

Modélisation par champs de markov cachés et son utilisation dans les problèmes inverses

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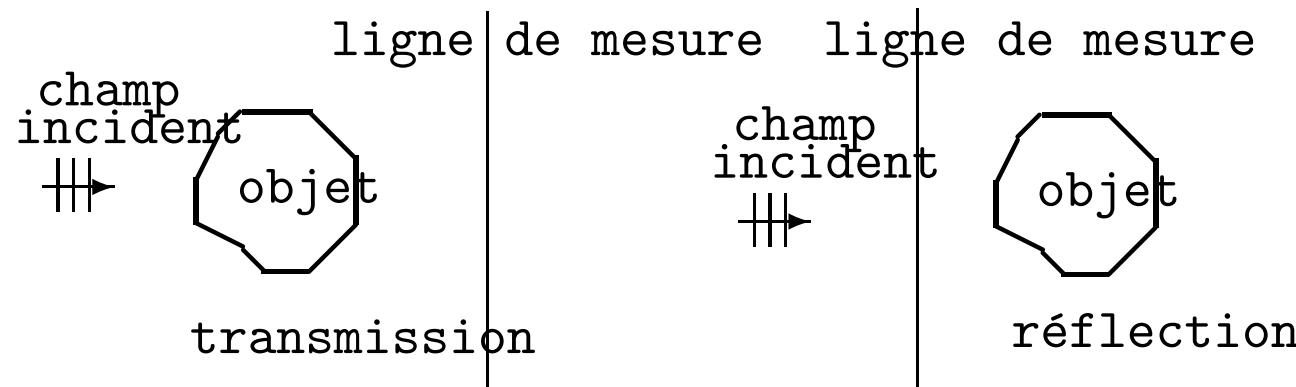
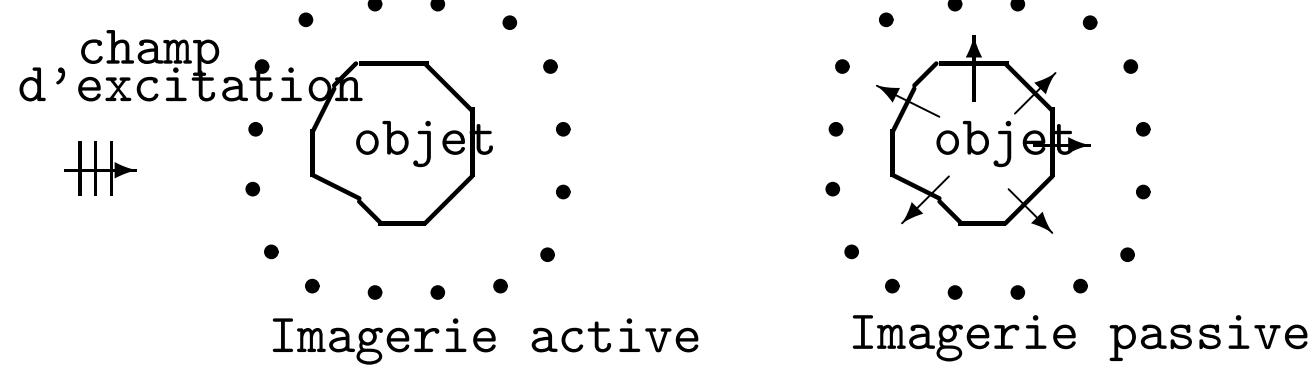
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PLAN DE L'EXPOSÉ

- Problèmes inverse en imagerie
- Inversion analytique/algébrique
- Inversion déterministe/probabiliste
- Approche bayésienne
- **Modélisations a priori des signaux et des images**
- Modèles séparables
- Modèles markoviens simples
- Modèles markoviens avec variables cachées lignes, contours et régions
- **Aspects mise en œuvres et calcul bayésien**
- **Exemples:** Tomographie X, Imagerie microondes, Fusion d'images, Super-résolution, Séparation de sources, Imagerie hyperspectrale
- Conclusions

PROBLÈMES INVERSES EN IMAGERIE: TOMOGRAPHIE

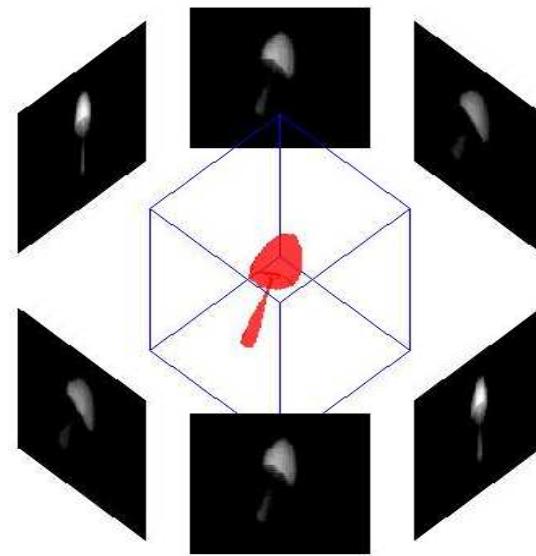


Problème directe: Connaisant l'**objet** calculer les **mesures**

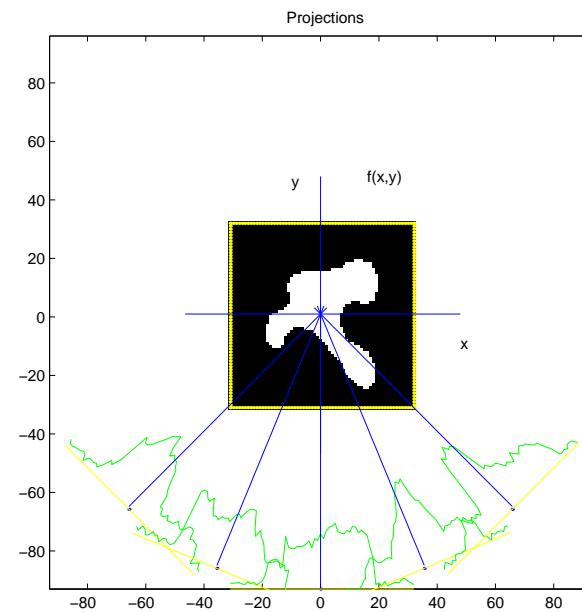
Problème inverse: Connaisant les **mesures** trouver l'**objet**

TOMOGRAPHIE X

3D



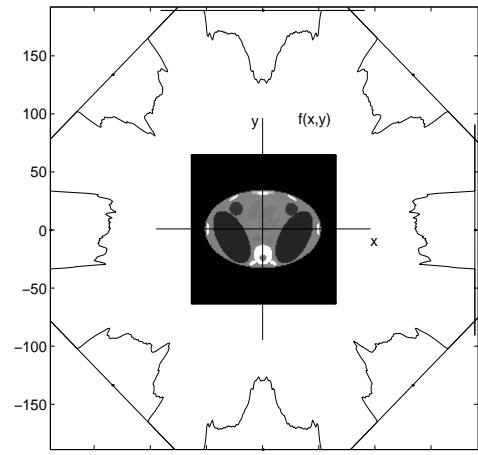
2D



$$g_\phi(r_1, r_2) = \int_{\mathcal{L}_{r_1, r_2, \phi}} f(x, y, z) \, dl \quad g_\phi(r) = \int_{\mathcal{L}_{r, \phi}} f(x, y) \, dl$$

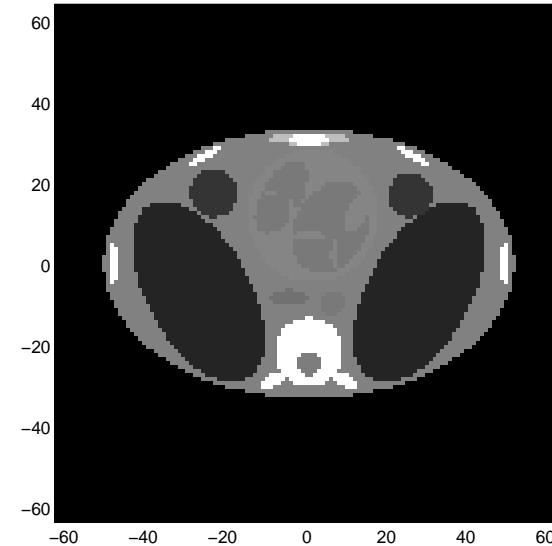
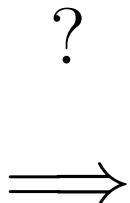
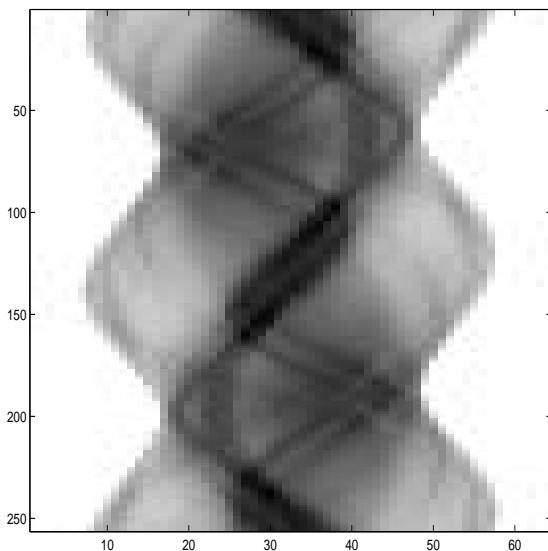
- Problème directe: $f(x, y)$ ou $f(x, y, z) \rightarrow g_\phi(r)$ or $g_\phi(r_1, r_2)$
- Problème inverse: $g_\phi(r)$ ou $g_\phi(r_1, r_2) \rightarrow f(x, y)$ or $f(x, y, z)$

TOMOGRAPHIE X ET TRANSFORMÉE DE RADON



$$g(r, \phi) = \int_{L_{r,\phi}} f(x, y) \, dl$$

$$g(r, \phi) = \iint_D f(x, y) \delta(r - x \cos \phi - y \sin \phi) \, dx \, dy$$



INVERSION ANALYTIQUE

$$f(x, y) = \left(-\frac{1}{2\pi^2} \right) \int_0^\pi \int_{-\infty}^{+\infty} \frac{\frac{\partial}{\partial r} g(r, \phi)}{(r - x \cos \phi - y \sin \phi)} dr d\phi$$

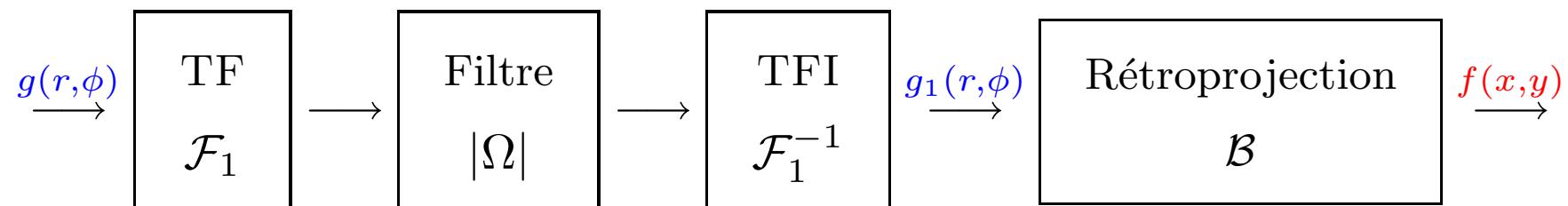
Dérivation \mathcal{D} : $\bar{g}(r, \phi) = \frac{\partial g(r, \phi)}{\partial r}$

Transformée de Hilbert \mathcal{H} : $g_1(r', \phi) = \frac{1}{\pi} \int_0^\infty \frac{\bar{g}(r, \phi)}{(r - r')} dr$

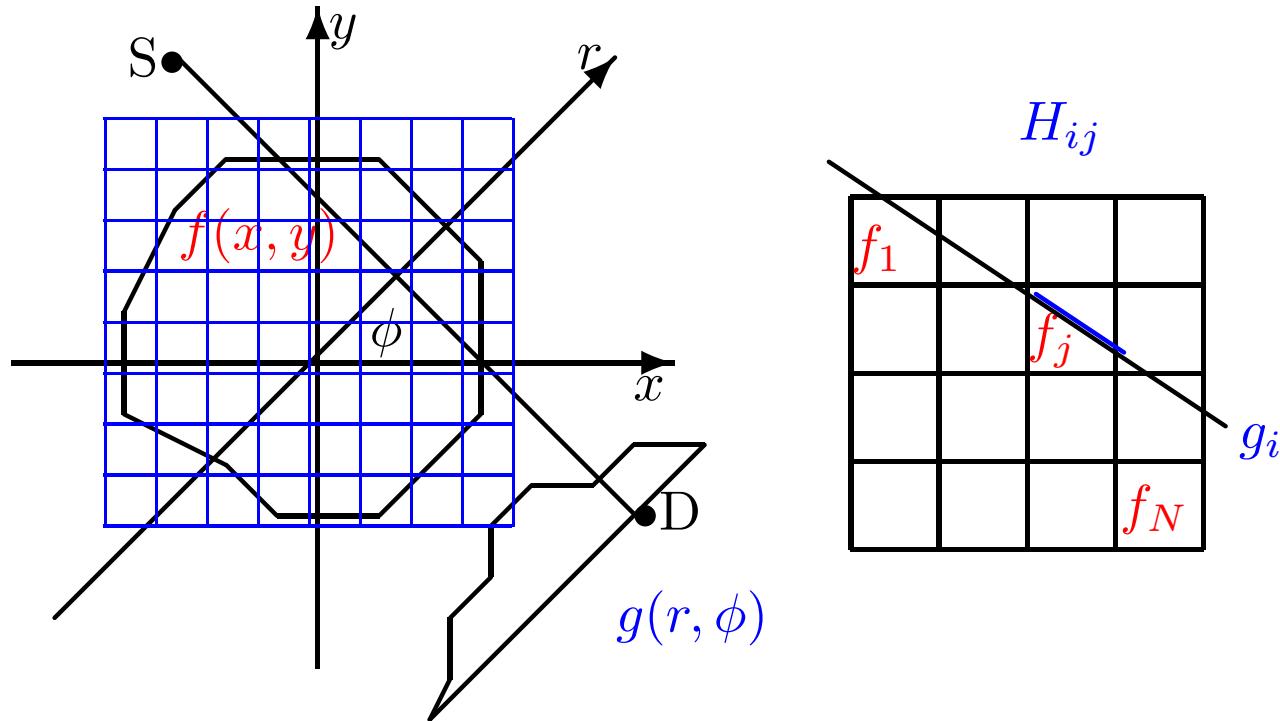
Rétroprojection \mathcal{B} : $f(x, y) = \frac{1}{2\pi} \int_0^\pi g_1(r' = x \cos \phi + y \sin \phi, \phi) d\phi$

$$f(x, y) = \mathcal{B} \mathcal{H} \mathcal{D} g(r, \phi) = \mathcal{B} \mathcal{F}_1^{-1} |\Omega| \mathcal{F}_1 g(r, \phi)$$

- Rétroprojection des projections filtrées:



INVERSION ALGÉBRIQUE



$$g(r, \phi) = \int_L f(x, y) \, dl$$

$$g_i = \sum_{j=1}^N H_{i,j} f_j + \epsilon_i$$

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$

IMAGERIE À ONDES DIFFRACTÉES

Mesures : champ diffracté par objet $g(\mathbf{r}_i) = \phi_d(\mathbf{r}_i)$

Grandeur inconnue : $f(\mathbf{r}) = k_0^2(n^2(\mathbf{r}) - 1)$

Grandeurs intermédiaires : $\phi(\mathbf{r})$

$$g(\mathbf{r}_i) = \iint_D G_m(\mathbf{r}_i, \mathbf{r}') \phi(\mathbf{r}') f(\mathbf{r}') d\mathbf{r}', \quad \mathbf{r}_i \in S$$

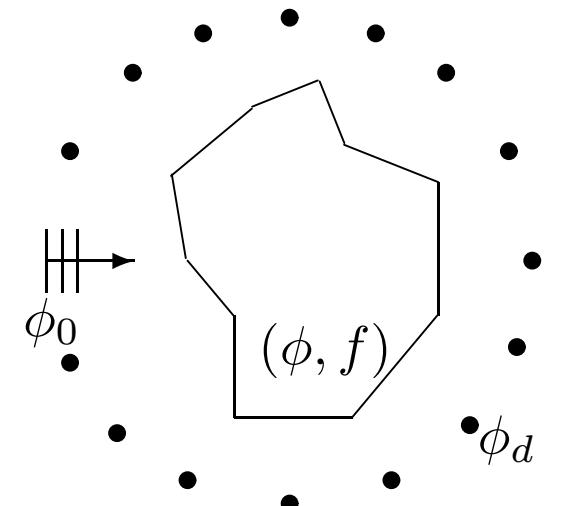
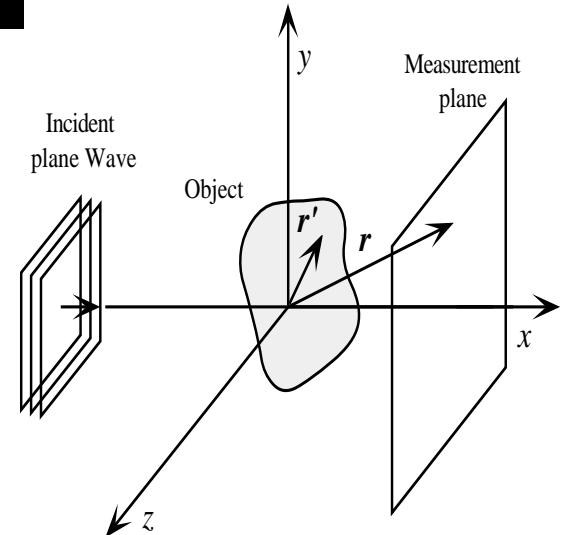
$$\phi(\mathbf{r}) = \phi_0(\mathbf{r}) + \iint_D G_o(\mathbf{r}, \mathbf{r}') \phi(\mathbf{r}') f(\mathbf{r}') d\mathbf{r}', \quad \mathbf{r} \in D$$

Approximation de Born ($\phi(\mathbf{r}') \simeq \phi_0(\mathbf{r}')$):

$$g(\mathbf{r}_i) = \iint_D G_m(\mathbf{r}_i, \mathbf{r}') \phi_0(\mathbf{r}') f(\mathbf{r}') d\mathbf{r}', \quad \mathbf{r}_i \in S$$

Discréétisation : notations: $\mathbf{F} = \text{diag}(\mathbf{f})$ et $\Phi = \text{diag}[\phi]$

$$\begin{cases} \mathbf{g} = \mathbf{G}_M \mathbf{F} \phi = \mathbf{G}_M \Phi \mathbf{f} \\ \phi = \phi_0 + \mathbf{G}_O \mathbf{F} \phi \end{cases} \xrightarrow{\quad} \begin{cases} \mathbf{g} = \mathbf{H}(\mathbf{f}) \text{ avec} \\ \mathbf{H}(\mathbf{f}) = \mathbf{G}_M \mathbf{F} (\mathbf{I} - \mathbf{G}_O \mathbf{F})^{-1} \phi_0 \end{cases}$$



MÉTHODES D'INVERSION DÉTERMINISTE

Ingédients de base :

- Un modèle directe : $\mathbf{g} = \mathbf{H}(\mathbf{f}) + \epsilon$
- Une mesure d'adéquation aux données : $Q(\mathbf{g}, \mathbf{H}(\mathbf{f}))$
- Une paramétrisation $\mathbf{f}(\boldsymbol{\theta})$ ou une fonctionnelle de régularisation $\Phi(\mathbf{f})$

Méthodes d'inversion standard :

- Estimation paramétrique : $\hat{\boldsymbol{\theta}} = \arg \min_{\boldsymbol{\theta}} \{Q(\mathbf{g}, \mathbf{H}(\mathbf{f}(\boldsymbol{\theta}))) = \|\mathbf{g} - \mathbf{H}(\mathbf{f}(\boldsymbol{\theta}))\|^2\}$
- Optimisation sous contraintes :

$$\hat{\mathbf{f}} = \arg \min_{\mathbf{f}} \{\Phi(\mathbf{f})\} \quad \text{sous contrainte } \mathbf{g} = \mathbf{H}(\mathbf{f}) \quad \text{ou} \quad Q(\mathbf{g}, \mathbf{H}(\mathbf{f})) < \epsilon$$

- Critère régularisé :

$$\hat{\mathbf{f}} = \arg \min_{\mathbf{f}} \{Q(\mathbf{g}, \mathbf{H}(\mathbf{f})) + \lambda \Phi(\mathbf{f})\}$$

MÉTHODES PROBABILISTES

Maximum de vraisemblance (MV)

$$\left\{ \begin{array}{l} \text{Modèle d'observation : } \quad \mathbf{g} = \mathbf{H}(\mathbf{f}) + \boldsymbol{\epsilon} \quad \longrightarrow p(\mathbf{g}|\mathbf{f}) \\ \text{Caractéristiques du bruit : } \quad p_{\boldsymbol{\epsilon}}(\boldsymbol{\epsilon}) \end{array} \right.$$

$$\widehat{\mathbf{f}} = \arg \max_{\mathbf{f}} \{p(\mathbf{g}|\mathbf{f})\} = \arg \min_{\mathbf{f}} \{-\ln p(\mathbf{g}|\mathbf{f})\}$$

- Cas linéaire gaussien : \longrightarrow Moindres Carrés
- Méthode rarement satisfaisante pour les problèmes inverses
- Maximum de vraisemblance pénalisée

$$\widehat{\mathbf{f}} = \arg \min_{\mathbf{f}} \{-\ln p(\mathbf{g}|\mathbf{f}) + \Phi(\mathbf{f})\}$$

APPROCHE ESTIMATION BAYÉSIENNE

Ingédients de base :

- Modèle directe : $\mathbf{g} = \mathbf{H}(\mathbf{f}) + \boldsymbol{\epsilon}$
- Mesure d'adéquation aux données : $p(\mathbf{g}|\mathbf{f}) = p_{\boldsymbol{\epsilon}}(\mathbf{g} - \mathbf{H}(\mathbf{f}))$
- Information *a priori* $p(\mathbf{f})$
- Règle de Bayes : $p(\mathbf{f}|\mathbf{g}) = \frac{p(\mathbf{g}|\mathbf{f}) p(\mathbf{f})}{p(\mathbf{g})} \propto p(\mathbf{g}|\mathbf{f}) p(\mathbf{f})$
- Choix d'un estimateur $\hat{\mathbf{f}} = \arg \min_{\mathbf{f}'} \left\{ \int C(\mathbf{f}' - \mathbf{f}) p(\mathbf{f}'|\mathbf{g}) \, d\mathbf{f}' \right\}$

Lien avec régularisation :

- Estimation au sens du maximum *a posteriori* (MAP) :

$$\hat{\mathbf{f}} = \arg \max_{\mathbf{f}} \{ \ln p(\mathbf{f}|\mathbf{g}) = \ln p(\mathbf{g}|\mathbf{f}) + \ln p(\mathbf{f}) \}$$

avec $Q(\mathbf{g}, \mathbf{H}(\mathbf{f})) = \ln p(\mathbf{g}|\mathbf{f})$ et $\Phi(\mathbf{f}) = \ln p(\mathbf{f})$

CAS LINÉAIRE ET MODÈLES A PRIORI GAUSSIENS

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$

- Hypothèse sur les erreurs: $\boldsymbol{\epsilon} \sim \mathcal{N}(\mathbf{0}, \sigma_\epsilon^2 \mathbf{I})$
 $\rightarrow \mathbf{g}|\mathbf{f} \sim \mathcal{N}(\mathbf{H}\mathbf{f}, \beta\mathbf{I}) \rightarrow p(\mathbf{g}|\mathbf{f}) \propto \exp[-\beta\|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2], \quad \beta = \frac{1}{2\sigma_\epsilon^2}$
- Modèle a priori \mathbf{f} :
 $\mathbf{f} \sim \mathcal{N}(\mathbf{0}, \sigma_f^2 (\mathbf{D}^t \mathbf{D})^{-1}) \rightarrow p(\mathbf{f}) \propto \exp[-\alpha\|\mathbf{D}\mathbf{f}\|^2], \quad \alpha = \frac{1}{2\sigma_f^2}$
- A posteriori:
 $p(\mathbf{f}|\mathbf{g}) \propto \exp[-\|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2 + \lambda\|\mathbf{D}\mathbf{f}\|^2] \quad \lambda = \frac{\sigma_\epsilon^2}{\sigma_f^2} = \frac{\alpha}{\beta}$
- MAP : $\widehat{\mathbf{f}} = \arg \max_{\mathbf{f}} \{p(\mathbf{f}|\mathbf{g})\} = \arg \min_{\mathbf{f}} \{J(\mathbf{f})\}$
avec $J(\mathbf{f}) = \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2 + \lambda\|\mathbf{D}\mathbf{f}\|^2$
- Avantage : caractérisation de la solution

$$\mathbf{f}|\mathbf{g} \sim \mathcal{N}(\widehat{\mathbf{f}}, \widehat{\mathbf{P}}) \quad \text{avec} \quad \widehat{\mathbf{f}} = \widehat{\mathbf{P}} \mathbf{H}^t \mathbf{g}, \quad \widehat{\mathbf{P}} = (\mathbf{H}^t \mathbf{H} + \lambda \mathbf{D}^t \mathbf{D})^{-1}$$

Estimation au sens du MAP ou régularisation:

$$\widehat{\mathbf{f}} = \arg \min_{\mathbf{f}} \{J(\mathbf{f})\} \quad \text{avec} \quad J(\mathbf{f}) = \|\mathbf{g} - \mathbf{H}(\mathbf{f})\|^2 + \lambda \Phi(\mathbf{f})$$

- Lois gaussiennes: $\Phi(\mathbf{f})$ quadratique
 $\longrightarrow J(\mathbf{f})$ quadratique $\longrightarrow \widehat{\mathbf{f}}$ fonction linéaire de \mathbf{g} \longrightarrow Algorithmes rapides
- Lois non gaussiennes mais $\Phi(\mathbf{f})$ convexe:
Exemples: $\Phi(\mathbf{f}) = \sum_j \phi(f_j)$ ou $\Phi(\mathbf{f}) = \sum_j \phi(f_j - f_{j-1})$
avec $\phi(t) = \{|t|^p, \quad (|t| \ln |t| - |t|), \quad (\ln(1 + |t|))\}$
 $\longrightarrow J(\mathbf{f})$ convexe \longrightarrow Estimation nonlinéaire mais facile à calculer
- Lois non gaussiennes et $\Phi(\mathbf{f})$ non convexe:
Exemples:
 $\Phi(t) = \begin{cases} |t|^2 & \text{if } |t| < \alpha, \\ \alpha^2 & \text{else,} \end{cases}, \quad \begin{cases} t^2 & \text{if } |t| < \alpha, \\ 2\alpha t - \alpha^2 & \text{else,} \end{cases}, \quad \frac{\alpha^2 t^2}{1+t^2}, \quad \log \cosh(t/\alpha)$
 $\longrightarrow J(\mathbf{f})$ non convexe \longrightarrow Minima locaux \longrightarrow Optimisation globale

MODÉLISATIONS A PRIORI DES SIGNAUX ET DES IMAGES

- Modèles séparables

$$p(\mathbf{f}) = \prod_j p_j(f_j) \propto \exp \left[-\beta \sum_j \phi(f_j) \right] \text{ ou}$$

$$p(\mathbf{f}) \propto \exp \left[-\beta \sum_{\mathbf{r} \in \mathcal{R}} \phi(f(\mathbf{r})) \right]$$

- Modèles markoviens simples

$$p(f_j | f_{j-1}) \propto \exp [-\beta \phi(f_j - f_{j-1})] \text{ ou}$$

$$p(\mathbf{f}) \propto \exp \left[-\beta \sum_{\mathbf{r} \in \mathcal{R}} \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \phi(f(\mathbf{r}), f(\mathbf{r}')) \right]$$

- Modèles markoviens avec variables cachées

$z(\mathbf{r})$ (lignes, contours, frontières et régions)

$$p(\mathbf{f}|\mathbf{z}) \propto \exp \left[-\beta \sum_{\mathbf{r} \in \mathcal{R}} \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \phi(f(\mathbf{r}), f(\mathbf{r}'), z(\mathbf{r}), z(\mathbf{r}')) \right]$$

MODÈLES SÉPARABLES IID

- Gaussienne:

$$p(f_j) \propto \exp[-\alpha|f_j|^2] \longrightarrow \Omega(\mathbf{f}) = \alpha \sum_j |f_j|^2$$

- Gaussienne généralisée:

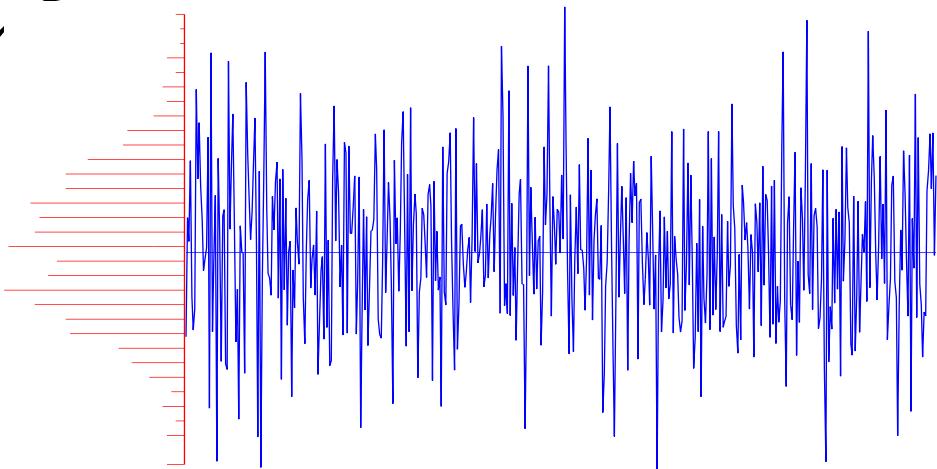
$$p(f_j) \propto \exp[-\alpha|f_j|^p], \quad 1 \leq p \leq 2 \longrightarrow \Phi(\mathbf{f}) = \alpha \sum_j |f_j|^p,$$

- Gamma: $f_j > 0$

$$p(f_j) \propto f_j^\alpha \exp[-\beta f_j] \longrightarrow \Omega(\mathbf{f}) = \alpha \sum_j \ln f_j + \beta \sum_j f_j,$$

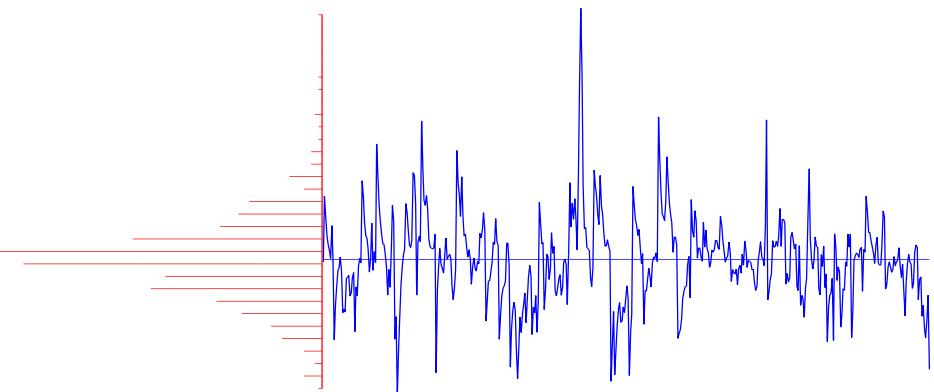
- Béta: $0 < f_j < 1$

$$p(f_j) \propto f_j^\alpha (1-f_j)^\beta \longrightarrow \Omega(\mathbf{f}) = \alpha \sum_j \ln f_j + \beta \sum_j \ln(1-f_j),$$



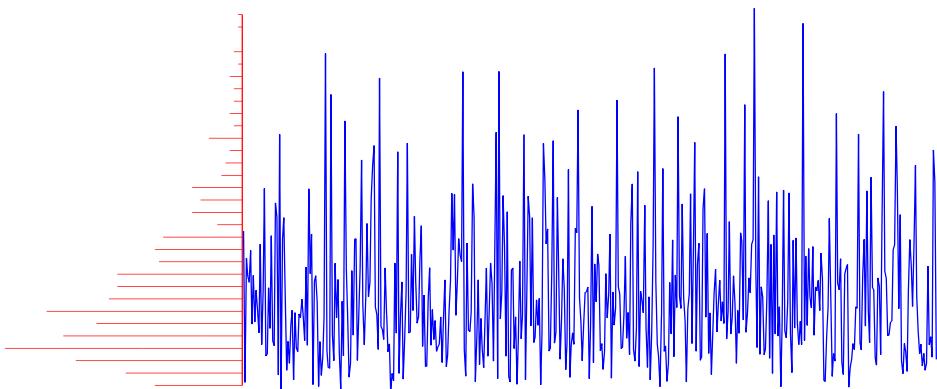
Gaussienne

$$p(f_j) \propto \exp [-\alpha |f_j|^2]$$



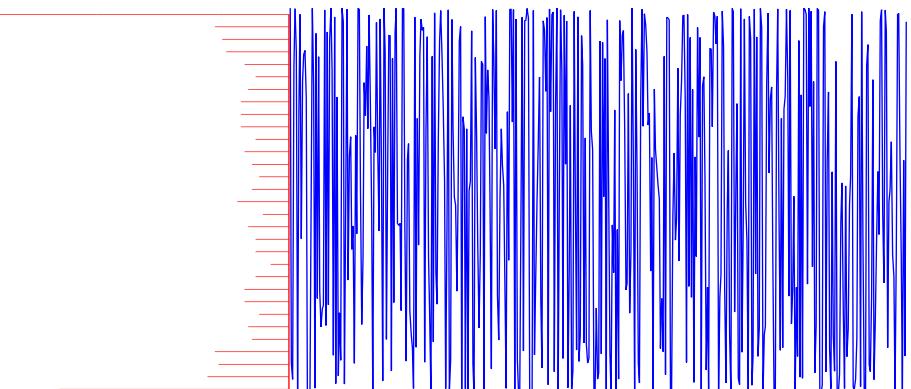
Gaussienne généralisée

$$p(f_j) \propto \exp [-\alpha |f_j|^p], \quad 1 \leq p \leq 2$$



Gamma

$$p(f_j) \propto f_j^\alpha \exp [-\beta f_j]$$



Beta

$$p(f_j) \propto f_j^\alpha (1 - f_j)^\beta$$

MODÈLES MARKOVIENS SIMPLES

$$p(f_j | \mathbf{f}) \propto \exp \left[-\alpha \sum_{i \in v_j} \phi(f_j, f_i) \right] \longrightarrow \quad \Phi(\mathbf{f}) = \alpha \sum_j \sum_{i \in V_j} \phi(f_j, f_i)$$

- Cas 1D et un seul voisin $V_j = j - 1$:

$$\Phi(\mathbf{f}) = \alpha \sum_j \phi(f_j - f_{j-1})$$

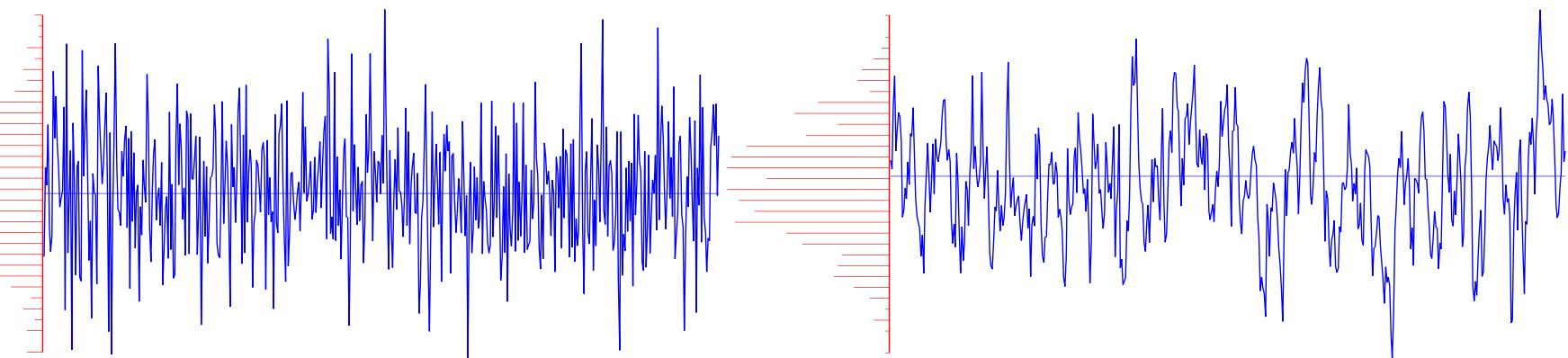
- Cas 1D et deux voisins $V_j = \{j - 1, j + 1\}$:

$$\Phi(\mathbf{f}) = \alpha \sum_j \phi(f_j - \beta(f_{j-1} + f_{j+1}))$$

- Cas 2D et les quatres voisins:

$$\Phi(\mathbf{f}) = \alpha \sum_{\mathbf{r} \in \mathcal{R}} \phi \left(f(\mathbf{r}) - \beta \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} f(\mathbf{r}') \right)$$

- $\phi(t) = |t|^\gamma$: Gaussienne généralisée



Gaussienne iid

$$p(f_j) \propto \exp [-\alpha |f_j|^2]$$

Gauss-Markov

$$p(f_j|f_{j-1}) \propto \exp [-\alpha |f_j - f_{j-1}|^2]$$

Gaussienne généralisée iid

$$p(f_j) \propto \exp [-\alpha |f_j|^p]$$

Gaussienne généralisée markovienne

$$p(f_j|f_{j-1}) \propto \exp [-\alpha |f_j - f_{j-1}|^p]$$

MODÈLES GAUSSIENS EN TERME DE VARIABLES CACHÉES

$$\left\{ \begin{array}{l} \mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon} \\ \text{avec } \mathbf{f} \sim \mathcal{N}\left(\mathbf{0}, \sigma_f^2 (\mathbf{D}^t \mathbf{D})^{-1}\right) \end{array} \right. = \left\{ \begin{array}{l} \mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon} \\ \mathbf{f} = \mathbf{C}\mathbf{f} + \mathbf{z} \text{ avec } \mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma_f^2 \mathbf{I}) \\ \text{et } \mathbf{D} = (\mathbf{I} - \mathbf{C}) \end{array} \right.$$

$$\mathbf{f}|\mathbf{g} \sim \mathcal{N}(\hat{\mathbf{f}}, \hat{\mathbf{P}}) \text{ avec } \hat{\mathbf{f}} = \hat{\mathbf{P}} \mathbf{H}^t \mathbf{g}, \quad \hat{\mathbf{P}} = (\mathbf{H}^t \mathbf{H} + \lambda \mathbf{D}^t \mathbf{D})^{-1}$$

$$\left\{ \begin{array}{l} \mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon} \\ \text{avec } \mathbf{f} \sim \mathcal{N}\left(\mathbf{0}, \sigma_f^2 (\mathbf{D} \mathbf{D}^t)\right) \end{array} \right. = \left\{ \begin{array}{l} \mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon} \\ \mathbf{f} = \mathbf{D}\mathbf{z} \text{ avec } \mathbf{z} \sim \mathcal{N}(\mathbf{0}, \sigma_f^2 \mathbf{I}) \end{array} \right.$$

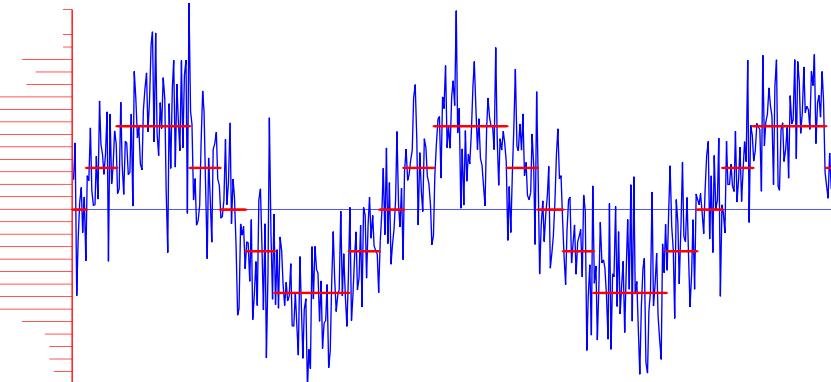
$$\mathbf{z}|\mathbf{g} \sim \mathcal{N}(\hat{\mathbf{z}}, \hat{\mathbf{P}}) \text{ avec } \hat{\mathbf{z}} = \hat{\mathbf{P}} \mathbf{D}^t \mathbf{H}^t \mathbf{g}, \quad \hat{\mathbf{P}} = (\mathbf{D}^t \mathbf{H}^t \mathbf{H} \mathbf{D} + \lambda \mathbf{I})^{-1} \longrightarrow \hat{\mathbf{f}} = \mathbf{D}\hat{\mathbf{z}}$$

\mathbf{z} coefficients de décomposition sur une base (colonnes de \mathbf{D} forment une base)

SIGNALS NON STATIONNAIRES

Variance modulée Gauss iid

$$p(f_j|z_j) = \mathcal{N}(0, v(z_j))$$

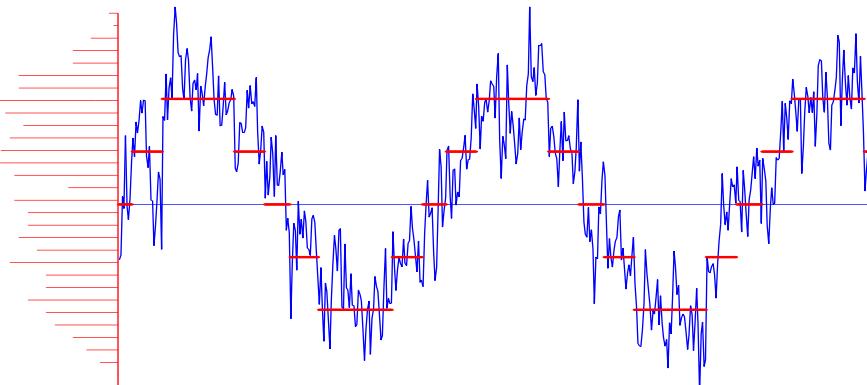


Amplitude modulée Gauss iid

$$p(f_j|z_j) = \mathcal{N}(a(z_j), 1)$$

Variance modulée Gauss-Markov

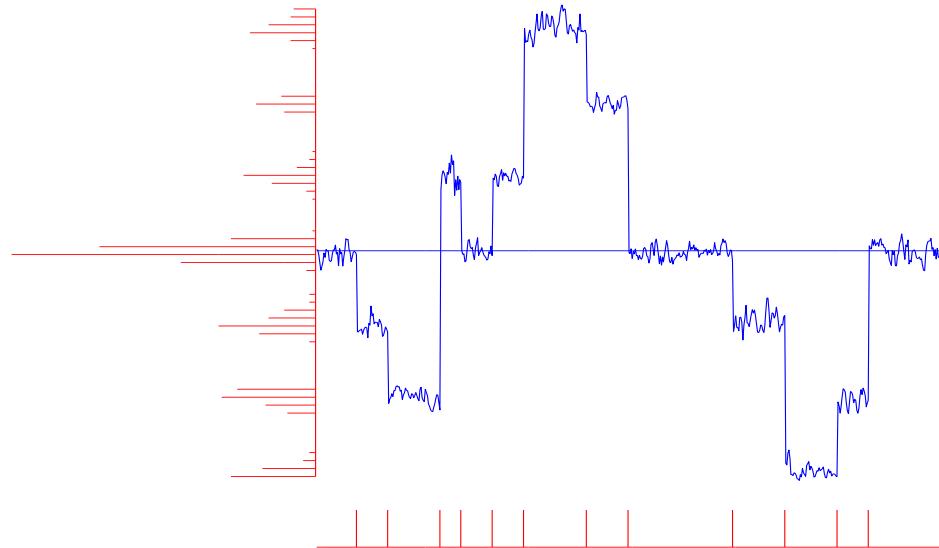
$$p(f_j|f_{j-1}, z_j) = \mathcal{N}(f_{j-1}, v(z_j))$$



Amplitude modulée Gauss-Markov

$$p(f_j|f_{j-1}, z_j) = \mathcal{N}(a(f_{j-1}, z_j), 1)$$

MODÈLES MARKOVIENS AVEC DES VARIABLES CACHÉES

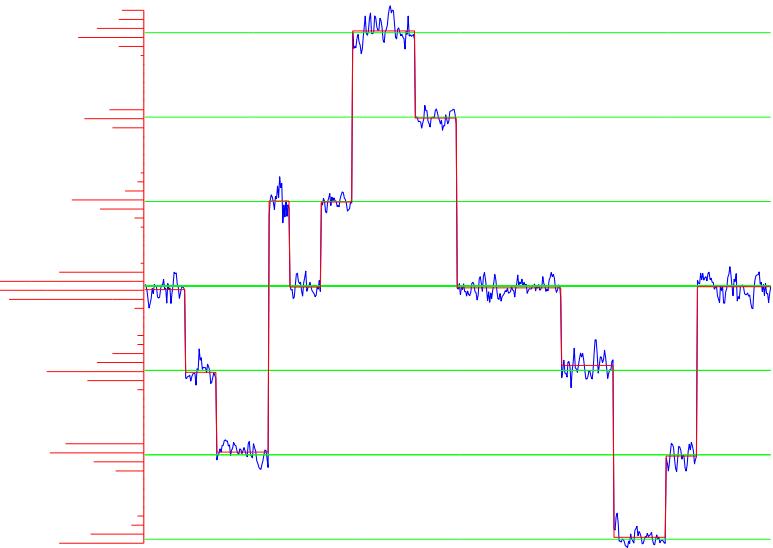


Gaussiennes par morceaux

(variables cachées ligne ou contours)

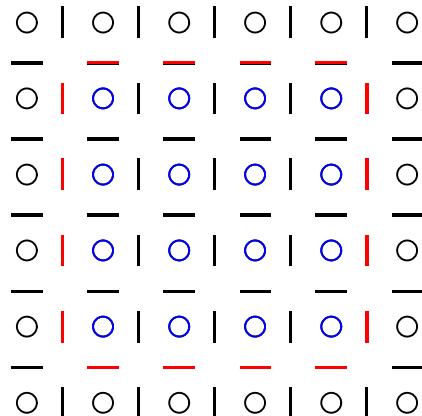
$$p(f_j|q_j, f_{j-1}) = \mathcal{N}((1 - q_j)f_{j-1}, \sigma_f^2) \quad p(f_j|z_j = k) = \mathcal{N}(m_k, \sigma_k^2) \text{ et } z_j \text{ markovien}$$

$$p(\mathbf{f}|\mathbf{q}) \propto \exp \left[-\alpha \sum_j |f_j - (1 - q_j)f_{j-1}|^2 \right] \quad p(\mathbf{f}|\mathbf{z}) \propto \exp \left[-\alpha \sum_k \sum_{j \in \mathcal{R}_k} \left(\frac{f_j - m_k}{\sigma_k} \right)^2 \right]$$



Mélange de gaussiennes

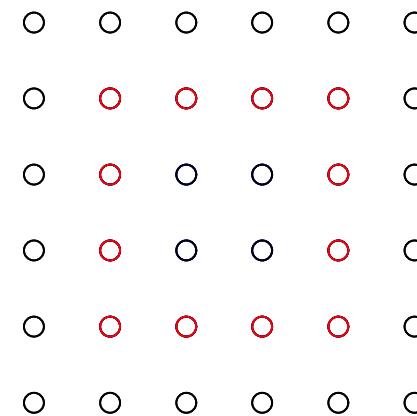
MODÈLES MARKOVIENS AVEC DES VARIABLES CACHÉES



$f(\mathbf{r})$ intensités

$l(\mathbf{r}, \mathbf{r}')$ lignes

$q(\mathbf{r}, \mathbf{r}')$ contours



$f(\mathbf{r})$ intensités

$z(\mathbf{r})$ étiquettes régions

$q(\mathbf{r})$ frontières des régions

$\mathcal{R} = \{(i, j) : 1 \leq i \leq m, 1 \leq j \leq n\}$ sites pixels ○ sites lignes —

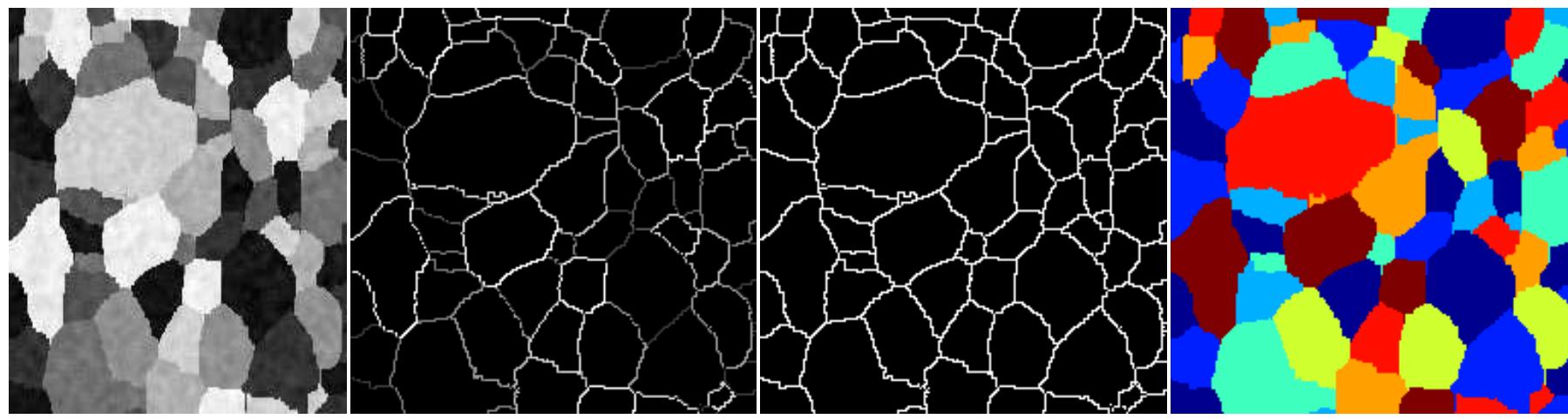
$\mathcal{R}_k = \{\mathbf{r} : \mathbf{r} \in \mathcal{R}_k\}$, $\cup_k \mathcal{R}_k = \mathcal{R}$ régions

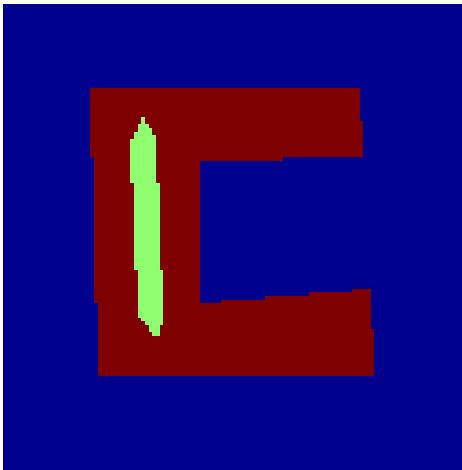
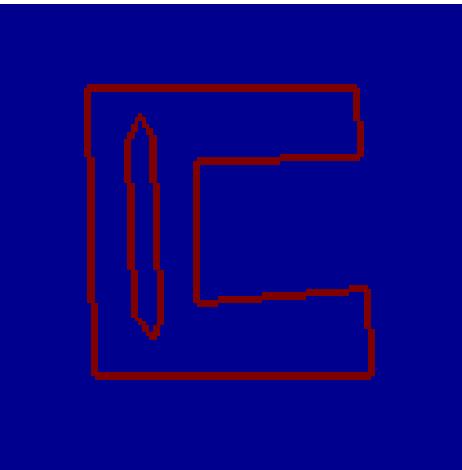
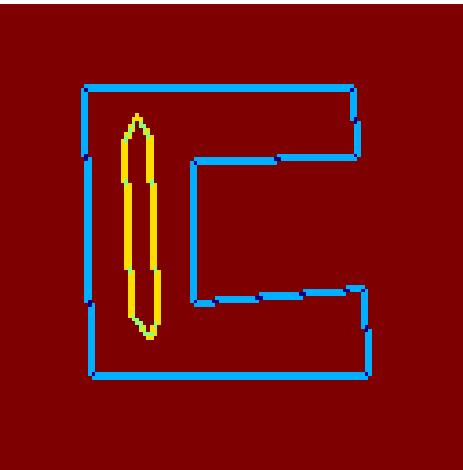
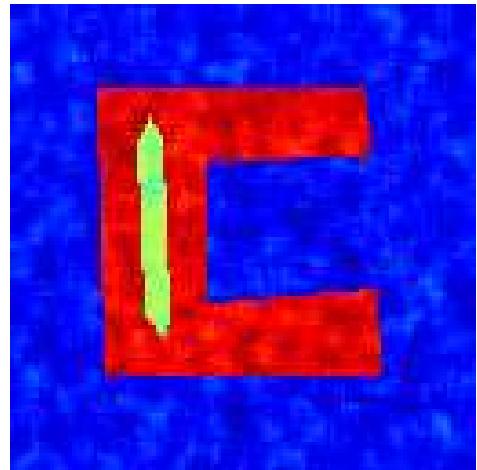
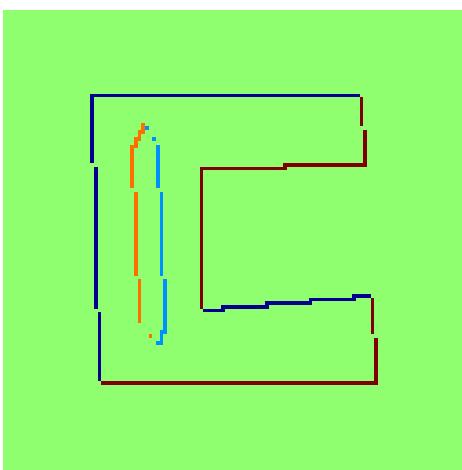
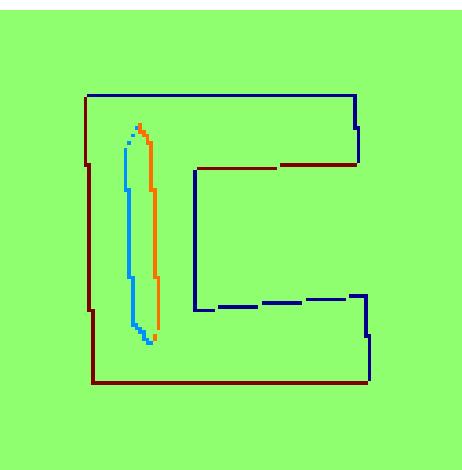
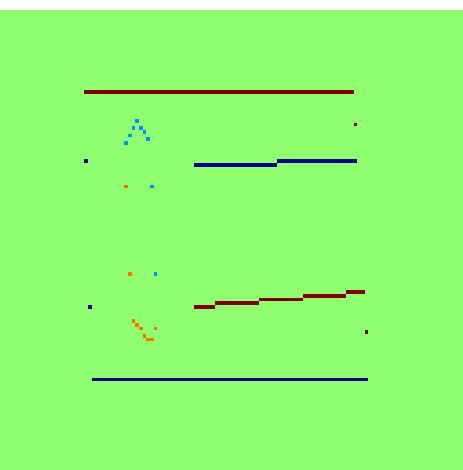
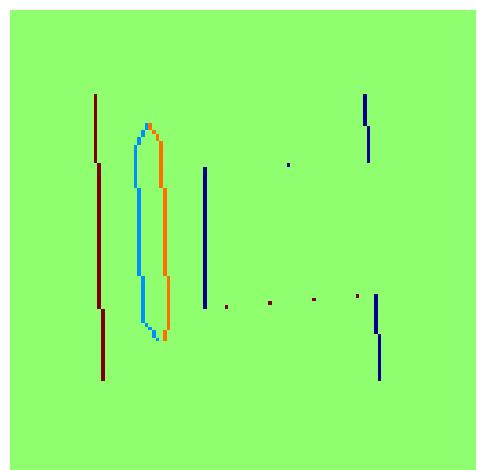
$f = \{f(\mathbf{r}), \mathbf{r} \in \mathcal{R}\}$ image (intensités)

$l = \{l(\mathbf{r}), \mathbf{r} \in \mathcal{R}\}$ (ou $l(\mathbf{r}, \mathbf{r}')$) lignes $l(\mathbf{r}) \in [0, 1]$

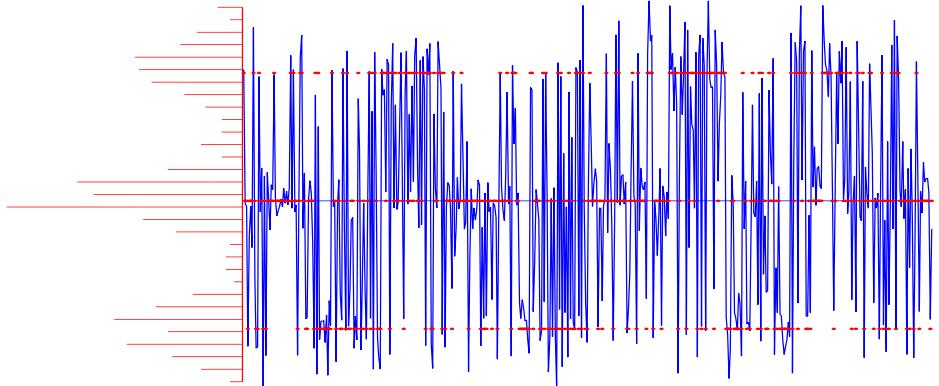
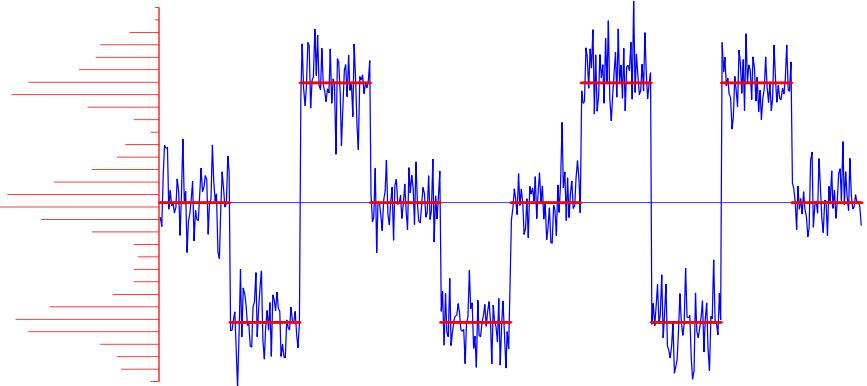
$q = \{q(\mathbf{r}), \mathbf{r} \in \mathcal{R}\}$ (ou $q(\mathbf{r}, \mathbf{r}')$) contours $q(\mathbf{r}) \in \{0, 1\}$

$z = \{z(\mathbf{r}), \mathbf{r} \in \mathcal{R}\}$ étiquette région $z(\mathbf{r}) \in \{0, \dots, K\}$

 $f(\mathbf{r})$ $l(\mathbf{r}) \in [0, 1]$ $q(\mathbf{r}) \in \{0, 1\}$ $z(\mathbf{r}) \in \{1, \dots, K\}$

 $f(\mathbf{r})$ $l(\mathbf{r})$ $q(\mathbf{r})$ $z(\mathbf{r})$  $l_x(\mathbf{r})$ $l_y(\mathbf{r})$ $l_{yx}(\mathbf{r})$ $l_{xy}(\mathbf{r})$

MODÈLES DE MÉLANGE DE LOIS

Mélange de gaussiennes iid (z iid)Mélange de gaussiennes iid (z Potts)

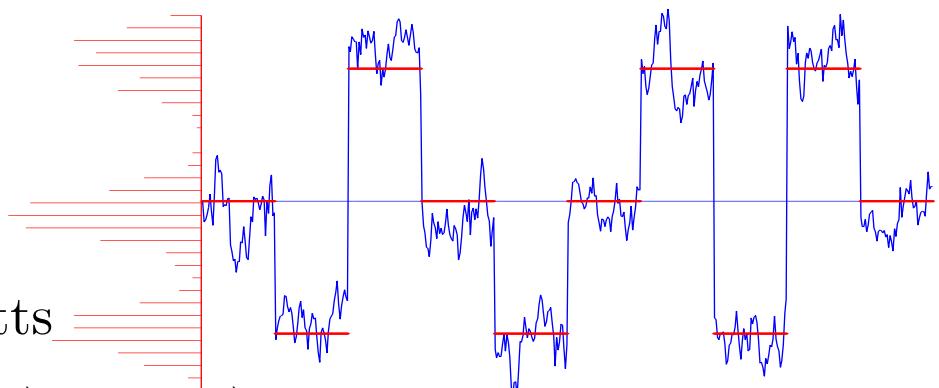
$$p(f_j|z_j = k) = \mathcal{N}(m_k, v_k)$$

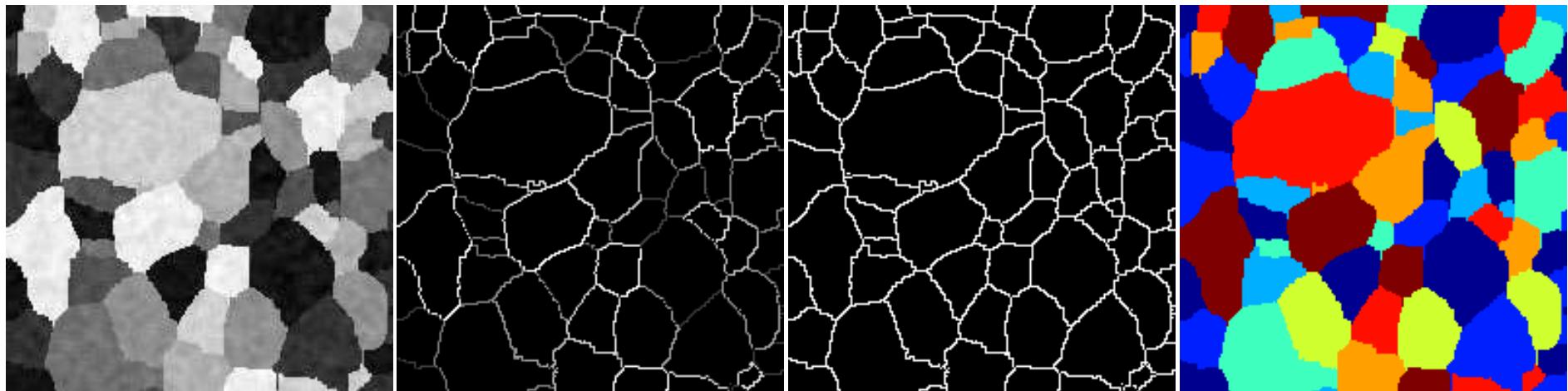
z_j iid ou

$$p(z_j|z_{j-1}) \propto \exp [-\alpha \delta(z_j - z_{j-1})] \text{ Potts}$$

$$p(f_j|z_j = k, f_{j-1}) = \mathcal{N}(q_j m_k + (1 - q_j) f_{j-1}, v_k)$$

avec $q_j = \delta(z_j - z_{j-1})$

Mélange de Gauss-Markov (z Potts)



$$\textcolor{red}{f}(\boldsymbol{r})$$

$$l(\boldsymbol{r}) \in [0, 1]$$

$$q(\boldsymbol{r}) \in \{0, 1\}$$

$$\textcolor{red}{z}(\boldsymbol{r}) \in \{1, \dots, K\}$$

$$\delta(f(\boldsymbol{r}) - f(\boldsymbol{r}'))$$

$$\delta(z(\boldsymbol{r}) - z(\boldsymbol{r}'))$$

$$\textcolor{red}{z} = \{z(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{R}\}$$

$$p(f(\boldsymbol{r})|z(\boldsymbol{r}) = k) = \mathcal{N}(f(\boldsymbol{r})|\textcolor{red}{m}_k, \sigma_k^2) \longrightarrow p(f(\boldsymbol{r})) = \sum_k P(z(\boldsymbol{r}) = k) \mathcal{N}(f(\boldsymbol{r})|m_k, \sigma_k^2)$$

$$\textcolor{red}{f}_k = \{f(\boldsymbol{r}), \boldsymbol{r} \in \mathcal{R}_k\} \longrightarrow \textcolor{red}{f} = \{\textcolor{red}{f}_k, k = 1, \dots, K\}$$

$$p(\textcolor{red}{f}_k) = \mathcal{N}(\textcolor{red}{f}_k|m_k \mathbf{1}_k, \Sigma_{\textcolor{violet}{k}}) \longrightarrow p(\textcolor{red}{f}|z) = \prod_k p(\textcolor{red}{f}_k)$$

$$p(\textcolor{red}{z}) \propto \exp \left[\alpha \sum_{\boldsymbol{r} \in \mathcal{R}} \sum_{\boldsymbol{r}' \in \mathcal{V}(\boldsymbol{r})} \delta(z(\boldsymbol{r}) - z(\boldsymbol{r}')) \right] \quad \text{Champ de Potts}$$

MODÈLE A PRIORI CHOISI ET INFÉRENCE BAYÉSIENNE

- Modèle d'observation: $\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon} \longrightarrow p(\mathbf{g}|\mathbf{f}) = \mathcal{N}(\mathbf{H}\mathbf{f}, \sigma_\epsilon^2 \mathbf{I})$
- Modèle a priori $p(\mathbf{f}, \mathbf{z}) = p(\mathbf{f}|\mathbf{z}) p(\mathbf{z})$

$$p(\mathbf{f}|\mathbf{z}) = \prod_k p(\mathbf{f}_k) \quad \text{avec} \quad p(\mathbf{f}_k) = \mathcal{N}(\mathbf{f}_k | \mathbf{m}_k \mathbf{1}_k, \boldsymbol{\Sigma}_k)$$

$$p(\mathbf{z}) \propto \exp \left[\alpha \sum_{\mathbf{r} \in \mathcal{R}} \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \delta(z(\mathbf{r}) - z(\mathbf{r}')) \right]$$

- Hyperparamètres $\boldsymbol{\theta} = \{\sigma_\epsilon^2, (m_k, \sigma_k^2), k = 1, \dots, K\}$:

$$p(m_k) = \mathcal{N}(m_k | m_{k0}, \sigma_{k0}^2), \quad p(\sigma_k^2) = \mathcal{IG}(\sigma_k^2 | \alpha_{k0}, \beta_{k0}),$$

$$p(\boldsymbol{\Sigma}_k) = \mathcal{IW}(\boldsymbol{\Sigma}_k | \alpha_{k0}, \boldsymbol{\Lambda}_{k0}), \quad p(\sigma_{\epsilon_i}^2) = \mathcal{IG}(\sigma_{\epsilon_i}^2 | \alpha_0^{\epsilon_i}, \beta_0^{\epsilon_i}).$$

- Loi *a posteriori* conjointe de \mathbf{f} , \mathbf{z} et $\boldsymbol{\theta}$

$$p(\mathbf{f}, \mathbf{z}, \boldsymbol{\theta} | \mathbf{g}) \propto p(\mathbf{g}|\mathbf{f}, \boldsymbol{\theta}_1) p(\mathbf{f}|\mathbf{z}, \boldsymbol{\theta}_2) p(\mathbf{z}|\boldsymbol{\theta}_3) p(\boldsymbol{\theta})$$

Estimation conjointe de $(\mathbf{f}, \mathbf{z}, \boldsymbol{\theta})$ utilisant $p(\mathbf{f}, \mathbf{z}, \boldsymbol{\theta} | \mathbf{g})$

$$\left\{ \begin{array}{lcl} \widehat{\boldsymbol{\theta}} & \sim & p(\boldsymbol{\theta} | \widehat{\mathbf{f}}, \widehat{\mathbf{z}}, \mathbf{g}) \quad \text{ou} \quad p(\boldsymbol{\theta} | \widehat{\mathbf{z}}, \mathbf{g}) \\ \widehat{\mathbf{z}} & \sim & p(\mathbf{z} | \widehat{\mathbf{f}}, \widehat{\boldsymbol{\theta}}, \mathbf{g}) \quad \text{ou} \quad p(\mathbf{z} | \widehat{\boldsymbol{\theta}}, \mathbf{g}) \\ \widehat{\mathbf{f}} & \sim & p(\mathbf{f} | \widehat{\mathbf{z}}, \widehat{\boldsymbol{\theta}}, \mathbf{g}) \end{array} \right.$$

où \sim signifie soit $\arg \max$ soit échantillonner suivant

$$\left\{ \begin{array}{lcl} p(\mathbf{f} | \mathbf{z}, \boldsymbol{\theta}, \mathbf{g}) & = & p(\mathbf{g} | \mathbf{f}, \boldsymbol{\theta}_1) p(\mathbf{f} | \mathbf{z}, \boldsymbol{\theta}_2) / p(\mathbf{g} | \mathbf{z}, \boldsymbol{\theta}) & \text{Gaussienne} \\ p(\mathbf{z} | \mathbf{f}, \boldsymbol{\theta}, \mathbf{g}) & \propto & p(\mathbf{g} | \mathbf{f}, \boldsymbol{\theta}_1) p(\mathbf{f} | \mathbf{z}, \boldsymbol{\theta}_2) p(\mathbf{z}) & \text{Potts} \\ p(\mathbf{z} | \boldsymbol{\theta}, \mathbf{g}) & = & p(\mathbf{g} | \mathbf{z}, \boldsymbol{\theta}) p(\mathbf{z}) / p(\mathbf{g} | \boldsymbol{\theta}) & \text{Potts} \\ p(\boldsymbol{\theta} | \mathbf{f}, \mathbf{z}, \mathbf{g}) & = & p(\mathbf{g} | \mathbf{f}, \boldsymbol{\theta}_1) p(\mathbf{f} | \mathbf{z}, \boldsymbol{\theta}_2) p(\boldsymbol{\theta}) / p(\mathbf{g}) & \text{Gaussienne/Inverse Gamma} \\ p(\boldsymbol{\theta} | \mathbf{z}, \mathbf{g}) & = & p(\mathbf{g} | \mathbf{z}, \boldsymbol{\theta}) p(\boldsymbol{\theta}) / p(\mathbf{g}) & \text{Gaussienne/Inverse Gamma} \end{array} \right.$$

$$\boldsymbol{\theta}_1 = \sigma_\epsilon^2, \boldsymbol{\theta}_2 = \{(m_k, \sigma_k^2), k = 1, \dots, K\}, \boldsymbol{\theta} = (\boldsymbol{\theta}_1, \boldsymbol{\theta}_2)$$

Estimation de \mathbf{f} lorsque θ et z sont connus:

$$\text{MAP:} \quad \widehat{\mathbf{f}} = \arg \max_{\mathbf{f}} \{p(\mathbf{f}|\mathbf{g}, z, \theta)\} = \arg \min_{\mathbf{f}} \{J(\mathbf{f}|\mathbf{g}, z, \theta)\}.$$

- avec un modèle i.i.d. :

$$J(\mathbf{f}|\mathbf{g}, z, \theta) = \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2 + \lambda \sum_{k=1}^K \sum_{\mathbf{r} \in \mathcal{R}_k} \frac{\|\mathbf{f}(\mathbf{r}) - m_k\|^2}{\sigma_k^2}$$

- avec modèle markovien : $\tilde{\mathbf{f}}_k = \mathbf{f}_k - m_k \mathbf{1}$

$$\begin{aligned} J(\mathbf{f}|\mathbf{g}, z, \theta) &= \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2 + \lambda \sum_{k=1}^K \sum_{\mathbf{r} \in \mathcal{R}_k} \frac{1}{\sigma_k^2} \left(\tilde{\mathbf{f}}(\mathbf{r}) - \beta_{\mathbf{r}} \sum_{\mathbf{r}' \in (\mathcal{V}(\mathbf{r}) \cap \mathcal{R}_k)} \tilde{\mathbf{f}}(\mathbf{r}') \right)^2 \\ &= \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2 + \lambda \sum_{\mathbf{r} \in \mathcal{R}} (1 - q(\mathbf{r})) \left(\mathbf{f}(\mathbf{r}) - \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} (1 - q(\mathbf{r}')) \mathbf{f}(\mathbf{r}') \right)^2 \\ &\quad + \lambda \sum_{k=1}^K \sum_{\mathbf{r} \in \mathcal{R}_k} \left(\frac{\mathbf{f}(\mathbf{r}) - m_k}{\sigma_k} \right)^2 \end{aligned}$$

où $\tilde{\mathbf{f}}(\mathbf{r}) = \mathbf{f}(\mathbf{r}) - m(\mathbf{r})$, $\beta_{\mathbf{r}} = \frac{1}{n_{\mathbf{r}}}$, $n_{\mathbf{r}} = \text{Card}(\mathcal{V}(\mathbf{r}) \cap \mathcal{R}_k)$.

ASPECTS CALCULATOIRES

Trois grandes classes d'approximations:

- Approximation de Laplace:
 - On approxime la loi *a posteriori* par une Gaussienne équivalente
 - Souvent trop grossières
- Approximation numérique (MCMC)
 - On explore la loi *a posteriori* en générant des échantillons suivant cette loi
 - Acceptation-réjet, Métropolis-Hasting, échantilleur de Gibbs
 - Souvent nécessite grand nombre d'itérations
- Approche variationnelle (Approximations séparables)
 - On approxime la loi *a posteriori* par des lois plus simples à manipuler
 - On utilise la distance de Kullback-Leibler comme mesure d'approximation
 - Lorsqu'on a obtenue ces lois approchées, on les utilise pour l'estimation des différentes grandeurs.
 - Approximation en lois séparables, champs moyens
 - Bon compromis entre précision et coût de calcul

APPLICATIONS

- Tomographie X
- Tomographie microondes
- Fusion d'images
- Imagerie hyperspectrale
- Séparation de sources en imagerie satellitaires
- Segmentation d'une séquence vidéo
- Super-rsolution
- Restauration des documents anciens par sparation d'images
- Tomographie 3D en CND (coll. avec EDF)
- Tomographie 3D pour des micro structures (coll. avec CEA)

MODÈLES DE GAUSS-MARKOV (SIGNAUX ET IMAGES CONTINUS)

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$

$f(\mathbf{r})$ image continue : Gauss-Markov

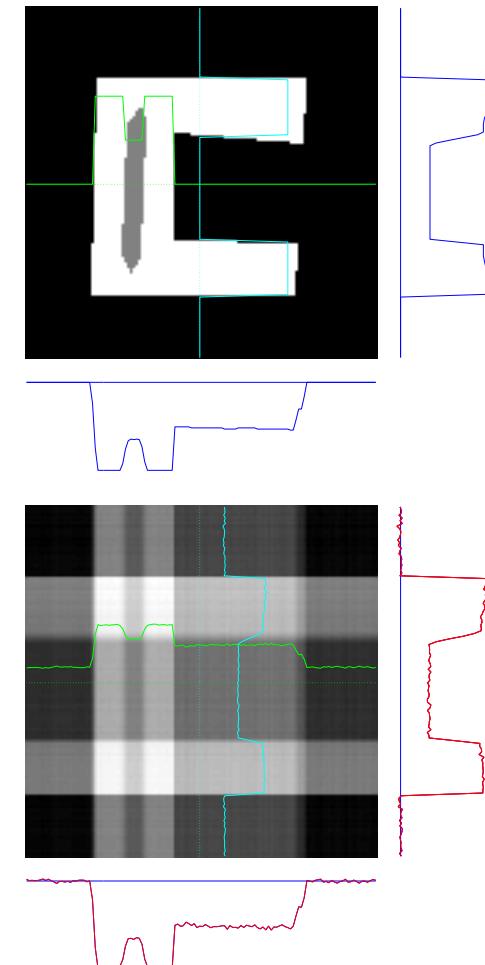
$$p(f(\mathbf{r})|f(\mathbf{r}')) = \mathcal{N} \left(\beta \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} f(\mathbf{r}'), \sigma_f^2 \right)$$

MAP :

$$\hat{\mathbf{f}} = \arg \max_{\mathbf{f}} \{p(\mathbf{f}|\mathbf{g})\} = \arg \min_{\mathbf{f}} \{J(\mathbf{f})\}$$

$$J(\mathbf{f}) = \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2 + \sum_{\mathbf{r}} \left(f(\mathbf{r}) - \beta \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} f(\mathbf{r}') \right)^2$$

MAP = Régularisation quadratique



MODÈLES MARKOVIENS AVEC VARIABLES CACHÉES LIGNES

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon},$$

$\mathbf{f}(\mathbf{r})$ images continues par morceaux :

Variables cachée lignes ou contours: $q(\mathbf{r})$

$$p(f(\mathbf{r})|q(\mathbf{r}), f(\mathbf{r}')) = \mathcal{N} \left(\beta(1 - q(\mathbf{r})) \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} f(\mathbf{r}'), \sigma_f^2 \right)$$

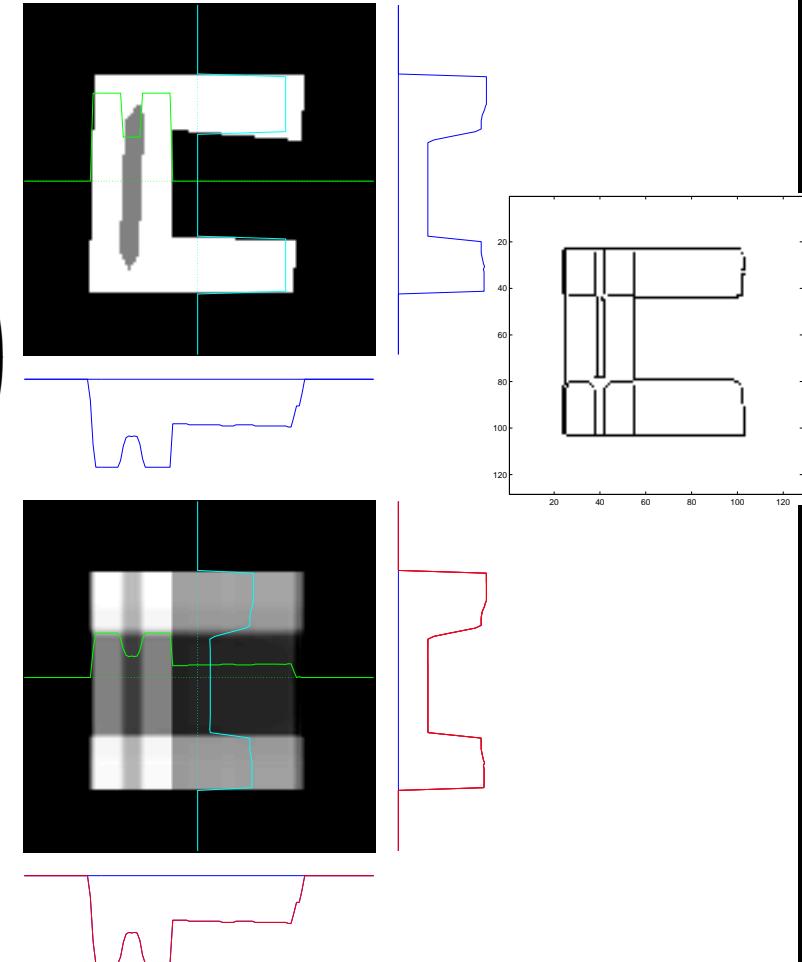
$$\text{MAP : } (\hat{\mathbf{f}}, \hat{\mathbf{q}}) = \arg \max_{\mathbf{f}, \mathbf{q}} \{p(\mathbf{f}, \mathbf{q} | \mathbf{g})\}$$

$$\hat{\mathbf{f}} = \arg \max_{\mathbf{f}} \{p(\mathbf{f} | \mathbf{g}, \mathbf{q})\} = \arg \min_{\mathbf{f}} \{J(\mathbf{f})\}$$

$$J(\mathbf{f}) = \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2$$

$$+ \sum_{\mathbf{r}} (1 - q(\mathbf{r})) \left(f(\mathbf{r}) - \beta \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} f(\mathbf{r}') \right)^2$$

$$\hat{\mathbf{q}} = \arg \max_{\mathbf{q}} \{p(\mathbf{q} | \mathbf{g})\}$$



MODÈLES MARKOVIENS AVEC VARIABLES CACHÉES LIGNES

Lien entre modèles à variables cachées et potentiels non convexes

$$(\hat{\mathbf{f}}, \hat{\mathbf{q}}) = \arg \max_{\mathbf{f}, \mathbf{q}} \{p(\mathbf{f}, \mathbf{q} | \mathbf{g})\}$$

$$\hat{\mathbf{f}} = \arg \max_{\mathbf{f}} \{p(\mathbf{f} | \mathbf{g}, \mathbf{q})\} = \arg \min_{\mathbf{f}} \{J(\mathbf{f})\}$$

$$J(\mathbf{f}) = \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2$$

$$+ \sum_{\mathbf{r}} (1 - q(\mathbf{r})) \left(f(\mathbf{r}) - \beta \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} f(\mathbf{r}') \right)^2 =$$

$$\hat{\mathbf{q}} = \arg \max_{\mathbf{q}} \{p(\mathbf{q} | \mathbf{g})\}$$

$$\hat{\mathbf{f}} = \arg \min_{\mathbf{f}} \{J_1(\mathbf{f} | \mathbf{g})\}$$

$$J_1(\mathbf{f}) = \|\mathbf{g} - \mathbf{H}\mathbf{f}\|^2$$

$$+ \sum_{\mathbf{r}} \phi \left(f(\mathbf{r}) - \beta \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} f(\mathbf{r}') \right)$$

$$\phi(t) = \begin{cases} |t|^2 & \text{if } |t| < \alpha, \\ \alpha^2 & \text{else,} \end{cases}$$

MODÈLES MARKOVIENS AVEC VARIABLES CACHÉES ÉTIQUETTES RÉGIONS

$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$

\mathbf{f} représente l'image d'un objet composé d'un nombre finie de matériaux homogènes

Variables cachée étiquettes

$$z(\mathbf{r}) = k, \quad k = 1, \dots, K$$

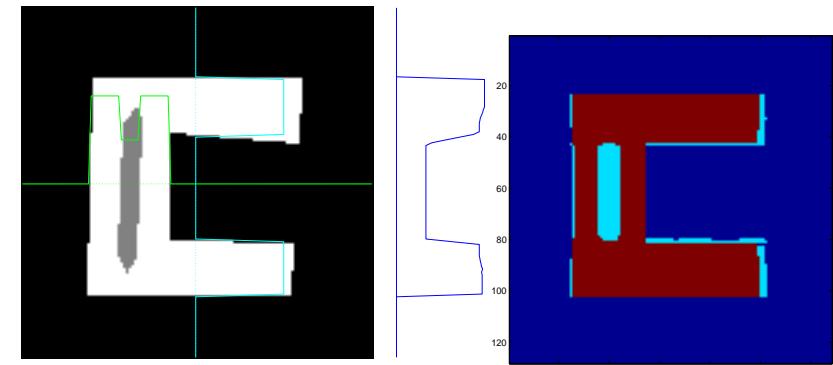
$$\mathcal{R}_k = \{\mathbf{r} : z(\mathbf{r}) = k\}, \quad \mathcal{R} = \cup_k \mathcal{R}_k$$

$$p(f(\mathbf{r})|z(\mathbf{r}) = k) = \mathcal{N}(f(\mathbf{r})|m_k, \sigma_k^2)$$

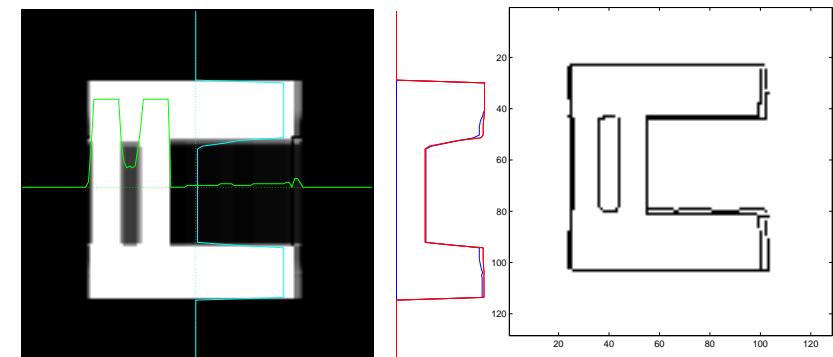
$\mathbf{z} = \{z(\mathbf{r}), \mathbf{r} \in \mathcal{R}\}$ image segmentée

Champ de Potts:

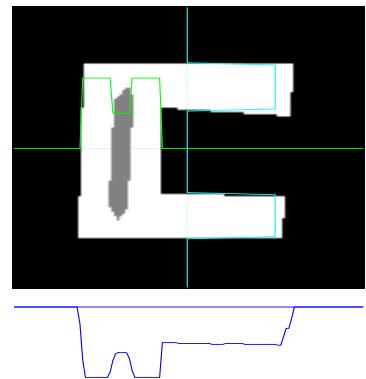
$$p(\mathbf{z}) \propto \exp \left[\alpha \sum_{\mathbf{r} \in \mathcal{R}} \sum_{\mathbf{r}' \in \mathcal{V}(\mathbf{r})} \delta(z(\mathbf{r}) - z(\mathbf{r}')) \right]$$



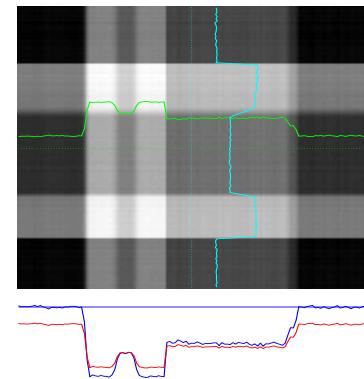
\mathbf{z}



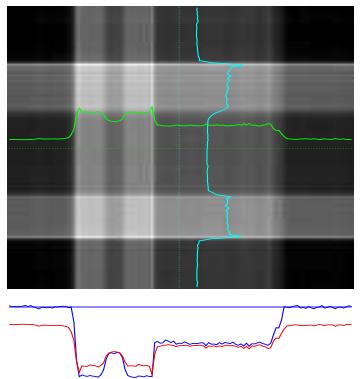
\mathbf{q}



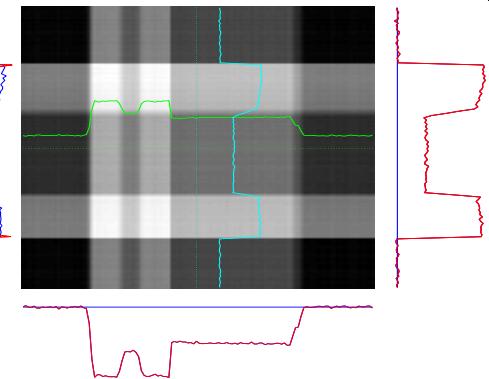
Original



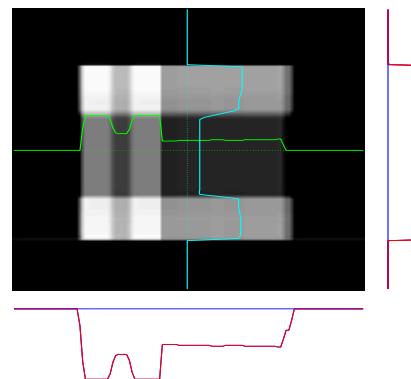
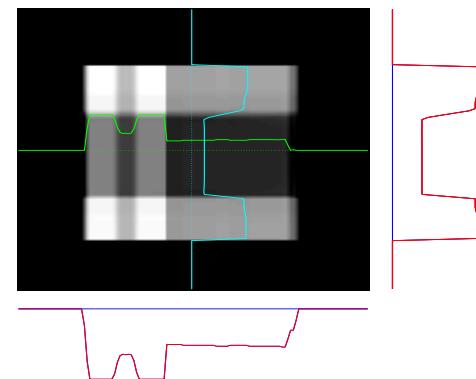
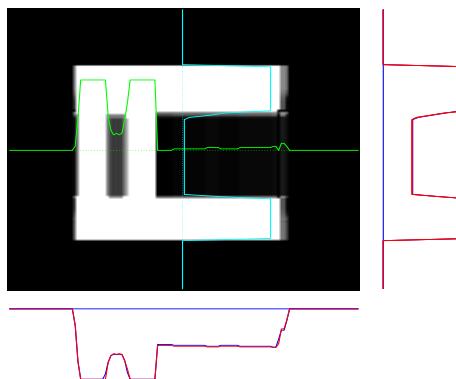
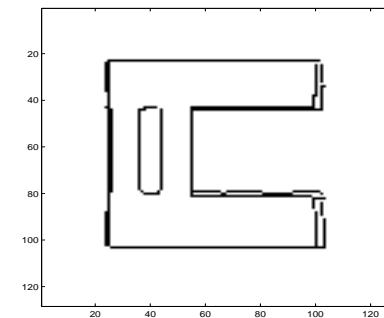
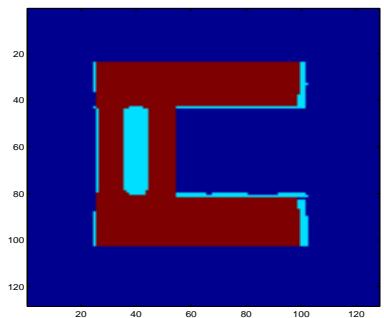
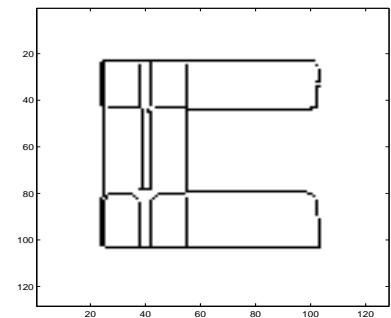
Backprojection



Filtered BP



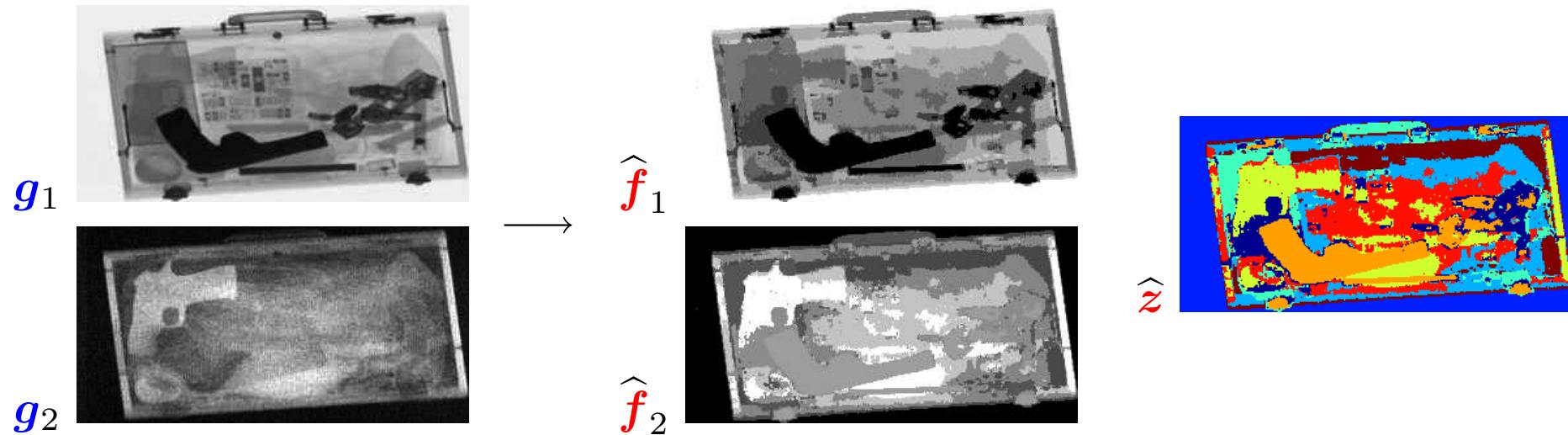
LS

Gauss-Markov+pos+supp f GM+Line process f, q GM+Label process f, z 

FUSION D'IMAGE ET SEGMENTATION JOINTE

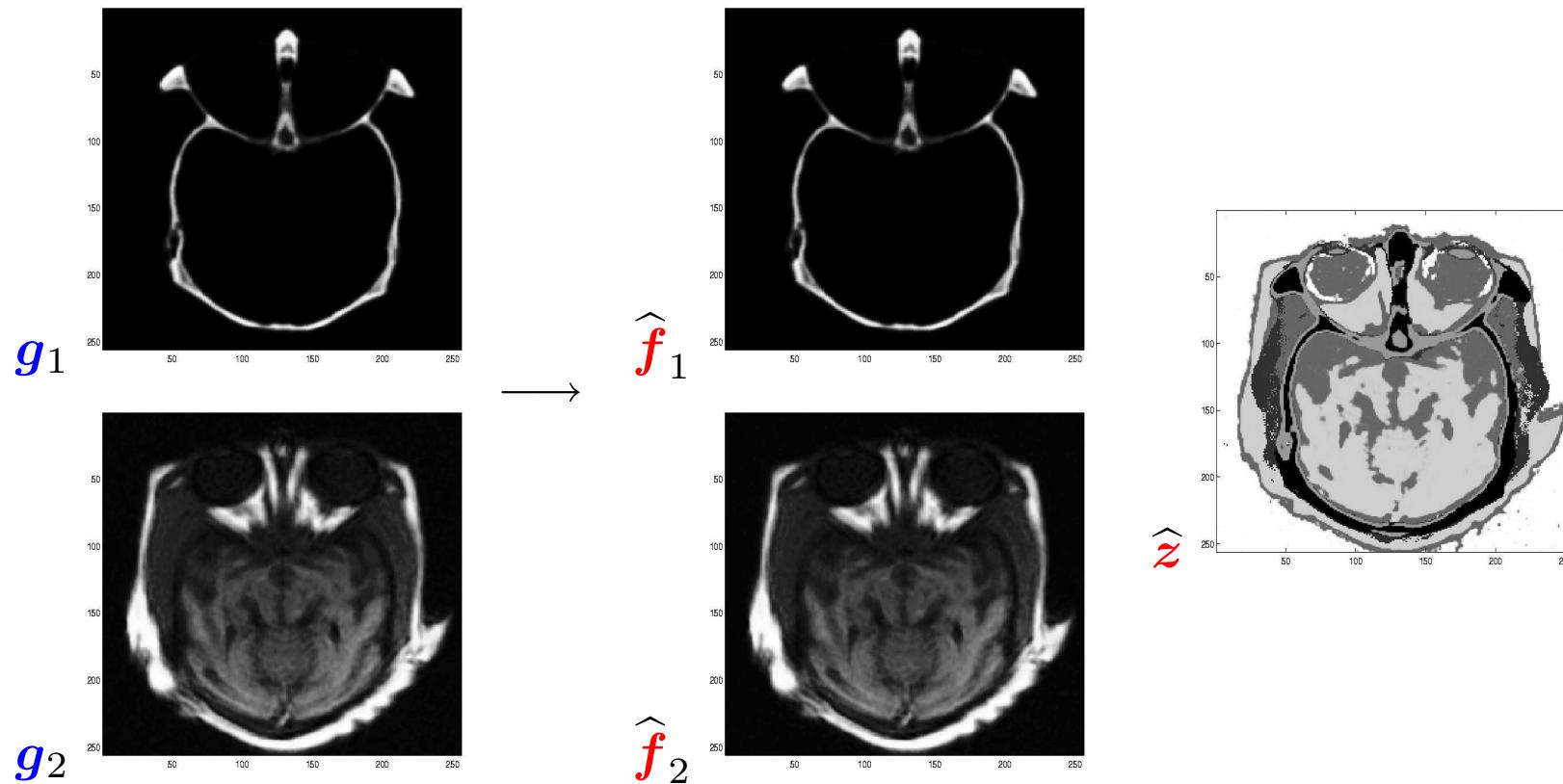
(Olivier FÉRON)

$$\begin{cases} g_i(\mathbf{r}) = f_i(\mathbf{r}) + \epsilon_i(\mathbf{r}) \\ p(f_i(\mathbf{r})|z(\mathbf{r}) = k) = \mathcal{N}(m_{ik}, \sigma_{ik}) \\ p(\underline{\mathbf{f}}|\underline{\mathbf{z}}) = \prod_i p(\mathbf{f}_i|\mathbf{z}) \end{cases}$$



FUSION D'IMAGE EN IMAGERIE MÉDICALE

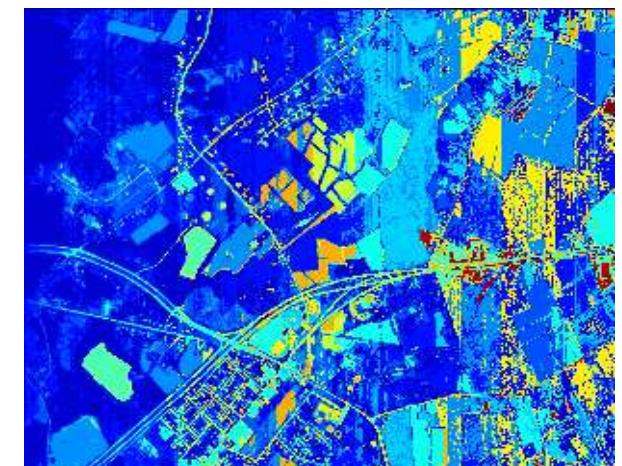
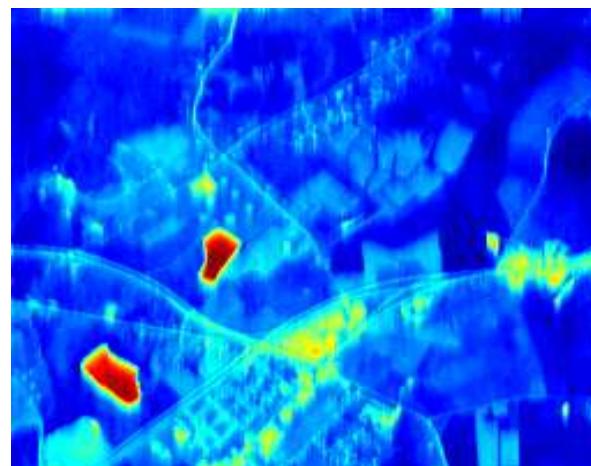
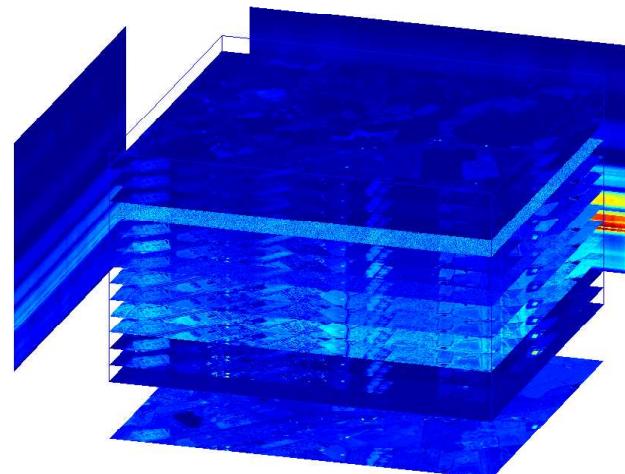
(Olivier FÉRON)



Ségmentation conjoint des images hyper-spectrales

(Adel MOHAMMADPOUR)

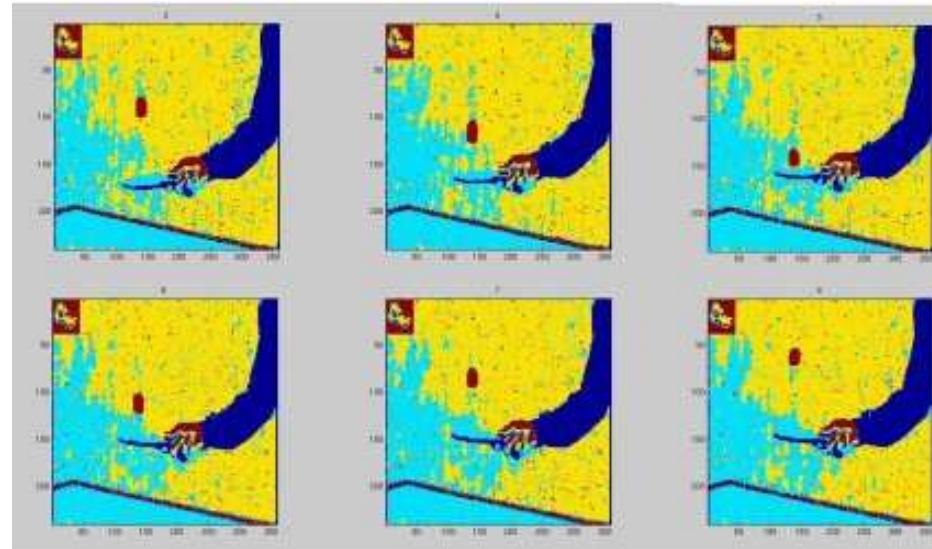
$$\left\{ \begin{array}{l} g_i(\mathbf{r}) = f_i(\mathbf{r}) + \epsilon_i(\mathbf{r}) \\ p(f_i(\mathbf{r})|z(\mathbf{r}) = k) = \mathcal{N}(m_{ik}, \sigma_{ik}), \quad k = 1, \dots, K \\ p(\underline{\mathbf{f}}|\underline{z}) = \prod_i p(\mathbf{f}_i|\mathbf{z}) \\ m_{ik} \text{ markovienne le long de l'index } i \end{array} \right.$$

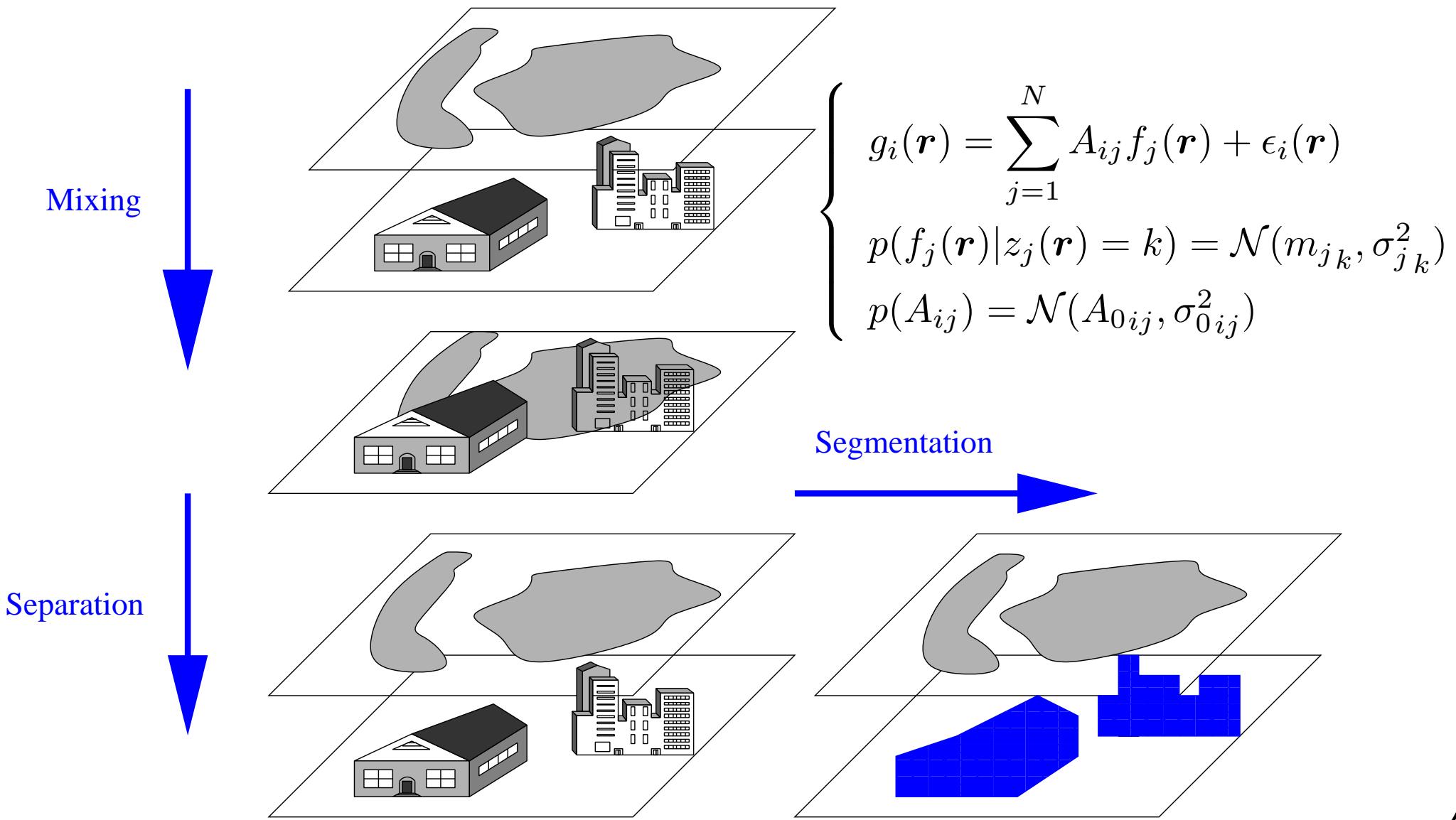


Ségmentation d'une séquence vidéo

(Patrice BRAULT)

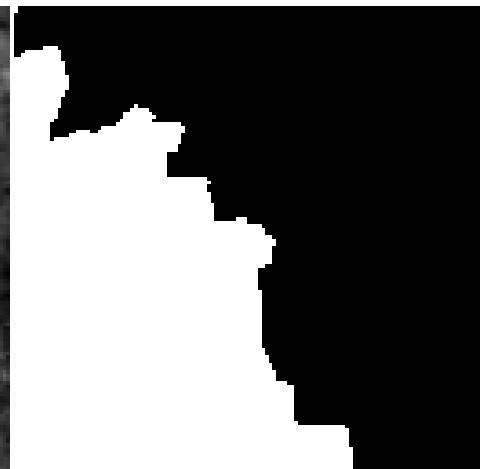
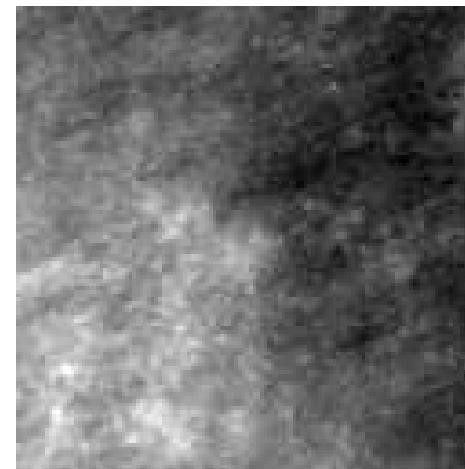
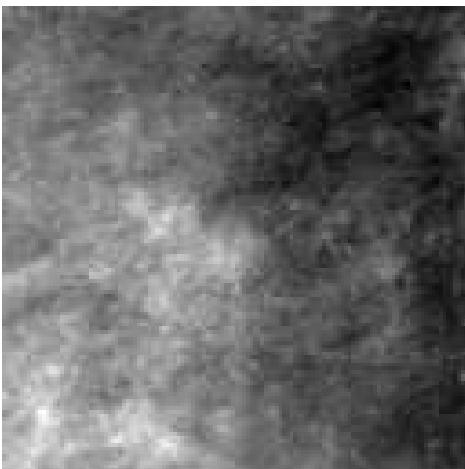
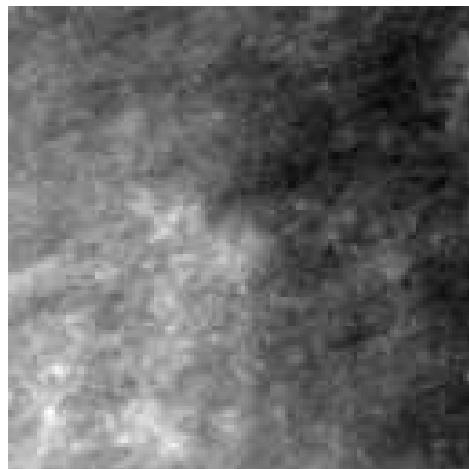
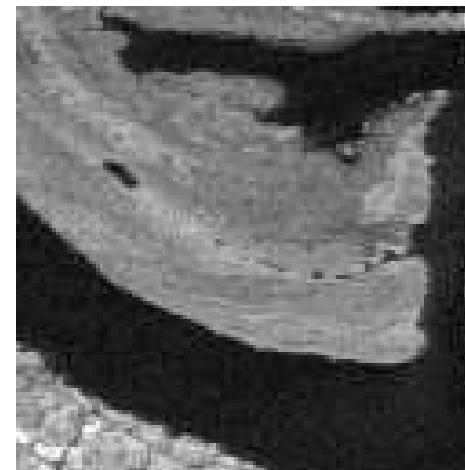
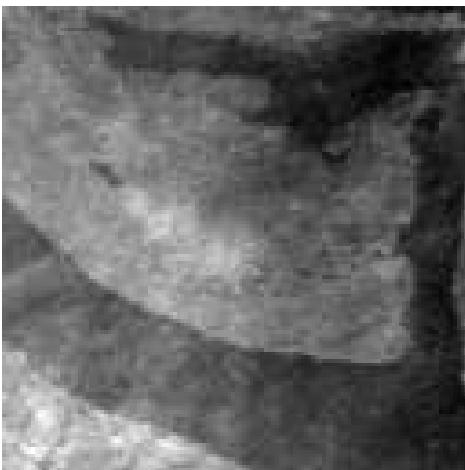
$$\left\{ \begin{array}{l} g_i(\mathbf{r}) = f_i(\mathbf{r}) + \epsilon_i(\mathbf{r}) \\ p(f_i(\mathbf{r})|z_i(\mathbf{r}) = k) = \mathcal{N}(m_{ik}, \sigma_{ik}), \quad k = 1, \dots, K \\ p(\underline{\mathbf{f}}|\mathbf{z}) = \prod_i p(\mathbf{f}_i|\mathbf{z}_i) \\ z_i(\mathbf{r}) \text{ markovien le long de l'index } i \end{array} \right.$$





SÉPARATION ET SÉGMENTATION DES IMAGES

(H. Snoussi)



f

g

\hat{f}

\hat{z}

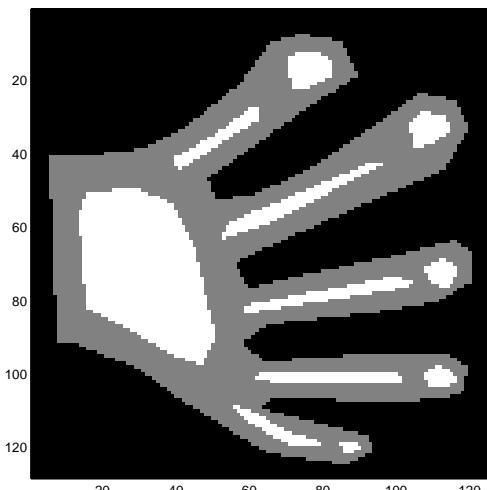
SYNTHÈSE DE FOURIER EN IMAGERIE MICROONDES

(O. Féron en collaboration avec DRÉ)

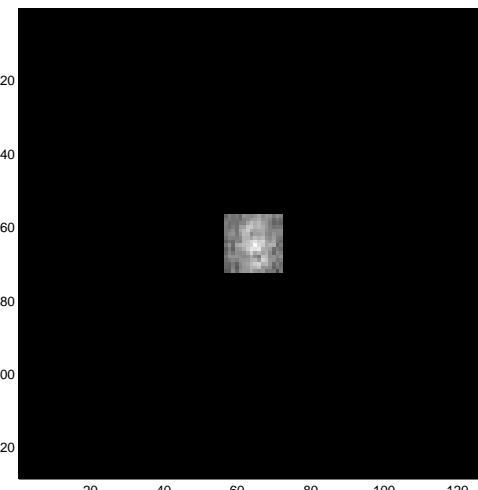
$$g(\omega) = \int f(r) \exp[-j(\omega \cdot r)] dr + \epsilon(\omega)$$

$$g(u, v) = \int f(x, y) \exp[-j(ux + vy)] dx dy + \epsilon(u, v)$$

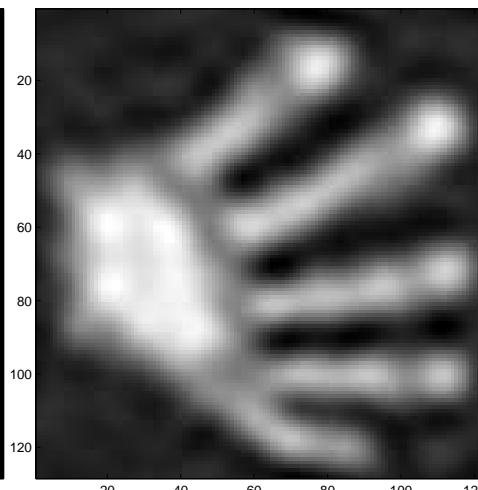
$$\mathbf{g} = \mathbf{H}\mathbf{f} + \boldsymbol{\epsilon}$$



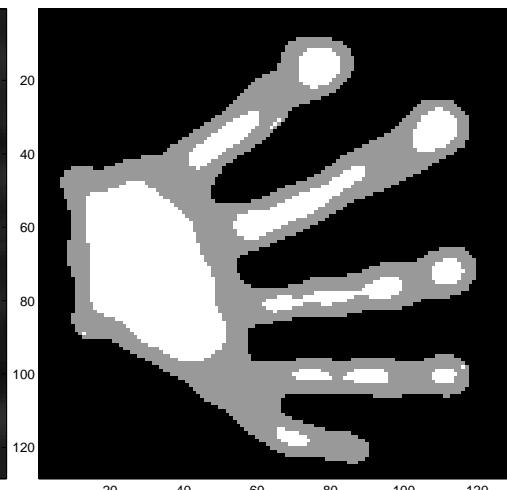
$f(x, y)$



$g(u, v)$



\hat{f} par FFTI



\hat{f} méthode proposée

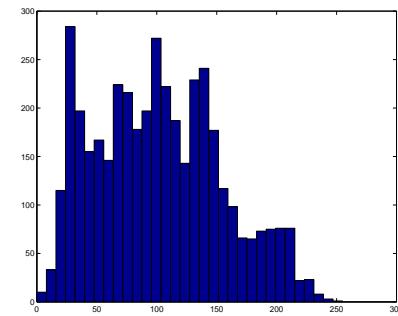
SÉPARATION D'IMAGES DANS LE DOMAINE DES ONDELETTES

$$g(r) = f(r) + \epsilon(r) \longrightarrow \text{WT} \longrightarrow \tilde{g}^j(r) = \tilde{f}^j(r) + \tilde{\epsilon}^j(r)$$

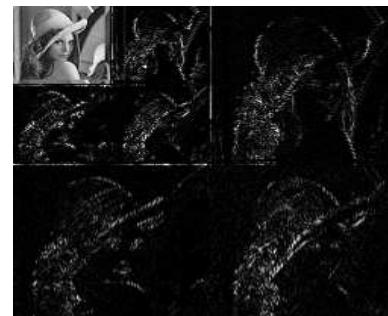
images



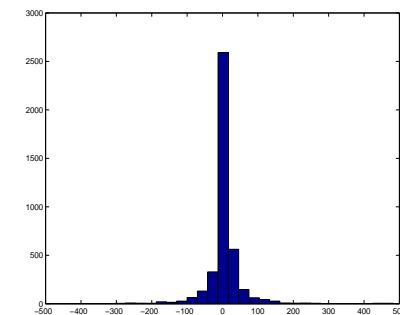
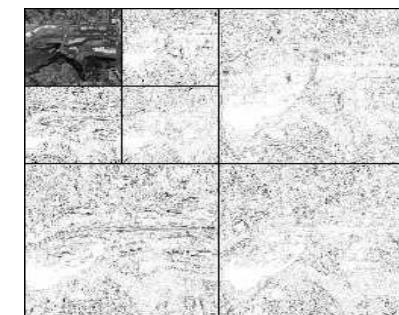
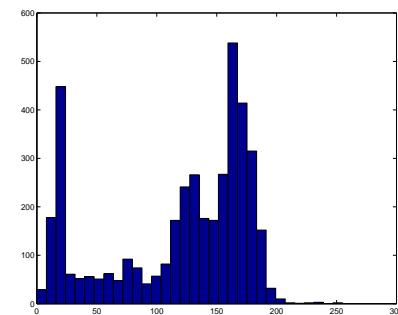
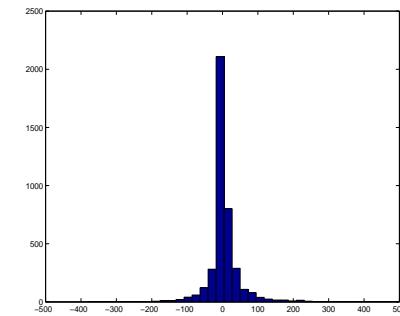
Hist. des images



Coéff. ondelettes



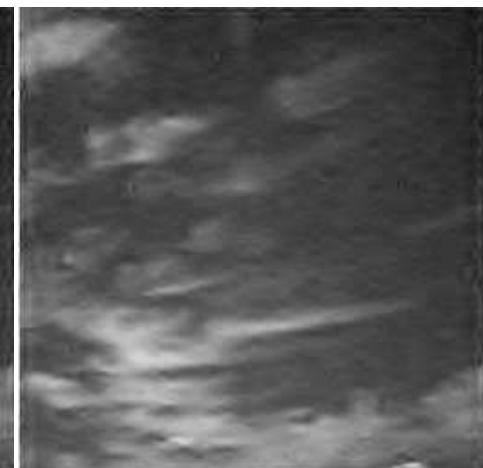
Hist. des coéff.



- multi-résolution
- Coefficients d'ondelettes peuvent souvent être classés en $K = 2$ classes

SÉPARATION D'IMAGES DANS LE DOMAINE DES ONDELETTES

(M. Ichir)



\underline{f}

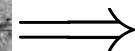
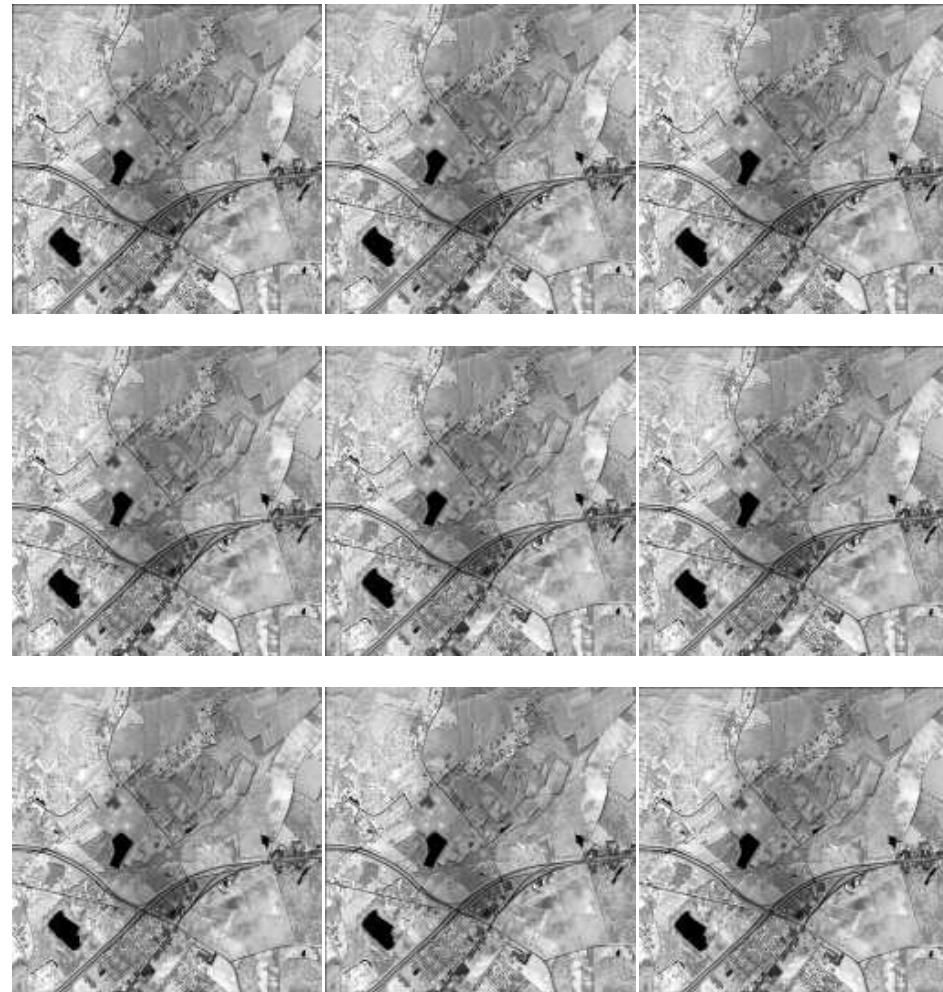
\underline{g}

$\widehat{\underline{f}}$ (GM)

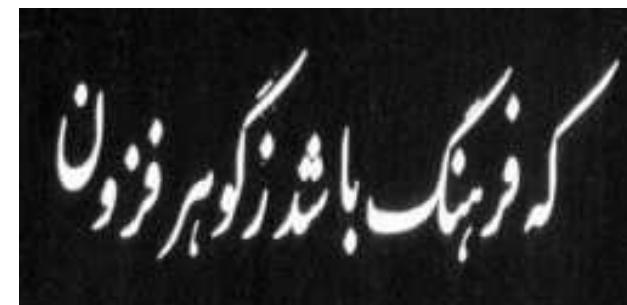
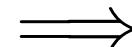
$\widehat{\underline{f}}$ (HMM)

SUPER-RÉSOLUTION

(F. Humblot)

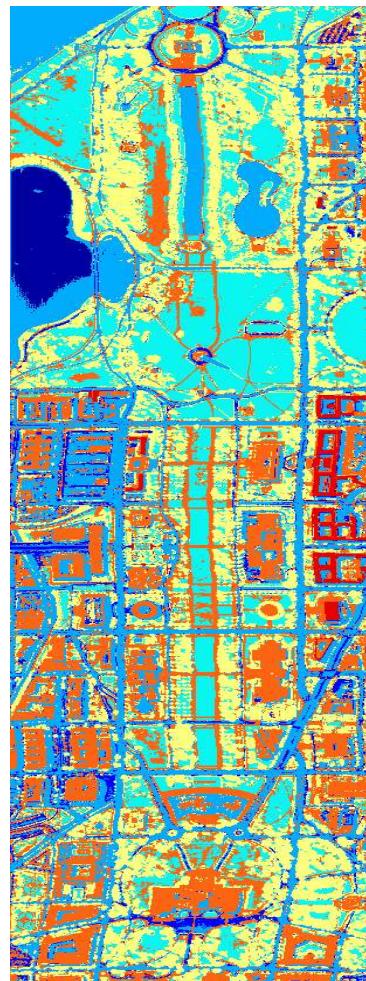


SÉPARATION DE SOURCES

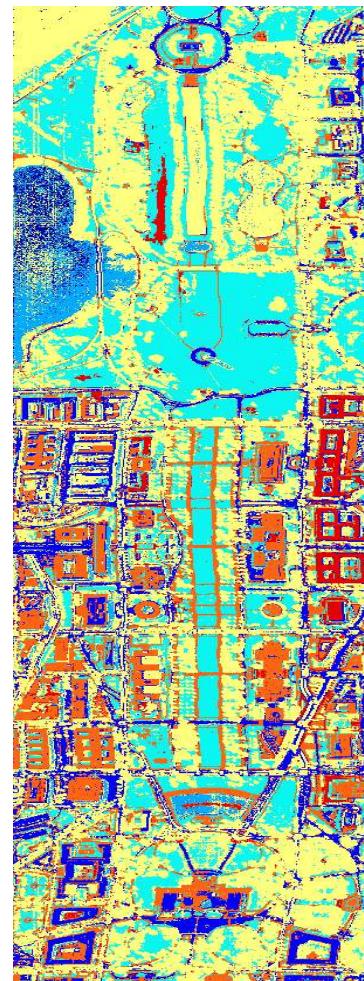


SÉGMENTATION ET RÉDUCTION DE DONNÉES EN IMAGERIE HYPER-SPECTRALE

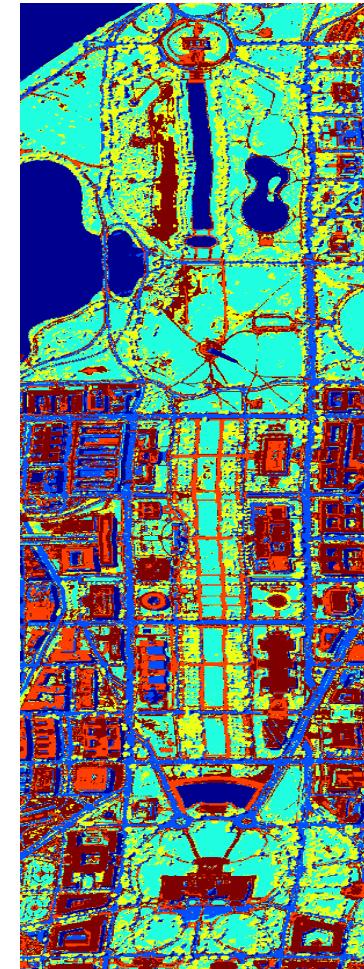
(N. Bali)



Originale



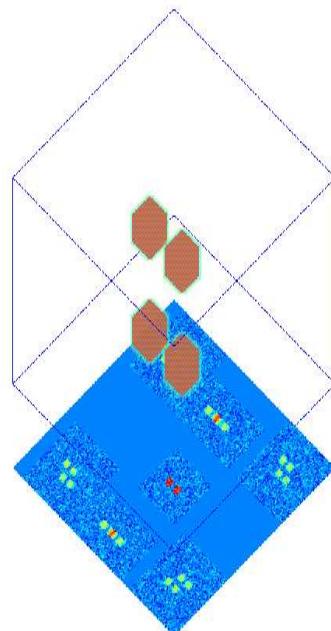
Classique (47%)



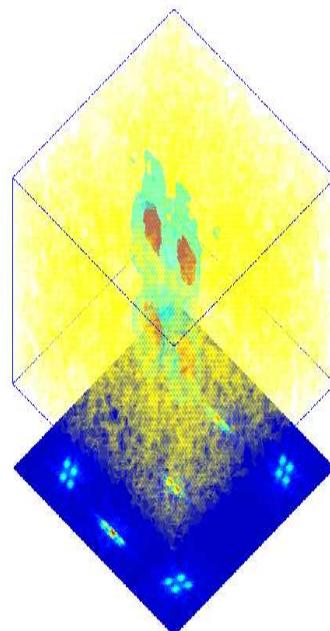
Méthode proposée (82%)

TOMOGRAPHIE 3D POUR CND

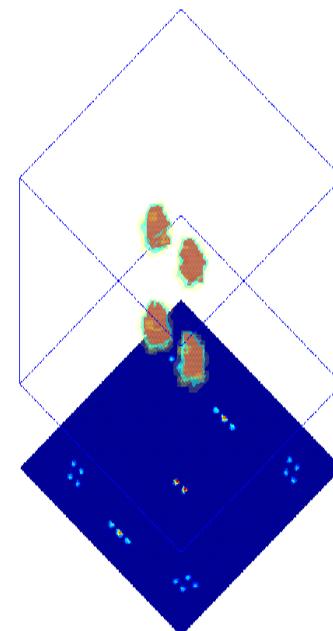
(en collaboration avec EDF)



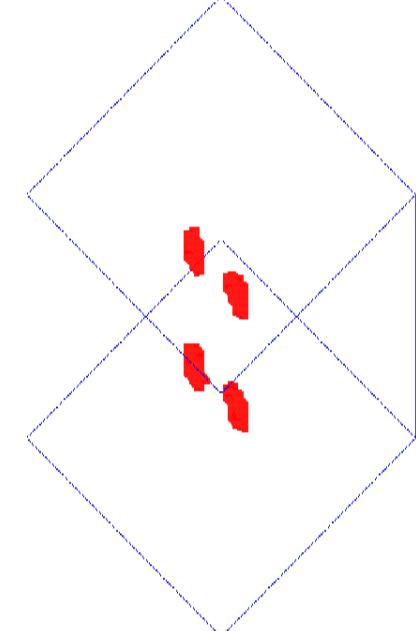
Originale



Rétroprojection



Méthode proposée f



Méthode proposée z

CONCLUSIONS

Approche bayésienne et les modèles markoviens avec des variables cachées sont des outils d'inférence bien appropriés pour grand nombre de problèmes inverses en traitement du signal et d'image

- H. Snoussi: Séparation de sources 1D et 2D
- M. Ichir: Séparation de sources dans le domaine des ondelettes
- S. Moussaoui: Séparation de sources positives et application en spéctrométrie
- O. Féron : Fusion d'image et problèmes inverses en imageries microondes
- P. Brault: Segmentation de séquence d'images
- A. Mohammadpour: Classification et segmentation d'images hyper-spectrales,
- F. Su: Restauration de documents anciens,
- F. Humblot: Super-résolution
- N. Bali : Séparation de sources pour la classification et réduction de données en imagerie hyper-spectrale
- S. Fekih-Salem: Tomographie 3D des micro structures (collaboration avec CEA)
- L. Robillard: Tomographie 3D en contrôle non destructif (CND)
(collaboration avec EDF)

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