpha, \beta > 0, \quad x > 0,$$

$$E[x] = \alpha^{1/\beta} \Gamma(1 + 1/\beta)$$

$$\text{Var}[x] = \alpha^{2/\beta} [\Gamma(1 + 2/\beta) - \Gamma^2(1 + 1/\beta)]$$

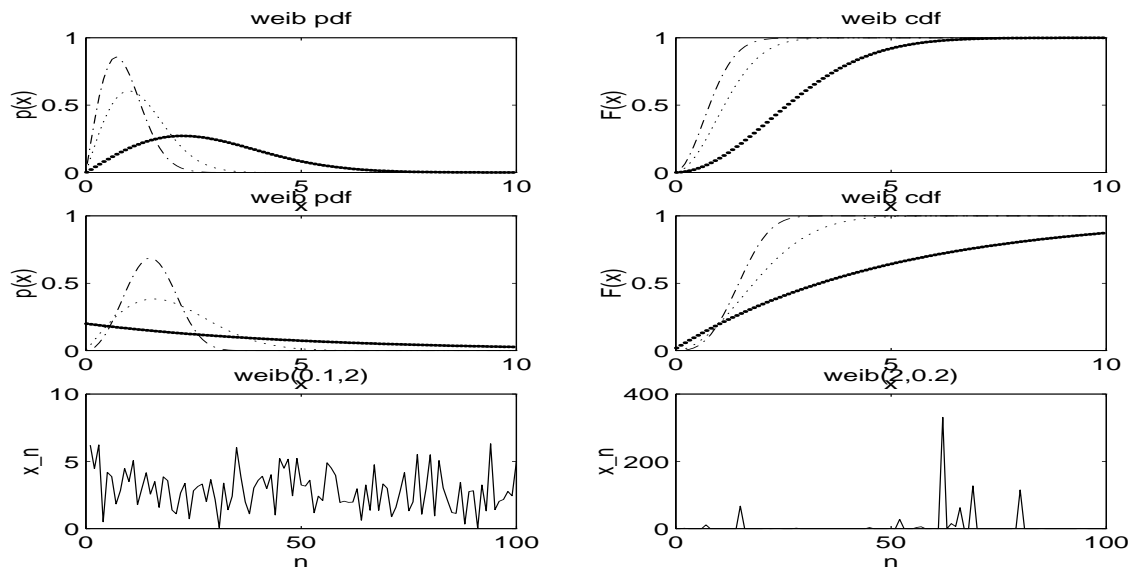
$$M[x] =$$

$$S[x] = ?$$

$$K[x] = ?$$

$$\text{Fonction de répartition: } F(x) = 1 - \exp[-x^\beta/\alpha], \quad F^{-1}(p) = (\alpha \ln \frac{1}{1-p})^{1/\beta}$$

$$H[x] = ?$$



Exponentielle double

$$\mathbf{Exd}(x|\lambda) = \frac{\lambda}{2} \exp[-\lambda|x|],$$
$$\lambda > 0, \quad x \in \mathbf{R},$$

$$\mathbf{E}[x] = 0,$$

$$\mathbf{Var}[x] = \frac{2}{\lambda^2},$$

$$\mathbf{M}[x] = 0$$

$$\mathbf{S}[x] = 0$$

$$\mathbf{K}[x] = ?$$

$$\mathbf{H}[x] = ?$$

Exponentielle tronquée

$$\mathbf{Ext}(x|\lambda) = \exp[-(x - \lambda)],$$

$$\lambda > 0, \quad x > \lambda,$$

$$E[x] =,$$

$$\text{Var}[x] =,$$

$$M[x] =$$

$$H[x] =?$$

Triangulaire

$$\text{Tri}(x|\theta) =,$$
$$\theta > 0, \quad ,$$

$$E[x] =,$$

$$\text{Var}[x] =,$$

$$M[x] =$$

$$H[x] = ?$$

du khi

$$\mathbf{Chi}(x|n) = \frac{1^{n/2}}{\Gamma(n/2)} x^{n/2-1} \exp\left[-\frac{x}{2}\right],$$
$$n > 0, \quad x > 0,$$

$$E[x] = n,$$

$$\text{Var}[x] = 2n,$$

$$M[x] = ?$$

$$S[x] = 2\sqrt{\frac{2}{n}}$$

$$K[x] = \frac{12}{n}$$

$$H[x] = ?$$

Famille exponentielle généralisée à un paramètre θ

$$\mathbf{Exf}(x|f, g, h, \phi, \theta) = f(x) g(\theta) \exp[\phi(\theta)h(x)],$$

$f(x)$	$g(\theta)$	$h(x)$	$\phi(\theta)$	$p(x \theta)$
1	$1 - \theta$	x	$\log \frac{\theta}{1-\theta}$	$\mathbf{Ber}(x \theta)$
$\frac{1}{x!}$	$\exp[-\theta]$	x	$\log(\theta)$	$\mathbf{Pn}(x \theta)$
1	θ	x	$-\theta$	$\mathbf{Ex}(x \theta)$
$\frac{1}{\sqrt{2\pi}}$	$\sqrt{\theta}$	x^2	$-\frac{1}{2}\theta$	$\mathbf{N}(x 0, \theta)$
1	$\frac{1}{\theta}$	0	θ	$\mathbf{Uni}(x 0, \theta)$
$\exp[-x]$	$\exp[\theta]$	0	θ	$\mathbf{Ext}(x \theta) = \exp[-(x - \theta)]$

Famille exponentielle à k paramètres

$$\mathbf{Exfk}(x|f, g, \mathbf{h}, \boldsymbol{\phi}, \boldsymbol{\theta}) = f(x) g(\boldsymbol{\theta}) \exp \left[\sum_{k=1}^K \phi_k(\boldsymbol{\theta}) \mathbf{h}_k(x) \right],$$

$f(x)$	$\boldsymbol{\theta}$	$g(\boldsymbol{\theta})$	$\mathbf{h}(x)$	$\boldsymbol{\phi}(\boldsymbol{\theta})$	$p(x \boldsymbol{\theta})$
$\frac{1}{\sqrt{2\pi}}$	(μ, λ)	$\sqrt{\lambda} \exp \left[-\frac{1}{2} \lambda \mu^2 \right]$	$\{x, x^2\}$	$\{\lambda \mu, -\frac{1}{2} \lambda\}$	$\mathbf{N}(x \mu, \lambda)$
$\frac{1}{\sqrt{2\pi}}$	(μ, σ^2)	$\frac{1}{\sigma} \exp \left[-\frac{1}{2} \frac{\mu^2}{\sigma^2} \right]$	$\{x, x^2\}$	$\left\{ \frac{\mu}{\sigma^2}, -\frac{1}{2\sigma^2} \right\}$	$\mathbf{N}(x \mu, \sigma^2)$
1	$(\theta - 2, \theta - 1)$	$\frac{1}{\theta_2 - \theta_1}$	$\{0, 0\}$	$\{0, 0\}$	$\mathbf{Uni}(x \theta_1, \theta_2)$
1	$\{\alpha_1, \dots, \alpha_k\}$	$\frac{\Gamma(\alpha_1) \dots \Gamma(\alpha_k)}{\Gamma(\alpha_1 + \dots + \alpha_k)}$	$\{\log x_1, \dots, \log x_k\}$	$\{\alpha_1, \dots, \alpha_k\}$	$\mathbf{Di}_k(\mathbf{x} \alpha_1, \alpha_k)$

Lois de probabilité à deux variables réelles

Normale-Gamma

$$\mathbf{NGam}(x, y|\mu, \lambda, \alpha, \beta) = \mathbf{N}(x|\mu, \lambda y) \mathbf{Gam}(y|\alpha, \beta),$$

$$\mu \in \mathbb{R}, \lambda, \alpha, \beta > 0,$$

$$x \in \mathbb{R}, y > 0,$$

$$\mathbf{E}[x] = \mu$$

$$\mathbf{E}[y] = \frac{\alpha}{\beta}$$

$$p(x) = \int_{-\infty}^{+\infty} p(x, y) dx = \int_{-\infty}^{+\infty} \mathbf{NGam}(x, y|\mu, \lambda, \alpha, \beta) dx = \mathbf{St}\left(x|\mu, \lambda \frac{\alpha}{\beta}, 2\alpha\right),$$

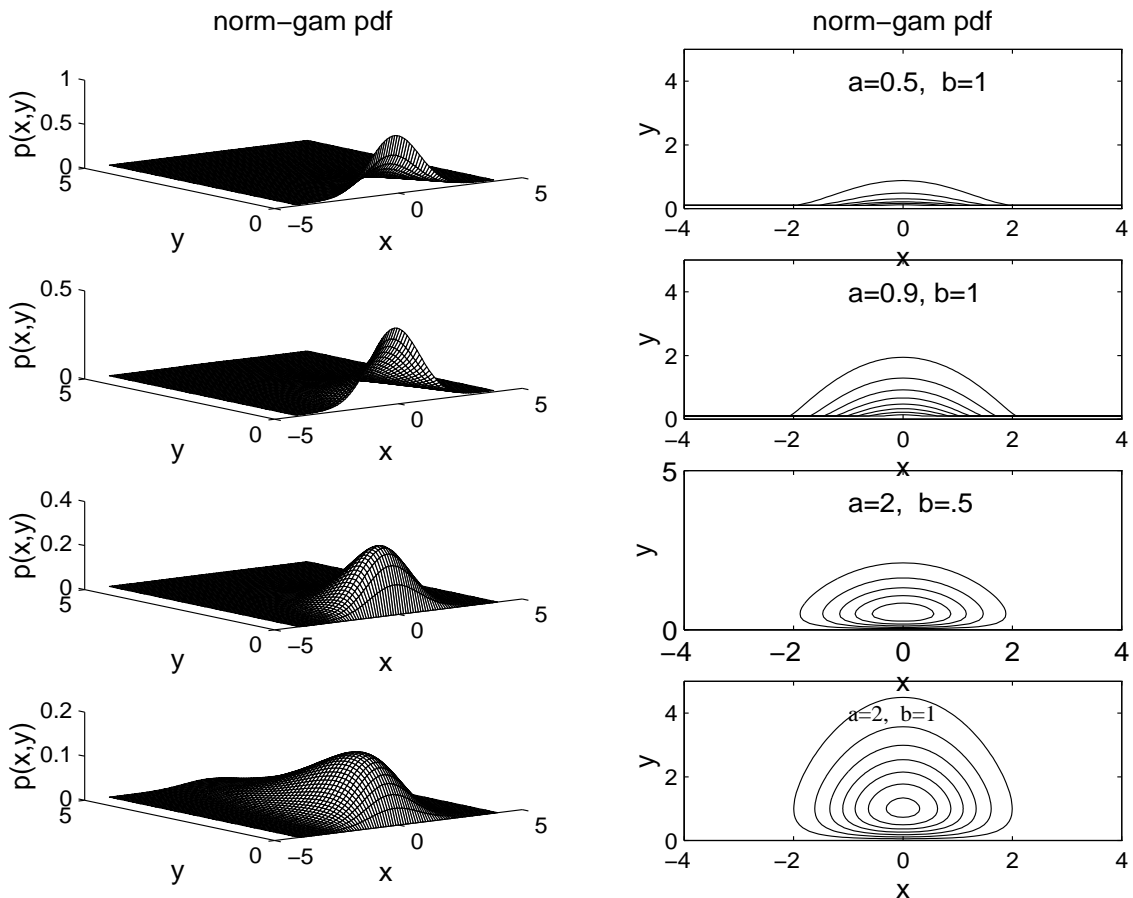
$$p(y) = \int_{-\infty}^{+\infty} p(x, y) dy = \int_0^{+\infty} \mathbf{NGam}(x, y|\mu, \lambda, \alpha, \beta) dy = \mathbf{Gam}(y|\alpha, \beta),$$

$$p(x|y) = \frac{p(x, y)}{p(y)} = \mathbf{N}(x|\mu, \lambda y),$$

$$\mathbf{E}[x] = \mu, \quad \text{si } 2\alpha > 1, \quad \text{Var}[x] = \frac{2\beta}{2\alpha - 2} \frac{1}{\lambda} \quad \text{si } \alpha > 1$$

$$\mathbf{E}[y] = \frac{\alpha}{\beta}, \quad \text{Var}[y] = \frac{\alpha}{\beta^2}$$

$$\mathbf{E}[xy] = \mu, \quad \text{Var}[x|y] = \lambda y$$



Pareto bivariable

$$\mathbf{Par}_2(x, y | \alpha, \beta_0, \beta_1) = \alpha(\alpha + 1)(\beta_1 - \beta_0)^\alpha (y - x)^{(\alpha+2)},$$

$$(\beta_0, \beta_1) \in \mathbf{R}^2, \beta_0 < \beta_1, \alpha > 0,$$

$$(x, y) \in \mathbf{R}^2, x < \beta_0, y > \beta_1,$$

$$E[x] = \frac{\alpha\beta_0 - \beta_1}{\alpha - 1}, \text{ si } \alpha > 1,$$

$$E[y] = \frac{\alpha\beta_1 - \beta_0}{\alpha - 1}, \text{ si } \alpha > 1,$$

$$\text{Var}[x] = \text{Var}[y] = \frac{\alpha(\beta_1 - \beta_0)^2}{(\alpha - 1)^2(\alpha - 2)}, \text{ si } \alpha > 2,$$

$$\text{Corr}[x, y] = -\frac{1}{\alpha},$$

$$t_1 = \beta_1 - x, t_2 = \beta_0 - y \sim \mathbf{Par}(t | \beta_1 - \beta_0, \alpha)$$

$$H[x] = ?$$

Lois de probabilité à n variables discrètes**Multinomiale**

$$\mathbf{Mu}_k(\mathbf{x}|\boldsymbol{\theta}, n) = \frac{n!}{\prod_{l=1}^{k+1} x_l!} \prod_{l=1}^{k+1} \theta_l^{x_l},$$

$$\mathbf{x} = \{x_1, \dots, x_k\}, \quad \boldsymbol{\theta} = \{\theta_1, \dots, \theta_k\}$$

$$x_{k+1} = n - \sum_{l=1}^k x_l, \quad \theta_{k+1} = 1 - \sum_{l=1}^k \theta_l,$$

$$0 < \theta_l < 1, \quad \sum_{l=1}^k \theta_l < 1, \quad n = 1, 2, \dots,$$

$$x_l = 0, 1, 2, \dots, \quad \sum x_l \leq n,$$

$$\mathbf{E}[x_i] = n\theta_i,$$

$$\mathbf{Var}[x_j] = n\theta_j(1 - \theta_j),$$

$$\mathbf{Cov}[x_i, x_j] = -n\theta_i\theta_j,$$

$$\mathbf{H}[x] = ?$$

$$n\theta_i < \mathbf{M}[x_i] < (n + k - 1)\theta_i, \quad i = 1, \dots, k$$

$$k = 1: \quad \mathbf{Mu}_1(x|\theta, n) = \mathbf{Bin}(x|\theta, n)$$

si $p(x_1, \dots, x_k) = \mathbf{Mu}_k(\mathbf{x}|\boldsymbol{\theta}, n)$, alors

$$p(x_1, \dots, x_m) = \mathbf{Mu}_m(x_1, \dots, x_m|\theta_1, \dots, \theta_m, n), \quad m \leq k, \text{ et}$$

$$p(x_1, \dots, x_m|x_{m+1}, \dots, x_k) = \mathbf{Mu}_m(x_1, \dots, x_m|\theta'_1, \dots, \theta'_m, n'),$$

avec $\theta'_i = \frac{\theta_i}{\sum_{j=1}^m \theta_j}$ et $n' = n - \sum_{j=m+1}^k \theta_j$

si on définit

$$y_1 = x_1 + \dots + x_{i_1},$$

$$y_2 = x_{i_1+1} + \dots + x_{i_2},$$

$$\vdots$$

$$y_t = x_{i_{t-1}+1} + \dots + x_{i_t}, \quad 1 \leq i_1 \leq \dots \leq i_t \leq k,$$

alors $p(y_1, \dots, y_t) = \mathbf{Mu}_t(y_1, \dots, y_t|\theta'_1, \dots, \theta'_t, n)$,

avec

$$\theta'_1 = \theta_1 + \dots + \theta_{i_1},$$

$$\theta'_2 = \theta_{i_1+1} + \dots + \theta_{i_2},$$

$$\vdots$$

$$\theta'_t = \theta_{i_{t-1}+1} + \dots + \theta_{i_t}.$$

si $\{x_1, \dots, x_m\} \sim \mathbf{Mu}_k(\mathbf{x}|\boldsymbol{\theta}, n_i), i = 1, \dots, k,$

alors $\mathbf{z} = \sum_{i=1}^m \mathbf{x}_i \sim \mathbf{Mu}_k(\mathbf{z}|\boldsymbol{\theta}, \sum n_i)$

si $\{x_1, \dots, x_k\} \sim \mathbf{Pn}(x_i|\lambda_i), i = 1, \dots, k,$

alors $(x_1, \dots, x_k | \sum x_i = n) \sim \mathbf{Mu}_k(\mathbf{x}|\boldsymbol{\theta}, n)$, avec $\theta_i = \frac{\lambda_i}{\sum \lambda_j}$

Dirichlet

$$\mathbf{Di}_k(\mathbf{x}|\boldsymbol{\alpha}) = \frac{\Gamma\left(\sum_{l=1}^{k+1} \alpha_l\right)}{\prod_{l=1}^{k+1} \Gamma(\alpha_l)} \prod_{l=1}^{k+1} x_l^{\alpha_l-1},$$

$$\mathbf{x} = \{x_1, \dots, x_k\}, \quad \boldsymbol{\alpha} = \{\alpha_1, \dots, \alpha_k, \alpha_{k+1}\}$$

$$\alpha_l > 0, l = 1, \dots, k+1$$

$$0 < x_l < 1, l = 1, \dots, k+1 \quad x_{l+1} = 1 - \sum_{l=1}^k x_l,$$

$$\mathbb{E}[x_i] = \frac{\alpha_i}{\sum_{l=1}^{k+1} \alpha_l},$$

$$\text{Var}[x_j] = \frac{\mathbb{E}[x_i](1 - \mathbb{E}[x_i])}{1 + \sum_{l=1}^{k+1} \alpha_l},$$

$$\text{Cov}[x_i, x_j] = \frac{\mathbb{E}[x_i]\mathbb{E}[x_j]}{1 + \sum_{l=1}^{k+1} \alpha_l},$$

$$\mathbb{M}[x_i] = \frac{\alpha_i - 1}{\sum_{l=1}^{k+1} \alpha_l - k - 1}, \quad i = 1, \dots, k$$

$$k = 1: \quad \mathbf{Di}_1(x|\alpha_1, \alpha_2) = \mathbf{Bet}(x|\alpha_1, \alpha_2)$$

si $\mathbf{x} = \{x_1, \dots, x_k\} \sim \mathbf{Di}_k(\mathbf{x}|\boldsymbol{\alpha})$ alors

$$p(x_1, \dots, x_m) = \mathbf{Di}_m\left(x_1, \dots, x_m | \alpha_1, \dots, \alpha_m, \sum_{j=m+1}^{k+1} \alpha_j\right), \quad m \leq k, \quad \text{et}$$

$$p(x'_1, \dots, x'_m | x_{m+1}, \dots, x_k) = \mathbf{Di}_m(x'_1, \dots, x'_m | \alpha_1, \dots, \alpha_m, \alpha_{k+1}),$$

$$\text{avec } x'_i = \frac{x_i}{1 - \sum_{j=m+1}^k x_j}$$

$$p(x'_i | x_{m+1}, \dots, x_k) = \mathbf{Bet}\left(x'_i | \alpha_i, \sum_{j=1}^m \alpha_j + \alpha_{k+1} - \alpha_i\right), \quad i = 1, \dots, m$$

si on définit

$$y_1 = x_1 + \dots + x_{i_1},$$

$$y_2 = x_{i_1+1} + \dots + x_{i_2},$$

$$\vdots$$

$$y_t = x_{i_{t-1}+1} + \dots + x_{i_t}, \quad 1 \leq i_1 \leq \dots \leq i_t \leq k,$$

alors $p(y_1, \dots, y_t) = \mathbf{Mu}_t(y_1, \dots, y_t | \alpha'_1, \dots, \alpha'_t)$,

avec $\alpha'_1 = \alpha_1 + \dots + \alpha_{i_1}$,

$$\alpha'_2 = \alpha_{i_1+1} + \dots + \alpha_{i_2},$$

$$\vdots$$

$$\alpha'_t = \alpha_{i_{t-1}+1} + \dots + \alpha_{i_t},$$

$$\alpha'_{t+1} = \alpha_{k+1}.$$

Multinomiale–Dirichlet

$$\mathbf{MuDi}_k(\mathbf{x}|\boldsymbol{\alpha}, n) = \frac{n!}{\sum_{l=1}^{k+1} \alpha_l^{[n]}} \prod_{l=1}^{k+1} \frac{\alpha_l^{[x_l]}}{x_l!},$$

$$\mathbf{x} = \{x_1, \dots, x_k\}, \quad \boldsymbol{\alpha} = \{\alpha_1, \dots, \alpha_k, \alpha_{k+1}\}$$

$$\alpha^{[s]} = \prod_{l=1}^s (\alpha + l - 1), \quad x_{k+1} = n - \sum_{l=1}^k x_l,$$

$$\alpha_l > 0, n = 1, 2, \dots,$$

$$x_l = 0, 1, 2, \dots, \quad \sum_{l=1}^k x_l < n,$$

$$\mathbb{E}[x_i] = np_i, \quad p_i = \frac{\alpha_i}{\sum_{l=1}^{k+1} \alpha_l},$$

$$\text{Var}[x_j] = \frac{n + \sum_{l=1}^{k+1} \alpha_l}{1 + \sum_{l=1}^{k+1} \alpha_l} np_j (1 - p_j),$$

$$\text{Cov}[x_i, x_j] = -\frac{n + \sum_{l=1}^{k+1} \alpha_l}{1 + \sum_{l=1}^{k+1} \alpha_l} np_i p_j,$$

si $\mathbf{x} = \{x_1, \dots, x_k\} \sim \mathbf{MuDi}_k(\mathbf{x}|\boldsymbol{\alpha}, n)$ alors

$$p(x_1, \dots, x_m) = \mathbf{MuDi}_m \left(x_1, \dots, x_m | \alpha_1, \dots, \alpha_m, \sum_{j=1}^{k+1} \alpha_j - \sum_{j=1}^m \alpha_j, n \right), \quad m \leq k, \quad \text{et}$$

$$p(x_i) = \mathbf{BinBet} \left(x_i | \alpha_i, \sum_{j=1}^{k+1} \alpha_j - \alpha_i, n \right), \quad i = 1, \dots, m \leq k$$

$$p(x_1, \dots, x_m | x_{m+1}, \dots, x_k) = \mathbf{MuDi}_m \left(x_1, \dots, x_m | \alpha_1, \dots, \alpha_m, \sum_{j=1}^{k+1} \alpha_j - \sum_{j=s+1}^k \alpha_j, n - \sum_{j=1}^s x_j \right),$$

$$p(x_i | x_{m+1}, \dots, x_k) = \mathbf{BinBet} \left(x_i | \alpha_i, \sum_{j=1}^m \alpha_j + \alpha_{k+1} - \alpha_i \right), \quad i = 1, \dots, m$$

$$p(x_{s+1}, \dots, x_k | x_1, \dots, x_s) = \mathbf{MuDi}_{k-s} (x_{s+1}, \dots, x_k | \alpha_{s+1}, \dots, \alpha_k, \sum_{j=1}^{k+1} \alpha_j - \sum_{j=s+1}^k \alpha_j, n - \sum_{j=1}^s x_j)$$

si on définit

$$y_1 = x_1 + \dots + x_{i_1},$$

$$y_2 = x_{i_1+1} + \dots + x_{i_2},$$

$$\vdots$$

$$y_t = x_{i_{t-1}+1} + \dots + x_{i_t}, \quad 1 \leq i_1 \leq \dots \leq i_t \leq k,$$

alors $p(y_1, \dots, y_t) = \mathbf{Mu}_t(y_1, \dots, y_t | \alpha'_1, \dots, \alpha'_t, n)$,

avec $\alpha'_1 = \alpha_1 + \dots + \alpha_{i_1}$,

$$\alpha'_2 = \alpha_{i_1+1} + \dots + \alpha_{i_2},$$

\vdots

$$\alpha'_t = \alpha_{i_{t-1}+1} + \dots + \alpha_{i_t}, \quad 1 \leq i_1 \leq \dots \leq i_t \leq k,$$

Lois de probabilité à n variables réelles**Exponentielle**

$$\mathbf{Exfc}_n(\mathbf{x}|a, b, \boldsymbol{\phi}) = a(\mathbf{x}) \exp[\boldsymbol{\phi}^t \mathbf{x} - b(\boldsymbol{\phi})],$$

$$\mathbf{x} = \{x_1, \dots, x_k\}, \quad x_i = h_i(x),$$

$$\boldsymbol{\phi} = \{\phi_1, \dots, \phi_k\}, \quad \phi_i = c_i \phi_i(\boldsymbol{\theta}), \quad i = 1, \dots, k$$

$$\mathbb{E}[\mathbf{x}] = \nabla b(\boldsymbol{\phi}), \quad \mathbb{E}[x_i] = \frac{\partial b(\boldsymbol{\phi})}{\partial \phi_i}$$

$$\text{Var}[\mathbf{x}] = \nabla^2 b(\boldsymbol{\phi}), \quad \text{Var}[x_i] = \frac{\partial^2 b(\boldsymbol{\phi})}{\partial \phi_i^2}$$

$$\text{Cov}[x_i, x_j] = \frac{\partial^2 b(\boldsymbol{\phi})}{\partial \phi_i \partial \phi_j}$$

si $\mathbf{x}_1, \dots, \mathbf{x}_n \sim \mathbf{Exfc}_n(\mathbf{x}_i|a, b, \boldsymbol{\phi})$

alors $\mathbf{y} = \sum_{i=1}^n \mathbf{x}_i \sim \mathbf{Exfc}_n(\mathbf{y}|a^{[n]}, nb, \boldsymbol{\phi}),$

avec

$$a^{[n]}(\mathbf{x}) = a(\mathbf{x}) * \dots * a(\mathbf{x}), \quad \text{et}$$

$$nb(\boldsymbol{\phi}) = \log \int a^{[n]}(\mathbf{s}) \exp[\boldsymbol{\phi}^t \mathbf{s}] \, d\mathbf{s}.$$

Normale (paramètres : $\boldsymbol{\mu}, \boldsymbol{\Lambda}$)

$$\mathbf{N}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}) = (2\pi)^{-\frac{k}{2}} |\boldsymbol{\Lambda}|^{\frac{1}{2}} \exp\left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Lambda}(\mathbf{x} - \boldsymbol{\mu})\right],$$

$$\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0,$$

$$\mathbf{x} \in \mathbb{R}^k,$$

$$\mathbb{E}[\mathbf{x}] = \boldsymbol{\mu},$$

$$\text{Var}[x_i] = (\boldsymbol{\Lambda}^{-1})_{ii},$$

$$\text{Cov}[x_i, x_j] = (\boldsymbol{\Lambda}^{-1})_{ij},$$

$$\mathbb{M}[\mathbf{x}] = \boldsymbol{\mu},$$

si $\mathbf{x} \sim \mathbf{N}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda})$, alors $z = (\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Lambda}(\mathbf{x} - \boldsymbol{\mu}) \sim \text{khi}^2(z|k)$

si $\mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} \sim \mathbf{N}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda})$

avec $\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \quad \boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} \\ \boldsymbol{\Lambda}_{21} & \boldsymbol{\Lambda}_{22} \end{pmatrix}$

alors

$$p(\mathbf{x}_1) = \mathbf{N}_{k_1}(\mathbf{x}_1|\boldsymbol{\mu}_1, \boldsymbol{\Lambda}_{11}),$$

$$p(\mathbf{x}_2) = \mathbf{N}_{k_2}(\mathbf{x}_2|\boldsymbol{\mu}_2, \boldsymbol{\Lambda}_{22}),$$

$$p(\mathbf{x}_1|\mathbf{x}_2) = \mathbf{N}_{k_1}(\mathbf{x}_1|\boldsymbol{\mu}', \boldsymbol{\Lambda}_{11}) \quad \text{avec} \quad \boldsymbol{\mu}' = \boldsymbol{\mu}_1 - \boldsymbol{\Lambda}_{11}^{-1} \boldsymbol{\Lambda}_{12}(\mathbf{x}_2 - \boldsymbol{\mu}_2),$$

$$p(\mathbf{x}_2|\mathbf{x}_1) = \mathbf{N}_{k_2}(\mathbf{x}_2|\boldsymbol{\mu}', \boldsymbol{\Lambda}_{22}) \quad \text{avec} \quad \boldsymbol{\mu}' = \boldsymbol{\mu}_2 - \boldsymbol{\Lambda}_{22}^{-1} \boldsymbol{\Lambda}_{21}(\mathbf{x}_1 - \boldsymbol{\mu}_1),$$

Normale (paramètres : $\boldsymbol{\mu}, \boldsymbol{\Sigma}$)

$$\mathbf{N}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma}) = (2\pi)^{-\frac{k}{2}} |\boldsymbol{\Sigma}|^{-\frac{1}{2}} \exp \left[-\frac{1}{2}(\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) \right],$$

$$\boldsymbol{\mu} \in \mathbf{R}^k, \boldsymbol{\Sigma} > 0,$$

$$\mathbf{x} \in \mathbf{R}^k,$$

$$\mathbb{E}[\mathbf{x}] = \boldsymbol{\mu},$$

$$\text{Var}[x_i] = \boldsymbol{\Sigma}_{ii},$$

$$\text{Cov}[x_i, x_j] = \boldsymbol{\Sigma}_{ij},$$

$$\mathbb{M}[\mathbf{x}] = \boldsymbol{\mu},$$

si $\mathbf{x} \sim \mathbf{N}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma})$, alors $z = (\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Sigma}^{-1}(\mathbf{x} - \boldsymbol{\mu}) \sim \text{Chi}_2(z|k)$

si $\mathbf{x} = \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} \sim \mathbf{N}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Sigma})$

avec $\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \quad \boldsymbol{\Sigma} = \begin{pmatrix} \boldsymbol{\Sigma}_{11} & \boldsymbol{\Sigma}_{12} \\ \boldsymbol{\Sigma}_{21} & \boldsymbol{\Sigma}_{22} \end{pmatrix}$

alors

$$p(\mathbf{x}_1) = \mathbf{N}_{k_1}(\mathbf{x}_1|\boldsymbol{\mu}_1, \boldsymbol{\Sigma}_{11}),$$

$$p(\mathbf{x}_2) = \mathbf{N}_{k_2}(\mathbf{x}_2|\boldsymbol{\mu}_2, \boldsymbol{\Sigma}_{22}),$$

$$p(\mathbf{x}_1|\mathbf{x}_2) = \mathbf{N}_{k_1}(\mathbf{x}_1|\boldsymbol{\mu}', \boldsymbol{\Sigma}_{11}) \quad \text{avec} \quad \boldsymbol{\mu}' = \boldsymbol{\mu}_1 - \boldsymbol{\Sigma}_{11} \boldsymbol{\Sigma}_{12}^{-1}(\mathbf{x}_2 - \boldsymbol{\mu}_2),$$

$$p(\mathbf{x}_2|\mathbf{x}_1) = \mathbf{N}_{k_2}(\mathbf{x}_2|\boldsymbol{\mu}', \boldsymbol{\Sigma}_{22}) \quad \text{avec} \quad \boldsymbol{\mu}' = \boldsymbol{\mu}_2 - \boldsymbol{\Sigma}_{22} \boldsymbol{\Sigma}_{21}^{-1}(\mathbf{x}_1 - \boldsymbol{\mu}_1),$$

Student

$$\mathbf{St}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha) = \frac{\Gamma((\alpha+k)/2)}{\Gamma(\alpha/2)(\alpha\pi)^{k/2}} (\boldsymbol{\Lambda})^{\frac{1}{2}} \left[1 + \frac{1}{\alpha} (\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Lambda} (\mathbf{x} - \boldsymbol{\mu}) \right]^{-(\alpha+k)/2},$$

$\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha > 0, \alpha$: nombre de degrés de liberté

$\mathbf{x} \in \mathbb{R}^k,$

$$\mathbb{E}[\mathbf{x}] = \boldsymbol{\mu},$$

$$\text{Var}[x_i] = (\boldsymbol{\Lambda}^{-1})_{ii},$$

$$\text{Cov}[x_i, x_j] = \frac{\alpha}{\alpha - 2} (\boldsymbol{\Lambda}^{-1})_{ij},$$

$$\mathbb{M}[x_i] =$$

si $\mathbf{x} \sim \mathbf{St}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha),$ alors $y = (\mathbf{x} - \boldsymbol{\mu})^t \boldsymbol{\Lambda} (\mathbf{x} - \boldsymbol{\mu}) \sim \mathbf{FS}_k(y|k, \alpha)$

si $\mathbf{x} = \begin{pmatrix} \mathbf{x}_1 \\ \mathbf{x}_2 \end{pmatrix} \sim \mathbf{St}_k(\mathbf{x}|\boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha)$

avec $\boldsymbol{\mu} = \begin{pmatrix} \boldsymbol{\mu}_1 \\ \boldsymbol{\mu}_2 \end{pmatrix}, \quad \boldsymbol{\Lambda} = \begin{pmatrix} \boldsymbol{\Lambda}_{11} & \boldsymbol{\Lambda}_{12} \\ \boldsymbol{\Lambda}_{21} & \boldsymbol{\Lambda}_{22} \end{pmatrix}$

alors

$$p(\mathbf{x}_1) = \mathbf{St}_{k_1}(\mathbf{x}_1|\boldsymbol{\mu}_1, \boldsymbol{\Lambda}_{11}, \alpha),$$

$$p(\mathbf{x}_2) = \mathbf{St}_{k_2}(\mathbf{x}_2|\boldsymbol{\mu}_2, \boldsymbol{\Lambda}_{22}, \alpha),$$

$$p(\mathbf{x}_1|\mathbf{x}_2) \sim \mathbf{St}_k(\mathbf{x}_1|\boldsymbol{\mu}', \boldsymbol{\Lambda}', \alpha + k_2)$$

$$\text{avec } \boldsymbol{\mu}' = \boldsymbol{\mu}_1 - \boldsymbol{\Lambda}_{11}^{-1} \boldsymbol{\Lambda}_{12} (\mathbf{x}_2 - \boldsymbol{\mu}_2),$$

$$\boldsymbol{\Lambda}' = \frac{\boldsymbol{\Lambda}_{11}}{\alpha + (\mathbf{x}_2 - \boldsymbol{\mu}_2)^t [\boldsymbol{\Lambda}_{22} - \boldsymbol{\Lambda}_{21} \boldsymbol{\Lambda}_{11}^{-1} \boldsymbol{\Lambda}_{12}]^{-1} (\mathbf{x}_2 - \boldsymbol{\mu}_2)} \boldsymbol{\Lambda}_{11}$$

$$p(\mathbf{x}_2|\mathbf{x}_1) \sim \mathbf{St}_k(\mathbf{x}_2|\boldsymbol{\mu}', \boldsymbol{\Lambda}', \alpha + k_1)$$

$$\text{avec } \boldsymbol{\mu}' = \boldsymbol{\mu}_2 - \boldsymbol{\Lambda}_{22}^{-1} \boldsymbol{\Lambda}_{21} (\mathbf{x}_1 - \boldsymbol{\mu}_1),$$

$$\boldsymbol{\Lambda}' = \frac{\boldsymbol{\Lambda}_{22}}{\alpha + (\mathbf{x}_1 - \boldsymbol{\mu}_1)^t [\boldsymbol{\Lambda}_{11} - \boldsymbol{\Lambda}_{12} \boldsymbol{\Lambda}_{22}^{-1} \boldsymbol{\Lambda}_{21}]^{-1} (\mathbf{x}_1 - \boldsymbol{\mu}_1)} \boldsymbol{\Lambda}_{22}$$

Wishart

$$\mathbf{Wi}_k(\mathbf{X}|\alpha, \mathbf{\Lambda}) = c |\mathbf{X}|^{\alpha-(k+1)/2} \exp[-\text{tr}(\mathbf{\Lambda X})],$$

$$c = \frac{|\mathbf{\Lambda}|^\alpha}{\Gamma_k(\alpha)}, \quad \text{avec} \quad \Gamma_k(\alpha) = \pi^{k(k-1)/4} \prod_{i=1}^k \Gamma\left(\frac{2\alpha+1-i}{2}\right)$$

$\mathbf{\Lambda}$ une matrice de dimensions $k \times k$,

\mathbf{X} une matrice symétrique d.p. de dimensions $k \times k$,

$$X_{i,j} = X_{j,i}, \quad i, j = 1, \dots, k,$$

$$2\alpha > k - 1,$$

$$E[\mathbf{X}] = \alpha \mathbf{\Lambda}^{-1},$$

$$E[\mathbf{X}^{-1}] = (\alpha - (k+1)/2)^{-1} \mathbf{\Lambda},$$

$$M[\mathbf{X}] =$$

$$k = 1: \quad \mathbf{Wi}_1(x|\alpha, \mathbf{\Lambda}) = \mathbf{Gam}(x|\alpha, \mathbf{\Lambda})$$

si $\mathbf{X} \sim \mathbf{Wi}_k(\mathbf{X}|\alpha, \mathbf{\Lambda})$, alors

$$\mathbf{Y} = \mathbf{A X A}^t \sim \mathbf{Wi}_m(\mathbf{Y}|\alpha, (\mathbf{A X}^{-1} \mathbf{A}^t)^{-1})$$

si $\{\mathbf{X}_1, \dots, \mathbf{X}_n\}$, $\mathbf{X}_i \sim \mathbf{N}(\mathbf{X}_i|\boldsymbol{\mu}, \mathbf{\Lambda})$, alors

$$\bar{\mathbf{X}}_i = \frac{1}{n} \sum \mathbf{X}_i, \sim \mathbf{N}(\bar{\mathbf{X}}_i|\boldsymbol{\mu}, n\mathbf{\Lambda}),$$

$$\mathbf{S} = (\mathbf{X}_i - \bar{\mathbf{X}}_i)^t (\mathbf{X}_i - \bar{\mathbf{X}}_i) \sim \mathbf{Wi}_k\left(\mathbf{S} \mid \frac{n-1}{2}, \frac{1}{2} \mathbf{\Lambda}\right)$$

$$\text{si} \quad \mathbf{X} = \begin{pmatrix} \mathbf{X}_{11} & \mathbf{X}_{12} \\ \mathbf{X}_{21} & \mathbf{X}_{22} \end{pmatrix}, \quad \mathbf{\Lambda}^{-1} = \begin{pmatrix} \mathbf{\Lambda}_{11} & \mathbf{\Lambda}_{12} \\ \mathbf{\Lambda}_{21} & \mathbf{\Lambda}_{22} \end{pmatrix},$$

alors

$$\mathbf{X}_{11} \sim \mathbf{Wi}_m(\mathbf{X}_{11}|\alpha, \mathbf{\Lambda}_{11}^{-1})$$

$$\int |\mathbf{x}|^{\alpha-(k+1)/2} \exp[-\text{tr}(\mathbf{\Lambda x})] d\mathbf{x} = c^{-1}$$

Lois de probabilité à $n + 1$ variables réelles (suite)**Normal-Gamma**

$$\mathbf{NGam}_k(\mathbf{x}, y | \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta) = \mathbf{N}_k(\mathbf{x} | \boldsymbol{\mu}, y \boldsymbol{\Lambda}) \mathbf{Gam}(y | \alpha, \beta),$$

$$\boldsymbol{\mu} \in \mathbb{R}^k, \boldsymbol{\Lambda} > 0, \alpha, \beta > 0,$$

$$\mathbf{x} \in \mathbb{R}^k, y > 0,$$

$$p(\mathbf{x}) = \int_{-\infty}^{+\infty} p(\mathbf{x}, y) d\mathbf{x} = \int_{-\infty}^{+\infty} \mathbf{NGam}_k(\mathbf{x}, y | \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta) d\mathbf{x} = \mathbf{St}\left(\mathbf{x} | \boldsymbol{\mu}, \frac{\alpha}{\beta} \boldsymbol{\Lambda}, 2\alpha\right),$$

$$p(y) = \int_0^{+\infty} p(\mathbf{x}, y) dy = \int_0^{+\infty} \mathbf{NGam}_k(\mathbf{x}, y | \boldsymbol{\mu}, \boldsymbol{\Lambda}, \alpha, \beta) dy = \mathbf{Gam}(y | \alpha, \beta),$$

$$p(\mathbf{x} | y) = \frac{p(\mathbf{x}, y)}{p(y)} = \mathbf{N}_k(\mathbf{x} | \boldsymbol{\mu}, y \boldsymbol{\Lambda}),$$

$$\mathbf{E}[\mathbf{x}] = \boldsymbol{\mu}, \quad \text{si } 2\alpha > 1, \quad \text{Var}[\mathbf{x}] = \frac{2\beta}{2\alpha - 2} \frac{1}{\boldsymbol{\Lambda}} \quad \text{si } \alpha > 1$$

$$\mathbf{E}[y] = \frac{\alpha}{\beta}, \quad \text{Var}[y] = \frac{\alpha}{\beta^2}$$

$$\mathbf{E}[\mathbf{x} \# 2] = \boldsymbol{\mu}, \quad \text{Var}[\mathbf{x} | y] = y \boldsymbol{\Lambda}$$

Normale-Wishart

$$\mathbf{NWi}_k(\mathbf{x}, \mathbf{Y} | \boldsymbol{\mu}, \lambda, \alpha, \mathbf{B}) = \mathbf{N}_k(\mathbf{x} | \boldsymbol{\mu}, \lambda \mathbf{Y}) \mathbf{Wi}_k(\mathbf{Y} | \alpha, \mathbf{B}),$$

$$\boldsymbol{\mu} \in \mathbf{R}^k, \lambda > 0, 2\alpha > k - 1, \mathbf{B} > 0,$$

$$\mathbf{x} \in \mathbf{R}^k, Y_{i,j} = Y_{j,i}, i, j = 1, \dots, k,$$

$$p(\mathbf{x} | \mathbf{Y}) = \mathbf{N}_k(\mathbf{x} | \boldsymbol{\mu}, \lambda \mathbf{Y}),$$

$$p(\mathbf{Y}) = \mathbf{Wi}_k(\mathbf{Y} | \alpha, \mathbf{B}),$$

$$p(\mathbf{x}) = \mathbf{St}_k(\mathbf{x} | \boldsymbol{\mu}, \lambda \alpha \mathbf{B}^{-1}, 2\alpha),$$

$$\mathbf{E}[\mathbf{x}] = \boldsymbol{\mu}, \quad \text{si } \alpha > 1, \quad \text{Var}[\mathbf{x}] = \frac{\alpha}{\alpha - 2} \frac{1}{\lambda} \quad \text{si } \alpha > 2$$

$$\mathbf{E}[Y] = \frac{\alpha}{B}, \quad \text{Var}[Y] = \frac{\alpha}{B^2}$$

$$\mathbf{E}[\mathbf{x}\#\mathbf{2}] = \boldsymbol{\mu}, \quad \text{Var}[\mathbf{x} | Y] = \lambda Y$$

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