Inverse Problems in imaging and computer vision: From regularization theory to Bayesian inference

Ali Mohammad-Djafari
Laboratoire des signaux et systèmes (UMR 8506 CNRS-SUPELEC-Univ Paris Sud)
SUPELEC, plateau de Moulon, 3 rue Joliot-Curie,
91192 GIF-SUR-YVETTE Cedex, France

The concept of inverse problems is now a familiar one to most scientists and engineers, particularly in the field of imaging systems and computer vision. Inverse problems arise whenever we want to infer an unknown quantity $f(\boldsymbol{r})$ which is not directly observable through a measurement system H which gives access to an observable quantity $g(\boldsymbol{s})$. The mathematical equations linking these two quantities $g(\boldsymbol{s})=H[f(\boldsymbol{r})](\boldsymbol{s})$ are called “forward model”. The problem of predicting $g(\boldsymbol{s})$ when $f(\boldsymbol{r})$ is assumed to be known is called “forward problem” and the one of inferring $f(\boldsymbol{r})$ from the observation of $g(\boldsymbol{s})$ is called “inverse problem”. Very often forward problems are “well-posed” and almost always the inverse problems are “ill-posed” [HAD 1901]. The classical mathematical tools for such inverse problems are either the deterministic regularization theory [TIK 1963, TIK 1976] or the probabilistic Bayesian inference and estimation theory [HAN 1983, TAR 1982].

In this keynote talk, first a few classical inverse problems such as: signal and image deconvolution, image reconstruction in X rays computed tomography (CT), microwave or optical tomography and Fourier synthesis which arise in many other medical imaging systems such as Magnetic Resonance Imaging (MRI) or Synthetic Aperture Radar (SAR) imaging or even radio-astronomical imaging, will be presented. Then, a few computer vision inverse problems such as Shape from shadows or 3D shape reconstruction from one or more photographies are also mentioned and a common mathematical abstraction for all these inverses problems will be presented.

By focusing on a simple linear forward model, first a synthetic analysis of the main deterministic methods (analytical inversion, parametric methods and regularization theory) used in inverse problems is presented and then the focus is given to the Bayesian inference approach. In a first step, the link between Maximum A Posteriori (MAP) and the regularization criteria is described and we will see how different prior modeling result to different regularization criteria. In particular the cases of separable Gaussian and Non-Gaussian, Gauss-Markov and more general Markovian prior models are considered.

Then, the advantages of the Bayesian approach to deterministic methods are highlighted through the possibilities of accounting more precisely for uncertainties of the data and model parameters, hyper parameter estimation, marginalization of nuisance parameters and the possibilities of the exploration of the space of the possible solutions by the Markov Chain Monte Carlo (MCMC) methods. One last advantage is the possibility of accounting for more specific prior knowledge through the Markovian or mixture models with hidden Markovian variables of contours and region labels, which do not have equivalent in deterministic methods.

These days, in most imagery techniques, the aim is not only to construct images, but also to directly access some geometric characteristics of those images. For this reason, we focus on imagery and vision problems for which the problem can clearly be written in terms of an inverse problem where an estimate for a function $f(\boldsymbol{r})$ and its geometrical attributes is sought, in other words its contours $q(\boldsymbol{r})$ or labels for its regions $z(\boldsymbol{r})$ are to be determined from the observation $g(\boldsymbol{s})$. To achieve this, not only wee need the forward model which links $g(\boldsymbol{s})$ to $f(\boldsymbol{r})$, but also the relations which link $f(\boldsymbol{r})$ to $q(\boldsymbol{r})$ and $z(\boldsymbol{r})$. As we will see, a class of Gauss-Markov-Potts prior models which we have developed and used effectively in many imaging applications will give the necessary probabilistic prior modeling to do this job effectively.

Also, as we will see, these models are also very useful in many inverse problems in imaging systems and in particular in image reconstruction in CT where we may know that the object under the test is composed of a finite number of materials, meaning that, the images to be reconstructed are composed of a finite number of homogeneous compact regions.

We use then these prior models in a Bayesian inference framework for imaging inverse problems which give us the possibility to jointly reconstruct the images and segment them in an optimal way. In this talk, first these prior models are presented in details, then appropriate MCMC or variational Bayesian methods are used to compute the mean posterior estimators.

Finally some results are presented to show the efficiency of the proposed methods for different imaging inverse problems, and in particular, in CT with limited angle and number of projections.

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